Healthcare or Smartphones: Inside Health inequalities and Economic Growth

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Abstract

Along with the unprecedented improvements in health outcomes such as life expectancy in the last decades, it remains huge disparities in terms of longevity across countries, and even from individual to another. This may have consequences on social equity and on the economic development of a country. In this paper, we introduce heterogenous agents in an overlapping generations model with pollution and private/public health expenditures that affect the agents' length-of-life. Heterogeneity stems from households preferences for health, as a consequence of a minimum consumption requirement on other goods. Therefore, households do not choose the same level of private health expenditures and this generates health inequalities. In addition, the contribution to capital accumulation differs across individuals. However, an appropriate environmental policy may reduce health inequalities.

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Keywords Longevity, Normal/Superior good, Minimal consumption, Health inequalities, Endogenous Growth.

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1 Introduction

Along with the outstanding increase in wealth production in the last decades, we have also noticed unprecedented improvements in health outcomes such as the average life expectancy. Nevertheless, it remains huge disparities in terms of longevity across countries and even from one individual to another. For instance, the gap in expected years of life between people with the highest level and those with the lowest level of education is, on average, 6 years in OECD countries in 2015 (OECD, 2017). Such a disparity may have detrimental economic consequence such as a greater social cost, a fall in investments and especially in savings. For those reasons, health inequalities are becoming a growing concern in most developed countries.

In this paper, we consider that longevity discrepancies across individuals stem from their preferences (at least for this preliminary step). In a setup where they have to choose to allocate their income between consumption and investment in their health, the value, in terms of utility, of healthcare/consumption differs from one household to another, even though they have the same income. Therefore, because people do not invest in the same way in health, we come up with longevity inequalities. Our focus is to investigate whether an environmental policy could be an efficient tool to close the gap in life expectancy between households.

At an aggregate level, Hall and Jones (2007) supports that the rising share of health spending in US can be explained by the decreasing marginal utility of consumption. As people get richer, they consume more but the pleasure derived from that is less and less important when consumption rises. On the contrary, they value more additional unit of life and therefore the share of health spendings increase as a consequence. But how can we implement this rationale at an individual level ? That is what we are going to do by specifying an utility function such that one additional unit of health expenditures is not valuated the same from one household to another. For a modest family, what matters most, buying medicines for flu or paying its rent ? The answer to this is the opposite for a wealthy household.

Of course, the point of this paper is not to claim that health inequalities stem only from differences in preferences for health. In the literature, an approach consists in explaining those inequalities by the socioeconomic backgrounds of households such as human capital or income. In other disciplines such as biology and medicine, genetics plays a major role to understand health inequalities. However, in this paper, we want to focus on the channels of health preferences and lifestyle behaviors as the underlying mechanism that makes people length-of-life to be different. We will begin by exploring the first channel of health preferences and the second one is tackled afterwards.

2 Model

This section presents economic agents in our framework and the main features of the model.

2.1 Households

We consider an overlapping generation model (OLG) with discrete time, indexed by $t = 0, 1, \cdots, +\infty$. Agents live for two periods. The length of both periods is 1. When young, individuals live the entire period and supply their labor force to produce the unique good of the economy. The number of workers at each period is constant and is normalised to 1. In return, they receive a wage $\omega_{i,t}$ they use for private health expenditure ($x_{i,t}$) and for saving ($s_{i,t}$). We put aside the consumption choices when young because we want to to focus on the link between pollution and state of health. However, an household just lives a length ϕ_t ($0 \leq \phi_{i,t} \leq 1$) when old, a timespan during which he uses his remunerated savings (at a per unit rental price of r_{t+1}) to consume ($C_{i,t+1}$). Let us also mention that if the second period of time is indivisible, $\phi_{i,t}$ can be interpreted as the probability the household lives the second period. Alternatively, $\phi_{i,t}$ captures the life expectancy or longevity in our model. The preference of the household i is given by the following utility function:[1]

$$U_{i,t} = \phi_{i,t}\, u_i(c_{i,t+1})$$

By the way, the function $\phi_{i,t}$ satisfies the usual properties of endogenous life expectancy which are $\phi(0) = 0$, $\phi(\infty) = b \leq 1$, $\phi'(0) < \infty$, $\phi' > 0$, $\phi'(\infty) = 0$, $\phi'' < 0$. Therefore, the life expectancy can be written as follows:[2]

$$\phi_{i,t} \equiv \phi(\theta_{i,t}) = \frac{b\,\theta_{i,t}}{1+\theta_{i,t}} \text{ with } \theta_{i,t} \equiv \frac{(x_{i,t})^{\sigma}(\eta_t)^{1-\sigma}}{D_t}. \tag{1}$$

where D_t is the environmental degradation, $\theta_{i,t}$ is the health status of the household i and η_t is the public health expenditures. The longevity is increasing and concave with the health status $\theta_{i,t}$. This means that a change in the health expenditure has a greater impact on life expectancy when the health status is low, and a more limited one otherwise. This can explain the little gain in longevity in western countries despite huge investments in health whereas less developed ones have experienced a huge increase in longevity (See the WHO report, 2003). This phenomenon is represented here by the parameter b.

As aforementioned, a household uses his available income to improve her health and to save. Furthermore, we assume a perfect annuity market. The financial intermediaries do not obtain any benefit from their activities and therefore, they gives back to the survivor households all the money they have collected. It implies that the effective interest rate on savings is equal to

[1] As Chakraborty(2004), Varvarigos (2010) and Raffin & Seegmuller (2014)

[2] see Blackburn and Cipriani (2002), Chakraborty(2004), Constant (2015), Raffin and Seegmuller (2014) and Varvarigos (2010)

$\frac{r_{t+1}}{\phi_{i,t}}$. Hence, the budget constraints faced by a household are:

$$\begin{cases} x_{i,t} + s_{i,t} = \omega_{i,t} \\ c_{i,t+1} = \dfrac{r_{t+1} s_{i,t}}{\phi_{i,t}} \end{cases}$$

The first equation expresses the traditional trade-off a household has to cope with between consumption and savings. The budget constraint over the life span is $x_{i,t} + \dfrac{\phi_{i,t}}{r_{t+1}} c_{i,t+1} = \omega_{i,t}$.

The optimal choices of an agent born at the period t must satisfy the following equation:

$$\frac{\partial \phi_{i,t}}{\partial x_{i,t}} u(c_{i,t+1}) = \phi_{i,t} \frac{\partial u(c_{i,t+1})}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial s_{i,t}} \tag{2}$$

The rationale behind this equation is to say that the household faces 2 opposite choices: either she invests in an additional unit of private health spendings or she devotes one more unit to the physical capital. In the first case, longevity increases by $\frac{\partial \phi_{i,t}}{\partial x_{i,t}}$ but the satisfaction she derives from old-age consumption decreases. In the second case, the latter increases instead by $\frac{\partial u(c_{i,t+1})}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial s_{i,t}}$ but the household lifespan is shorter. At the optimum, the benefits are the same for the agent whatever the options chosen.

From the previous equation, we deduce that at the optimum, we have:

$$\boldsymbol{\frac{x_{i,t}}{s_{i,t}} = \varepsilon_{x_{i,t}} \left(\frac{1}{\varepsilon_{c_{i,t+1}}} - 1\right)} \tag{3}$$

where $\varepsilon_{x_{i,t}}$ is the health production function elasticity ($\varepsilon_{x_{i,t}} = \dfrac{\partial \phi_{i,t}}{\partial x_{i,t}} \dfrac{x_{i,t}}{\phi_{i,t}}$) and $\varepsilon_{c_{i,t+1}}$ the consumption elasticity ($\varepsilon_{c_{i,t+1}} = \dfrac{\partial u}{\partial c_{i,t+1}} \dfrac{c_{i,t+1}}{u}$).

Then, the equation (3) suggests that the relative importance of healthcare with respect to saving/consumption depends on the two elasticities. As income rises, the share of healthcare in the household budget goes up if the elasticity of consumption falls relative to the elasticity of health.

The major point of our argumentation is, for some categories of the population, a rise in income benefits more to consumption than to health improvement because the elasticity of consumption falls relative to the elasticity of health. For other categories, the consumption is favored for the opposite reason.

In other words, before investing in having an extra year of life or in savings, the household takes into account both her actual expected longevity and her current consumption. Then, she chooses either of them, depending on which brings relatively more in terms of years of life or utility.

A psychological (individual) minimum consumption

In our framework, each individual has an idea of the composition of the minimum basket of goods she desires. For instance, the size and the area of her accommodation, the number and the (quality of) education of her offspring, etc. Therefore, the individual optimal level of consumption has to ensure at least that minimal standard of living. That entails heterogeneity across households and depending on many factors such as the family background, the level of education, households do not have the same minimum consumption requirement. For instance, wealthy families are prone to send their children in private and onerous schools than the rest of the population. That is to say that the minimal consumption constraint tends to to higher for people who have experienced better material conditions.

In this paper, we introduce heterogeneity, related to the way the consumption elasticity varies with consumption. For that purpose, we define the per-period utility $u(c_{i,t+1})$ such that $u(c_{i,t+1}) = \dfrac{(c_{i,t+1}+d_i)^{1-\gamma}}{1-\gamma}$ and we have $\varepsilon_{c_{i,t+1}} = (1-\gamma)\dfrac{c^i_{t+1}}{d_i + c^i_{t+1}}$.[3] We note that $\varepsilon_{c_{i,t+1}}$ is decreasing in c_{t+1} for $d_i < 0$ and increasing for $d_i > 0$. Therefore, we consider 2 kinds of household $(i = 1, 2)$ having the same utility function, except for the value of the parameter d_i which is positive for one category and negative for the other. When $d_i < 0$, the consumer faces a minimum consumption of non-health goods equal to $-d_i$.

It is clear that the health elasticity $\varepsilon_{x_{i,t}}$ is equal to $\dfrac{\sigma}{1+\theta_{i,t}}$ and is decreasing in $x_{i,t}$. Therefore, from the equation (3), we derive the optimal level of healthcare service the household should purchase:

$$x_{i,t} = \frac{\sigma}{1+\theta_{i,t}}\left(\frac{d_i}{1-\gamma}\frac{\phi_{i,t}}{r_{t+1}} + \frac{\gamma}{1-\gamma}s_{i,t}\right) \tag{4}$$

Rising income, households preferences and healthcare services

From the equation (4), when $d_i < 0$, the higher the minimum consumption requirement of non-health goods the lower private health expenditures. That is to say that the more the basket of non-health goods appears important for the household, the more she saves to ensure that at least that level of consumption when old. Consequently, they devote less money for private health spending.

Similarly, an exogenous rise in life expectancy leads to a reduction of private health expenditures and to increase savings. This underlines the complementarity between private healthcare and all other measures that could improve household health status. Proofs of these mechanism are given below.

[3] This elasticity is lower than 1 by construction because we assume there are no negative savings.

Proof. From the equation (4), we have $\underbrace{\frac{d_i}{r_{t+1}}\phi_{i,t}}_{=f_1(x_{i,t})} = \underbrace{\frac{1-\gamma}{\sigma}x_{i,t}(1+\theta_{i,t})-\gamma s_{i,t}}_{=f_2(x_{i,t})}$. Then, $f_2(0) = -\gamma\omega_{i,t} < 0$, $f_2(\omega_{i,t}) > 0$, $f_1(0) = 0$, $f_1(\omega_{i,t}) > 0$, in addition to $f_1'(x_t) < 0$ and $f_2'(x_t) = \frac{1-\gamma}{\sigma}(1+(1+\sigma)\theta_{i,t})+\gamma > 0$ imply that for $d_i < 0$, there exists a unique $x_{i,t} = x_t(d_i) \in]0, \omega_{i,t}[$ solution of the equation (4). For a given $x_{i,t}$, $f_1(x_{i,t})$ increases with d_i, therefore $x_{i,t}$, the solution of (4) is increasing in d_i.
The equation (4) is equivalent to $F(x_t,\lambda_t) \equiv x_{i,t} - \frac{\sigma}{1+\theta_{i,t}}\left(\frac{d_i}{1-\gamma}\frac{\phi_{i,t}}{r_{t+1}} + \frac{\gamma}{1-\gamma}s_{i,t}\right) = 0$ where λ_t is an exogenous parameter different from $x_{i,t}$ that can affect $\phi_{i,t}$ such as σ the share of private health in total health spendings or the public health expenditures. Then, $\frac{\mathrm{d}s_{i,t}}{\mathrm{d}\lambda_t}$ has the sign of $\frac{\partial F/\partial\theta_{i,t}}{\partial F/\partial x_{i,t}}$. We have $\frac{\partial F}{\partial x_{i,t}}(x_t,\lambda_t) = \frac{1}{1+\theta_{i,t}}\left[1 - \frac{\sigma}{1-\gamma}\left(\frac{d_i}{r_{t+1}}\frac{\partial\phi_{i,t}}{\partial x_{i,t}} - \gamma\right) + \gamma\theta_{i,t}\right] > 0$ and $\frac{\partial F}{\partial\theta_{i,t}}(x_t,\lambda_t) = -\frac{\sigma}{1-\gamma}\frac{1}{(1+\theta_{i,t})^3}\left[\frac{d_i}{r_{t+1}}b - \gamma s_{i,t} - \left(\frac{d_i}{r_{t+1}}b + \gamma s_{i,t}\right)\theta_{i,t}\right] > 0$. That is why savings are increasing with longevity. ■

From the equation (4), we can derive the nature of healthcare good for a given household. When the consumer faces a minimum consumption requirement, the amount of healthcare good she demands goes up with the rising income. In this case, healthcare is either a normal or a superior good. This leads to the following lemma.

Lemma 1. ***For household that has a minimum consumption requirement ($d < 0$), healthcare services are:***

- ***superior good when the household income ($\omega_{i,t}$) is low;***
- ***normal good when the household income ($\omega_{i,t}$) is high enough.***

Proof. From the equation (4), we have $\frac{\partial x_{i,t}}{\partial\omega_{i,t}} = \frac{\sigma}{1-\gamma}\frac{\left(\gamma\frac{\partial s_{i,t}}{\partial\omega_{i,t}} + \frac{d_i}{r_{t+1}}\frac{\partial\phi_{i,t}}{\partial\omega_{i,t}}\right)(1+\theta_{i,t}) - (\gamma s_{i,t} + \frac{d_i}{r_{t+1}}\phi_{i,t})\frac{\partial\theta_{i,t}}{\partial\omega_{i,t}}}{(1+\theta_{i,t})^2}$.
This leads to $\frac{\partial x_{i,t}}{\partial\omega_{i,t}} = \gamma\left[\gamma + \frac{1-\gamma}{\sigma}(1+\theta_{i,t}) - \frac{d_i}{r_{t+1}}\frac{\partial\phi_{i,t}}{\partial x_{i,t}} + (1-\gamma)\theta_{i,t}\right]^{-1} > 0$ for $d_i < 0$. Therefore, $\frac{\partial x_{i,t}}{\partial\omega_{i,t}} - \frac{\omega_{i,t}}{x_{i,t}}$ has the sign of $f(x_{i,t}) \equiv \gamma\omega_{i,t} - x_{i,t}\left[\gamma + \frac{1-\gamma}{\sigma}(1+\theta_{i,t}) - \frac{d_i}{r_{t+1}}\frac{\partial\phi_{i,t}}{\partial x_{i,t}} + (1-\gamma)\theta_{i,t}\right]$.
$f(x_{i,t})$ is continuous and $f(\omega_{i,t}) = -\omega_{i,t}\left(\frac{1-\gamma}{\sigma}(1+\theta(\omega_{i,t})) - \frac{d_i}{r_{t+1}}\frac{\partial\phi}{\partial x_{i,t}}(\omega_{i,t}) + (1-\gamma)\theta(\omega_{i,t})\right) < 0$ and $f(0) = \gamma\omega_{i,t} > 0$. That lemma is proved. ■
Due to the consumption requirements, in a first step, all households secure a more or less important resources to finance their consumption when old, depending on their income. For poor households, this mandatory savings force them to devote little money for health improvements. Therefore, any additional resources goes more to healthcare than to daily consumption goods. The opposite mechanism occurs for wealthy households.

Similarly, we have $\gamma s_{i,t} = \frac{1-\gamma}{\sigma}x_{i,t}(1+\theta_{i,t}) - \frac{d_i}{r_{t+1}}\phi_{i,t}$. As expected, a higher minimum consumption requirement implies more savings to satisfy the living standard once retired. From that equation, we have $\gamma\frac{\partial s_{i,t}}{\partial \omega_{i,t}} = \left(\frac{1-\gamma}{\sigma}(1+(1+\sigma)\theta_{i,t}) - \frac{d_i}{r_{t+1}}\frac{\partial \phi_{i,t}}{\partial x_{i,t}}\right)\frac{\partial x_{i,t}}{\partial \omega_{i,t}} > 0$ and $\gamma r_{t+1}\frac{\partial (s_{i,t}/\phi_{i,t})}{\partial \omega_{i,t}} = \gamma\frac{\partial c_{i,t+1}}{\partial \omega_{i,t}} = r_{t+1}\frac{1-\gamma}{\sigma\,\phi_{i,t}}(1-\sigma+(1+\sigma)\phi_{i,t})\frac{\partial x_{i,t}}{\partial \omega_{i,t}} > 0$. That is to say that a rising income benefits both to private health expenditures and to consumption.

Consequently, the utility the household derives from old-age consumption is decreasing with income, so does the efficiency of private health spendings in improving longevity. Then, between healthcare and physical capital, a richer household invests more in the one whose elasticity reduces less compared to the elasticity of the other, according to (3). We are also interested in how the value of the minimum consumption of non-health good may influence the budget breakdown of the household. These two questions are the objects of the following lemma.

Lemma 2. ***For $d_i < 0$,***

- ***There exists a threshold income $\omega^\star(d_i) > 0$ such that:***
  - ***If $\omega_{i,t} < \omega^\star(d_i)$, then $\dfrac{\partial (s_{i,t}/x_{i,t})}{\partial \omega_{i,t}} < 0$;***
  - ***If $\omega_{i,t} > \omega^\star(d_i)$, then $\dfrac{\partial (s_{i,t}/x_{i,t})}{\partial \omega_{i,t}} > 0$.***
- ***$\omega^\star(d_i)$ is decreasing with d_i and $\omega^\star(0) = 0$.***

Proof. From (4), we have:

$$\gamma\frac{s_{i,t}}{x_{i,t}} = \frac{1-\gamma}{\sigma}(1+\theta_{i,t}) - \frac{d_i}{r_{t+1}}\frac{\phi_{i,t}}{x_{i,t}} \tag{5}$$

Then, we have $\frac{\partial (s_{i,t}/x_{i,t})}{\partial \omega_{i,t}} = \frac{1}{x_{i,t}}\frac{\partial x_{i,t}}{\partial \omega_{i,t}}\frac{\phi_{i,t}}{x_{i,t}}\frac{1-\gamma+\theta_{i,t}}{1+\theta_{i,t}}\left((1-\gamma)\theta_{i,t}\frac{x_{i,t}}{\phi_{i,t}}\frac{1+\theta_{i,t}}{1-\gamma+\theta_{i,t}} + \frac{d_i}{r_{t+1}}\right)$. Therefore, $\frac{\partial (s_{i,t}/x_{i,t})}{\partial \omega_{i,t}}$ has the sign of $f(x_{i,t}) = (1-\gamma)\theta_{i,t}\frac{x_{i,t}}{\phi_{i,t}}\frac{1+\theta_{i,t}}{1-\gamma+\theta_{i,t}} + \frac{d_i}{r_{t+1}} = \frac{1-\gamma}{b}x_{i,t}\frac{(1+\theta_{i,t})^2}{1-\gamma+\theta_{i,t}} + \frac{d_i}{r_{t+1}}$. We have $f(0) = \frac{d_i}{r_{t+1}} < 0$ and $f'(x_{i,t}) > 0$. Hence, there exists, if any, a unique $x_i^\star > 0$ such that $f(x_i^\star) = 0$ and for all $x_{i,t} < x_i^\star$, $f(x_{i,t}) < 0$ and for all $x_{i,t} > x_i^\star$, $f(x_{i,t}) > 0$. Because, $\dfrac{\partial x_{i,t}}{\partial \omega_{i,t}} > 0$, to $x_i^\star$ corresponds a $\omega^\star(d_i)$.
It is obvious that for $d_i = 0$, we have $f(0) = 0 \Rightarrow x_i^\star = 0$ and then $\omega^\star(0) = 0$. In addition, $f'(x_{i,t}) > 0$ and $\frac{\partial f(x_{i,t})}{\partial d_i} > 0$. Therefore, the solution of $f(x_{i,t}) = 0$ is decreasing with d_i, whence $x_i^\star$ and $\omega^\star(d_i)$ is higher when d_i is lower. The lemma is proved. ■

As aforementioned, private health spendings and consumption of non-health goods increase as income rises. However, the satisfaction a poor household derives from one additional unit of consumption of non-health goods falls more rapidly than the benefit from spending one more unit of resources for health improvement when income goes up. Therefore, they choose to save less and to expand their lifespan when they get richer. In addition, the more their constraint consumption is high, they more such a behaviour prevails. On the contrary, rich enough people instead devote more resources to consume than to improve their health when income goes up.

Let note that this heterogeneity in behaviour lies in the parameter d_i. Also, a household with no consumption requirement ($d = 0$) always favours consumption with respect to health when income increases.

2.2 Firms

The unique final good in the economy, which can also be used as capital good, is produced by perfectly competitive firms that combines labor offered by young and capital from financial intermediaries. The economy is populated by are $l_{1,t}$ and $l_{2,t}$ individuals of type 1 and 2 respectively. Each individual does not have the same productivity, according to his type. Wealthy households have a productivity ρ_1 and the others ρ_2. Therefore, the labor force in the economy is $L_t = \rho_1 l_{1,t} + \rho_2 l_{2,t}$. The production technology is given by: $Y_t = F(K_t, \bar{K}_t L_t) = AK_t^{\alpha}(\bar{K}_t L_t)^{1-\alpha}$ with $A > 0$ and $0 < \alpha < 1$. Y_t is the aggregate output, K_t the aggregate level of capital. In addition, we assume that capital is fully depreciated after one period.

With $\omega_{i,t}$ and r_{t+1} denoting respectively the real wage of individual of type i and the real interest rate, the optimal choices for the firms are described by:

$$\omega_{i,t} = \rho_i(1-\alpha)(1-\tau)A\bar{K}_t^{1-\alpha}\left(\frac{K_t}{L_t}\right)^{\alpha} \tag{6}$$

$$r_{t+1} = \alpha(1-\tau)A\bar{K}_t^{1-\alpha}\left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{7}$$

2.2.1 Assumption on the productivity of wealthy households

As mentioned in previous subsection, due to the consumption constraint, wealthy households, that is, those with higher $|d|$ choose to save more and to consume less private health services than the poor, for a given labor income. Given the positive relationship between private health expenditures and income, we assume as from now, that the labor income of rich households($i = 1$) is greater enough to ensure that the their optimal health choices are bigger than those of the poor individuals($i = 2$), at least at the first period of time.

Assumption 1. $\rho_1 >>> \rho_2$ *such that* $x_{1,1} > x_{2,1}$.

This assumption ensures that rich households live longer that poor households at the first period.

2.3 Government

In our setup, the government budget is balanced and is financed by imposing a proportional tax τ on the outcome Y_t. With that revenue, public authorities provide 2 kinds of services:

- Public health care denoted η_t. This encompasses the cost of building new hospital, prevention campaigns, the budget of medical research, etc. Curative health services represent an outcome tax of $(1-\mu)\tau$. Therefore, the public health expenditures account for $\eta_t = (1-\mu)\,\tau Y_t$ with $0 < \mu < 1$.[4]

- Pollution abatement activities denoted A_t. This incorporates the cost of all public environmental maintenance activities, which includes the domestic garbage collection, the maintenance of green areas which improve the air quality and so on. We also include all recycling activities of industrial wastes. This represents an outcome tax of $\mu\,\tau$ and therefore environmental policy is endowed with a budget of $A_t = \mu\,\tau\,Y_t$.

2.4 Environment degradation

The environment degradation is fueled by the production activity[5] whereas cleaning activities supported by the government reduce it. In this paper, the dynamics of the environmental degradation is defined as follows

$$D_t = a_1 Y_t - a_2 A_t = (a_1 - \mu\,\tau\,a_2)Y_t \tag{8}$$

where $a_1 > 0, a_1 > 0$. However, these two opposite effects are linked because the more we produce, on the one hand the more polluted the environment is, and on the other hand, the more public authorities provide cleaning activities by levying more taxes on labor.

Assumption 2. $\boldsymbol{a_1 > a_2}$.
This assumption is based on the fact that the pollution net flow is still positive.

3 Equilibrium

Because the population size is normalized to 1 and the share of wealthy households is ϵ, the workforce L_t involved in the production is equal to $L \equiv \rho_1\epsilon + \rho_2(1-\epsilon)$ at the equilibrium in the labor market. Therefore, equations (6) and (7) become

$$\omega_{i,t} = \frac{\rho_i}{L^\alpha}(1-\alpha)(1-\tau)AK_t \tag{9}$$

$$r_{t+1} = \alpha A(1-\tau) \tag{10}$$

[4]This condition, along with the strictly positivity of the stock of capital, rules out the case of a zero length-of-life. This happens when there is no public health care provided.

[5]See Van Oort et *al.*(2007) and Huwart et *al.*(2012)

Production is given by $Y_t = AK_t$. By substituting the preceding equations in (3), the household i optimal choices become :

$$\boldsymbol{s_{i,t}} = \frac{\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_{i,t})\omega_{i,t} - \dfrac{d_i}{\gamma}\dfrac{\phi_{i,t}}{\alpha(1-\tau)A}}{1+\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_{i,t})} \tag{11}$$

where the health status is given by

$$\theta_{i,t} = \Gamma_0\,(x_{i,t})^{\sigma}\,K_t^{-\sigma} \tag{12}$$

with $\Gamma_0 = ((1-\mu)\tau)^{1-\sigma}\,(a_1 - \mu\tau a_2)^{-1}\,A^{-\sigma}$. Γ_0 encompasses the resulting effects of the public health spending and of the level of pollution on longevity of the household. At this point, let us notice that we always have a positive amount of savings in the case of minimum consumption requirements ($d \leq 0$).

4 Households have the same preferences with $d = 0$

The intertemporal equilibrium in the asset market is given by $K_{t+1} = s_t$. Let $g_t \equiv \dfrac{K_{t+1}}{K_t}$.

The equation (11) leads to:

$$K_{t+1} = \frac{\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)}{1+\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)}\,\omega_{i,t}$$

$$g_t = \frac{\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)}{1+\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)}\,(1-\alpha)(1-\tau)A \equiv \mathcal{H}(\theta_t(g_t)) \tag{13}$$

where $\theta_t = \Gamma_0\,((1-\alpha)(1-\tau)A - g_t)^{\sigma}$.

Definition 1. *Given the initial condition $K_0 \geq 0$, the intertemporal equilibrium is a sequence $(K_t)_{t\in\mathcal{N}}$ such that the equation (13) is satisfied for all $t \geq 0$ with $\theta_t = \Gamma_0\,((1-\alpha)(1-\tau)A - g_t)^{\sigma}$.*

As expected, the saving rate is positively affected by the household health status, because the longer they live, the more they will enjoy the benefit of their investment. It is also noteworthy that the growth rate of capital is the same at each period of time, and equal to g, the solution of the dynamics equation.

Proposition 1. *There exists a unique g such that $g_t \equiv \dfrac{K_{t+1}}{K_t} = g\;\forall\, t \geq 0$, $K_0 > 0$ is given. In addition, there exists a threshold value of the global productivity A_0 such that:*

1. *For $A < A_0$, $g < 1$ and the stock of capital shrinks over time, but it never reaches 0;*

2. *For $A > A_0$, $g > 1$ and the stock of capital is steadily increasing period after period.*

Proof. Because the private health expenditures x_t is positive, we always have $g_t = g \leq (1-\alpha)(1-\tau)A$ for all $t \geq 0$. $\mathcal{H}(\theta_t(g_t))$ is a decreasing function of g_t, and we have $\mathcal{H}(\theta_t(g_t)) < (1-\alpha)(1-\tau)A$, especially for g_t close to $(1-\alpha)(1-\tau)A$. Hence, we deduce that there exists a unique g such that $0 < g < (1-\alpha)(1-\tau)A$ that satisfies the equation (13). This means that the stock of capital evolves at the same rate over time, and depending on whether the growth factor of capital g is lower or greater than 1, the capital accumulation is explosive or converges to 0. $\mathcal{H}(\theta(1)) - 1$ has the sign of $f(A) \equiv \frac{1-\gamma}{\gamma\sigma}\left(1+\theta_A(1)\right)\left((1-\alpha)(1-\tau)A-1\right) - 1 > 0$. Because $\theta_A(1)$ is increasing in A, $f(A)$ is an increasing function. We have $f(0) = -\infty$, $f(\frac{1}{(1-\tau)(1-\alpha)}) = -1 < 0$ and $f(+\infty) = +\infty$. Therefore, there exists a unique $A_0 > \frac{1}{(1-\tau)(1-\alpha)}$ such that $f(A_0) = 0$. For $A < A_0$, $\mathcal{H}(\theta(1)) - 1 < 0$ and $g < 1$. For $A > A_0$, $\mathcal{H}(\theta(1)) - 1 > 0$ and $g > 1$. The proposition is proved. ■

The growth factor of capital is constant over time. Therefore, longevity is the same for each generation and the effects of health expenditures are offset in the same way by the effects of pollution. Private health spendings grow/decrease at the same rate from one period to another. A_0 is a threshold of the global productivity factor that makes the stock of capital to be constant over time. If A is lower that A_0, then the household income is too low to invest enough resources in physical capital that maintains the stock of capital. Hence, the latter decreases over time, so do private health expenditures. However, the latter decreases less than savings as income falls. At the end, healthcare turn out to be a superior good. If A is above A_0, the household is rich enough to save more and more money as income rises and the saving rate is constant over time. Also, consumption is favoured with respect to health, and this strengthens the dynamics. Healthcare becomes a normal good at the end.

Let us focus now on the effect of the tax policy on the growth rate of capital. For that purpose, let us first analyse the effect on longevity.

Lemma 3. ***Under Assumption 1, if $\sigma + \tau < 1$, then there exists $g_\tau^\star > 0$ such that for $g_t < g_\tau^\star$, $\frac{\partial\theta_t}{\partial\tau} > 0$ and for $g_t > g_\tau^\star$, $\frac{\partial\theta_t}{\partial\tau} < 0$.***

Proof. $\theta_t = \Gamma_0\left((1-\alpha)(1-\tau)A - g_t\right)^\sigma = \Gamma_0(\frac{x_{i,t}}{K_t})^\sigma$, with $\Gamma_0 = \left((1-\mu)\tau\right)^{1-\sigma}\left(a_1 - \mu\tau a_2\right)^{-1}A^{-\sigma}$. We have $\frac{\partial\Gamma_0}{\partial\tau} = (1-\mu)^{1-\sigma}A^{-\sigma}\tau^{-\sigma}\frac{(1-\sigma)(a_1-\mu\tau a_2)+\mu\tau a_2}{(a_1-\mu\tau a_2)^2} > 0$.

$$\frac{\partial\theta_t}{\partial\tau} = (1-\mu)^{1-\sigma}A^{-\sigma}\tau^{-\sigma}\frac{((1-\alpha)(1-\tau)A-g_t)^{\sigma-1}}{(a_1-\mu\tau a_2)^2}\overbrace{\left[\left((1-\sigma)(a_1-\mu\tau a_2)+\mu\tau a_2\right)\left((1-\alpha)(1-\tau)A-g_t\right) - \sigma\tau(1-\alpha)A(a_1-\mu\tau a_2)\right]}^{f(g_t)}.$$

$\frac{\partial\theta_t}{\partial\tau}$ has the sign of $f(g_t)$, which is decreasing and we have $f(0) = (1-\alpha)A\left((1-\sigma-\tau)a_1 + \mu\tau\sigma a_2\right)$ and $f(g_t^{max} = (1-\tau)(1-\alpha)A) < 0$. If $\sigma + \tau < 1$, then there exists $g_\tau^\star > 0$ such that for $g_t < g_\tau^\star$, $\frac{\partial\theta_t}{\partial\tau} > 0$, for $g_t > g_\tau^\star$, $\frac{\partial\theta_t}{\partial\tau} > 0$.

■

The tax policy has 2 opposite effects on the life expectancy. On the one hand, a rise in τ

allows the government to increase pollution abatement activities and financing more public health programs. On the other hand, such a move reduces the labor income and therefore the amount of resources the agents devote to health. Due to the concavity in x_t, the marginal effect of the tax policy on private health spendings is bigger as x_t is lower. Therefore, the first effect prevails for low growth rates of capital, that is, when saving rate is low and the second effect is dominating for high value of the growth factor.

5 Households have the same preferences with $d < 0$: Minimum consumption requirements

From the equation (11), we can notice that the amount of savings s_t is always positive. At the equilibrium in the asset market ($K_{t+1} = s_t$), by introducing the growth factor of capital g_t in that equation, we obtain:

$$g_t = \frac{\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)(1-\alpha)(1-\tau)A - \dfrac{d}{\gamma}\dfrac{\phi_t}{\alpha(1-\tau)A\,K_t}}{1 + \dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)} \tag{14}$$

where $\theta_t = \Gamma_0\left((1-\alpha)(1-\tau)A - g_t\right)^\sigma$ and $\phi_t = \dfrac{b\,\theta_t}{1+\theta_t}$.

Range of values of the growth factor

From the previous equation, we deduce that:

$$\frac{1}{K_t} = \frac{\gamma\alpha(1-\tau)A}{d\,\phi_t}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_t)(1-\alpha)(1-\tau)A - g_t\left(1 + \frac{1-\gamma}{\gamma\sigma}(1+\theta_t)\right)\right] \tag{15}$$

Hence, because consumption occurs through savings, the consumption requirements on non-health goods ($d < 0$) ensures that the growth factor is lower-bounded by g_0, defined as the solution of the equation $g_t = \mathcal{H}(\theta_t(g_t))$. Indeed, from (15), $d < 0$ implies that we have $\psi(g_t) \equiv \dfrac{1}{\phi_t}\left[\dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)(1-\alpha)(1-\tau)A - g_t\left(1 + \dfrac{1-\gamma}{\gamma\sigma}(1+\theta_t)\right)\right] \leq 0$ for all $t \geq 0$. In addition, in the expression of θ_t, because savings has to be lower than available income, we have $\boldsymbol{g_0 \leq g_t < (1-\alpha)(1-\tau)A}$. Note that g_t can be close to them, but never reaches them.

Definition 2. *Given the initial condition $K_0 > 0$, the intertemporal equilibrium is a sequence $(g_t)_{t\in\mathcal{N}}$ such that the following dynamic equation is satisfied for all $t \geq 0$, $\psi(g_{t+1}) = \dfrac{1}{g_t}\psi(g_t)$ with $g_t \equiv \dfrac{K_{t+1}}{K_t}$, $\phi_t = \dfrac{b\,\theta_t}{1+\theta_t}$, $\theta_t = \Gamma_0\left((1-\alpha)(1-\tau)A - g_t\right)^\sigma$ and g_0 is given by (14).*

Note that g_t is a *pre-determined variable*, with g_0 given by (14). Also, the dynamics does not depend on the value of $d < 0$. Instead of that, the second period consumption constraint rather affects the stock of capital over time. The higher $|d|$ is, the more important the physical capital becomes. In fact, if a household considers that her list of necessary goods when old is large, that entails saving significant resources in order to satisfy that consumption at the second period. Using (15), we have:

$$g_t = \frac{K_{t+1}}{K_t} = \frac{\phi_{t+1}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_t)(1-\alpha)(1-\tau)A - g_t\left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_t)\right)\right]}{\phi_t\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_{t+1})(1-\alpha)(1-\tau)A - g_{t+1}\left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{t+1})\right)\right]} = \frac{\psi(g_t)}{\psi(g_{t+1})} \tag{16}$$

Steady states analysis

The steady states are characterized by $g_{t+1} = g_t = g$. When the later holds, the equation (16) leads to $g = 1$ if $\psi(g) \neq 0$. Otherwise, we have

$$\psi(g) = \frac{1}{\phi(g)}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta(g))(1-\alpha)(1-\tau)A - g\left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta(g))\right)\right] = 0 \tag{17}$$

Lemma 4. ***For $g_0 \leq g < (1-\alpha)(1-\tau)A$, $\psi(g)$ is negative and decreasing in g.***

Proof. $\phi(g) > 0$ is a decreasing function in g. The derivative of the expression into brackets is $\frac{1-\gamma}{\gamma\sigma}\left((1-\alpha)(1-\tau)A - g\right)\theta'(g) - 1 - \frac{1-\gamma}{\gamma\sigma}(1+\theta(g))$. Because $\theta'(g) < 0$, the term into brackets of $\psi(g)$ is negative and decreasing in g. Since $\psi(g)$ is positive and decreasing, then ψ declines with $g \in [g_0, (1-\alpha)(1-\tau)A[$. The lemma is proved. ■

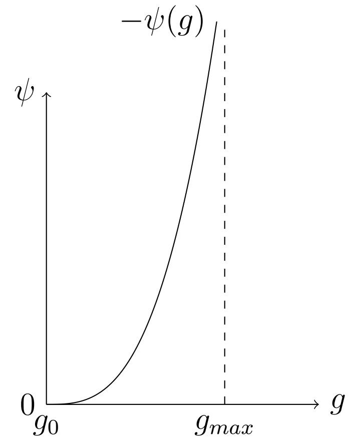


Figure 1: The dynamic function ψ

Proposition 2. ***There exists a unique threshold of the global productivity $A_0 > 0$ such that:***

1. ***For $A > A_0$, there exists a unique steady state $g_0 > 1$, which is stable. The growth factor g_t converges to that steady state but never reaches it.***

2. ***For $A < A_0$, there are two steady states $g_1 = g_0$ and $g_2 = 1$; the first one is unstable and the second is stable. In addition, we have $g_0 < 1$.***

Proof. For $g \neq 1$, we have $\psi(g) = 0 \Rightarrow g = \dfrac{\dfrac{1-\gamma}{\gamma\sigma}(1+\theta)(1-\alpha)(1-\tau)A}{1 + \dfrac{1-\gamma}{\gamma\sigma}(1+\theta)} = \mathcal{H}(\theta(g)) \Rightarrow g = g_0$.
Hence, for $A > A_0$, we have $g_0 > 1$ and g_0, the lower bound of g_t, is the unique steady state. For $A < A_0$, we have $g_0 < 1$, and in addition to g_0, $g = 1$ is also a steady state. Whatever the case, the steady state $g = g_0$ is never reached.

Let $g \in]g_0, (1-\alpha)(1-\tau)A[$. $\psi(g) - \frac{1}{g}\psi(g) = \frac{1}{\phi(g)}\left(1 - \frac{1}{g}\right)\left(1 + \frac{1-\gamma}{\gamma\sigma}(1+\theta(g))\right)(\mathcal{H}(\theta(g)) - g)$. Since $g > g_0$, we have $\mathcal{H}(\theta(g)) - g < 0$. Then, for $A > A_0$, $g > g_0 > 1$ and $\psi(g) - \frac{1}{g}\psi(g) < 0$. We deduce that the unique steady state is stable. For $A < A_0$, if $g_0 < g < 1$, then $\psi(g) - \frac{1}{g}\psi(g) > 0$ and if $g > 1$ then $\psi(g) - \frac{1}{g}\psi(g) < 0$. We deduce that the steady state $g = 1$ is stable and $g = g_0$ is unstable. ■

In case 1 of Proposition 2, the lower bound of the growth factor g_t is greater than 1, meaning that the productivity of agents allow them for saving more over time, no matters the initial value of the stock of capital. The capital accumulation is enhanced by the minimum consumption requirement of nonhealth goods through investment in physical capital. However, this dynamics slows down over time and this benefits to household health status, as a consequence of accelerating private health spendings. Nonetheless, it remains that in the long run, consumption is more valuated than health as income increases and the stock of capital is more and more greater. Health is a normal good. At the steady state g_0, the global health is constant and the stock of capital is infinite.

In case 2,

Comparative Statics: Effects of the tax (τ), environmental policies (μ) and minimum consumption requirement (d_i) on the long run equilibria

6 Heterogeneous agents

Let us consider two kinds of households in the economy, those with the preferences parameter $d < 0$ ($i = 1$) and the others with $d = 0$ ($i = 2$). Let us remind also that the first ones have a consumption elasticity that goes up with the level of consumption, whereas the second category has a constant consumption elasticity.

By the way, if ϵ denotes the share of the first category in the economy and $K_{i,t+1} \equiv s_{i,t}$, the market clearing condition is $K_{t+1} = \epsilon\, s_{1,t} + (1-\epsilon)\, s_{2,t} = \epsilon\, K_{1,t+1} + (1-\epsilon)\, K_{2,t+1}$, that is:

$$K_{t+1} = \epsilon \frac{\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\omega_{i,t} - \frac{d}{\gamma}\frac{\phi_{1,t}}{\alpha(1-\tau)A}}{1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})} + (1-\epsilon)\frac{\frac{1-\gamma}{\gamma\sigma}(1+\theta_{2,t})\omega_{i,t}}{1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{2,t})} \tag{18}$$

where $\phi_{i,t}$, $\theta_{i,t}$ $(i = 1, 2)$ and $\omega_{i,t}$ are given by (1), (12) and (9) respectively.

6.1 The Intertemporal Equilibrium

Let us define $g_{i,t} = \frac{K_{i,t+1}}{K_t}$. We have $g_{2,t} = \frac{\frac{1-\gamma}{\gamma\sigma}(1+\theta_{2,t})}{1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{2,t})}\frac{\rho_2}{L^\alpha}(1-\alpha)(1-\tau)A = \mathcal{H}(\theta_{2,t}(g_{2,t}))$ and $0 < g_{2,t} < \frac{\rho_2}{L^\alpha}(1-\tau)(1-\alpha)A$. As in section 4, we obtain that $g_{2,t} = g_0$ for all $t \geq 0$[6]. The effort of households of type 2 to the aggregate growth is the same over time. Regarding the contribution to growth of wealthy households $(i = 1)$, we have: (18) becomes

$$g_{1,t} = \frac{\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A - \frac{d}{\gamma}\frac{\phi_{1,t}}{\alpha(1-\tau)AK_t}}{1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})} \tag{19}$$

We also deduce that:

$$\frac{1}{K_t} = \frac{\gamma\alpha(1-\tau)A}{d}\psi(g_{1,t}) = \frac{\gamma\alpha(1-\tau)A}{d\,\phi_{1,t}}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\frac{\rho_1}{L^\alpha}(1-\alpha)(1-\tau)A - g_{1,t}\left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\right)\right] \tag{20}$$

As in the previous section, this implies that the contribution to growth of individuals of type 1, $g_{1,t}$, is bounded such that $\boldsymbol{g_0 \leq g_{1,t} < \frac{\rho_1}{L^\alpha}(1-\alpha)(1-\tau)A}$.

Definition 3. ***Given the initial condition $\boldsymbol{K_0 > 0}$, the intertemporal equilibrium is a sequence $\boldsymbol{(g_t)_{t\in\mathcal{N}}}$ such that the following dynamic system is satisfied for all $\boldsymbol{t \geq 0}$, $\boldsymbol{\psi\left(\frac{1}{\epsilon}g_{t+1} - \frac{1-\epsilon}{\epsilon}g_0\right) = \frac{1}{g_t}\psi\left(\frac{1}{\epsilon}g_t - \frac{1-\epsilon}{\epsilon}g_0\right)}$, that is:***

$$\begin{aligned}\boldsymbol{\frac{1}{\phi_{1,t+1}}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t+1})\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A - \left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t+1})\right)g_{1,t+1}\right]} &\boldsymbol{=}\\ \boldsymbol{= \frac{1}{g_t\phi_{1,t}}\left[\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A - \left(1+\frac{1-\gamma}{\gamma\sigma}(1+\theta_{1,t})\right)g_{1,t}\right]}\end{aligned}$$

with $\boldsymbol{\phi_{1,t} = \frac{b\,\theta_{1,t}}{1+\theta_{1,t}}}$, $\boldsymbol{\theta_{1,t} = \Gamma_0\left(\frac{\rho_1}{L^\alpha}(1-\alpha)(1-\tau)A - g_{1,t}\right)^\sigma}$, $\boldsymbol{g_{1,t} = \frac{1}{\epsilon}g_t - \frac{1-\epsilon}{\epsilon}g_0}$ and $\boldsymbol{g_{1,0}}$ is given by the equation (19).

[6] g_0 is the solution of the equation $g_t = \mathcal{H}(\theta_t(g_t))$.

6.2 Steady State analysis

A steady state g, defined by $g_{t+1} = g_t = g$, is characterized by $\left(1 - \frac{1}{g}\right)\psi(\frac{1}{\epsilon} g_t - \frac{1-\epsilon}{\epsilon} g_0) = 0$. The results are summarized in the following proposition.

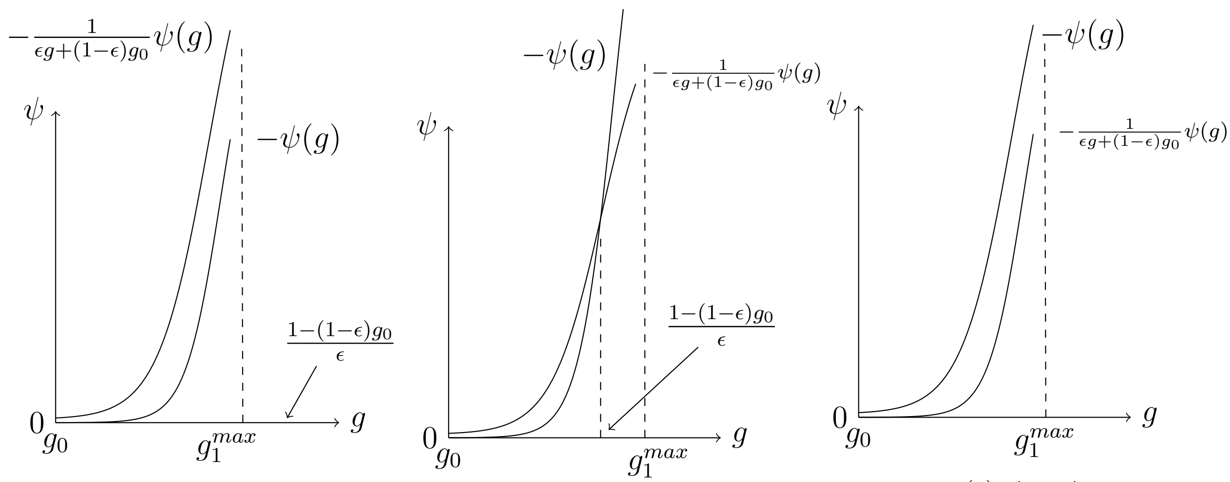
Proposition 3. ***Under Assumption 1,***

1. ***For $A < A_0$,***

   - ***If $\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A < \frac{1-(1-\epsilon)g_0}{\epsilon}$, then there exists 2 steady states, $g_a = g_0 < 1$ and $g_b = \epsilon\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A + (1-\epsilon)g_0 < 1$. The first one is unstable and the second is asymptotic and stable. Health inequalities declines over time.***

   - ***If $\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A > \frac{1-(1-\epsilon)g_0}{\epsilon}$, then there exists 3 steady states, $g_a = g_0 < 1$, $g_b = \epsilon\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A + (1-\epsilon)g_0 > 1$ and $g_c = 1$. g_a is unstable, g_b is asymptotic and unstable and g_c is stable.***

2. ***For $A > A_0$, there exists 2 steady states $g_a = g_0 > 1$ and $g_b = \epsilon\frac{\rho_1}{L^\alpha}(1-\tau)(1-\alpha)A + (1-\epsilon)g_0 > 1$. The first one is stable and the second is asymptotic and s unstable. The stock of capital increases steadily over time and health inequalities widen over time.***



(a) $A < A_0$ and $g_1^{max} < \frac{1-(1-\epsilon)g_0}{\epsilon}$ (b) $A < A_0$ and $g_1^{max} > \frac{1-(1-\epsilon)g_0}{\epsilon}$ (c) $A > A_0$

Figure 2: Dynamics of the economy

Proof. g is a steady state if and only if $g = 1$ or $\frac{g-(1-\epsilon)g_0}{\epsilon} = g_0 \Rightarrow g = g_0$. Since $g_{2,t} = g_0$ at steady states, $g = g_0$ entails that $g_{1,t} = g_0$ whereas $g = 1$ implies $g_{1,t} = \frac{1-(1-\epsilon)g_0}{\epsilon}$ in the long run. If $A > A_0$, then $g_0 > 1$ and there is a unique steady state which is $g = g_0$ because $g_{1,t} \geq g_0$ for all $t \geq 0$. Otherwise, there exists two steady states.

ψ is a negative function. If $A > A_0$, let consider $g_t > g_0$. Then, because $g_0 > 1$, we have

$(1-\frac{1}{g_t})\psi(g_{1,t})<0$. ■

6.2.1 Description++++

Note that in Case 2 of Proposition 3, we have $g_{1,t}=g_{2,t}=g_0$ at the unstable steady state, and $\boldsymbol{g_{1,t}=\frac{1-(1-\epsilon)g_0}{\epsilon}>1,\ g_{2,t}=g_0<1}$ at the stable one. In this configuration, health inequality occurs.

In the case 1 of the Proposition 3, the capital accumulation occurs at the steady state and the stock of capital increases continuously. This is also the case for each category of the population for which the saving rate is the same and the ratio of savings over capital is more than one and equal to g_0. In other words, both households devotes the same share of their budget to health as incomes increases.

The same configuration prevails in the case 2 of the proposition, when the aggregated growth factor is equal to g_0. Both households devotes the same share of income to improve their health.

6.3 Implications on health inequalities

When the aggregated growth factor is equal to g_b instead, agents with a minimum consumption requirements at the steady state devotes a greater share of their income to health than those with $d=0$. *That is why they are in poorer health than the latter.*

6.3.1 Effect of population distribution (ϵ) and g_0 on the health inequalities

6.4 An environmental tax to reduce health inequalities

7 Heterogeneous agents with endogenous preferences

In the previous sections, the consumption constraint is set exogenously. However, to more stick to the idea of a minimum standard of living dependent on social and family background, we can make d varies with the income. First, let's assume that it is proportional to the income, that is, $d_t=\zeta\omega_{t-1}$ with $\zeta<0$. Therefore, the equation (20) becomes:

$$\frac{1}{g_{t-1}}=\frac{\gamma\alpha(1-\tau)A}{\zeta}\psi(g_{1,t}) \tag{21}$$

7.1 Steady State analysis

A steady state g is characterized by $g_t=g=t-1=g$ and we have $\frac{1}{g}=\frac{\gamma\alpha(1-\tau)A}{\zeta}\psi(g)$.

Proposition 4. *Under Assumption* 1*, there exists a unique steady state* $g_a > g_0$*, which is stable.*

Proof. ∎

7.2 Implications on health inequalities

8 Model with early age consumption

The preference of the household i is given by the following utility function:[7]

$$U_{i,t} = u_1(c_{i,t}) + \phi_{i,t}\, u(c_{i,t+1}) \qquad \text{with } u_1(c_{i,t}) = u(c_{i,t} - d_i) \text{ and } d_i > 0$$

The budget constraints faced by a household are:

$$\begin{cases} c_{i,t} + x_{i,t} + s_{i,t} = \omega_{i,t} \\ c_{i,t+1} = r_{t+1} s_{i,t} \end{cases}$$

The optimal choices satisfy the two following equations:

$$u_1'(c_{i,t}) = \phi_{i,t}' u(c_{i,t+1}) \tag{22}$$

$$\phi_{i,t}' u(c_{i,t+1}) = r_{t+1}\phi_{i,t} u'(c_{i,t+1}) \tag{23}$$

Let $u(c_{i,t+1}) = \dfrac{c_{i,t+1}^{1-\gamma}}{1-\gamma}$. Therefore, from (23), we deduce that $s_{i,t} = \dfrac{1-\gamma}{\sigma} x_{i,t}(1+\theta_{i,t}) \equiv s(x_{i,t})$. Unlike the case with no consumption in first period, ***there is a positive relationship between private health spendings and savings***. In other words, people in good health save more than those with lower health status.

From (22), we have $c_{i,t} = \left(\dfrac{1-\gamma}{\sigma}\dfrac{x_{i,t}(1+\theta_{i,t})}{\phi_{i,t}} c_{i,t+1}^{\gamma-1}\right)^{1/\gamma} + d_i \equiv c(x_{i,t})$. If $\boldsymbol{\gamma > \sigma}$, then ***the early age consumption is increasing with the private health spending***. $\dfrac{\partial c_{i,t}}{\partial x_{i,t}}$ has the sign of $P(x_{i,t}) = x_{i,t}^{\gamma-\sigma-1}(1+\theta_{i,t})^{\gamma}\left(\gamma - \sigma + \gamma(1+\sigma)\theta_{i,t}\right)$. If $\gamma > \sigma$ then $\dfrac{\partial c_{i,t}}{\partial x_{i,t}} > 0$. Otherwise, for $\boldsymbol{\theta_{i,t} < \theta^\star \equiv \dfrac{\sigma - \gamma}{\gamma(1+\sigma)}}$, $\dfrac{\partial c_{i,t}}{\partial x_{i,t}} < 0$ and for $\boldsymbol{\theta_{i,t} > \theta^\star}$, $\dfrac{\partial c_{i,t}}{\partial x_{i,t}} > 0$.

Let's suppose $d_i = 0$. Then, we have $\dfrac{c_{i,t}}{x_{i,t}} = \left(\dfrac{1-\gamma}{\sigma}\dfrac{x_{i,t}^{1-\gamma}(1+\theta_{i,t})}{\phi_{i,t}} c_{i,t+1}^{\gamma-1}\right)^{1/\gamma}$ and $\dfrac{\partial(c_{i,t}/x_{i,t})}{\partial x_{i,t}}$ has the sign of $P_0(x_{i,t}) = \sigma x_{i,t}^{-\sigma-1}(1+\theta_{i,t})^{\sigma}(\sigma\theta_{i,t} - 1)$. Hence, **if $\boldsymbol{\theta_{i,t} < \dfrac{1}{\sigma}}$ then the ratio $\boldsymbol{c_{i,t}/x_{i,t}}$ is decreasing with $\boldsymbol{x_{i,t}}$**. Otherwise, **if $\boldsymbol{\theta_{i,t} > \dfrac{1}{\sigma}}$, the ratio of consumption over health spending is increasing with $\boldsymbol{x_{i,t}}$**.

The optimal private health spending is given by $\boldsymbol{c(x_{i,t}) + x_{i,t} + s(x_{i,t}) = \omega_{i,t}}$. If $c' > 0$, then private health spendings go up when income rises, that is $\boldsymbol{\dfrac{dx_{i,t}}{d\omega_{i,t}} > 0}$.

[7] As Chakraborty(2004), Varvarigos (2010) and Raffin & Seegmuller (2014)

In the case of a single agent, the market clearing condition gives:

$$g_t = \frac{1-\gamma}{\sigma}\frac{x_{i,t}}{K_t}(1+\theta_{i,t}) \text{ with } \theta_t = \Gamma_0(\frac{x_{i,t}}{K_t})^{\sigma}$$

References