What is Bitcoin? - Price Jumps, Demand vs. Supply, and Market Characteristics

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Abstract

This paper conducts the first detailed analysis of the dynamics of Bitcoin prices. The application of a number of both linear and non-linear GARCH models indicates that the role of extreme price movements seems to be particularly pronounced: GARCH models with student-\(t\) innovations as well as combined jump-GARCH models are among the models with the best fit. This is reflected in both a relative increase in model performance and also compared to behavior of crude oil and gold prices. In contrast, no evidence of leverage effects is found. Market features such as the fixed supply of Bitcoin imply that Bitcoin is reminiscent of an exhaustible resource commodity. Whereas the supply of gold and oil are uncertain, there are no uncertainties on the Bitcoin supply-side. Thus, the observed price movements are attributable to demand side factors.

Keywords: Bitcoins, GARCH, Leverage, Jump models, Commodity Pricing

JEL-Classification: C12, C22, C58, G12
1 Introduction

The virtual currency of Bitcoin emerged in 2008, developed by a group of anonymous programmers with the purpose to make possible online payments without involvement of a financial institution or other third parties; see Nakamoto (2008). Bitcoin is the most popular virtual currency and received considerable attention from both the general public and academia, mainly due to the spectacular price behaviour, its general novelty value and certainly also extraordinary events and scandals related to Bitcoin. It does not come as a surprise that monetary theorists as well as central banks are particularly interested in this phenomenon. Ali et al. (2014), for instance, discuss whether or not Bitcoin has the ability to perform the functions required of a fiat money. European Central Bank (2012) emphasises that virtual currencies generally can have the function of serving as medium of exchange within a specific community. Among the issues the very comprehensive paper by Boehme et al. (2015) discusses is whether or not Bitcoin can disrupt existing monetary systems.

Bitcoins can be obtained, first, by verifying transactions within the Bitcoin network - this process is commonly referred to as Bitcoin mining. Second, Bitcoins are also traded on various exchanges. The following figures illustrate that the Bitcoin market is economically highly relevant, and, thus, deserves the attention it currently receives. The market capitalisation just exceeded 70 billion USD, about 16.5 million Bitcoins are in circulation, almost 700,000 unique Bitcoin addresses are used per day and there are more
than 300,000 confirmed transactions per day.\textsuperscript{1} Daily trading volume can exceed 500 million USD. In addition to this, virtual currencies are generally a new phenomenon and are, at the same time, associated with the emergence of a new tradable entity and a new market place. Studying the price behaviour of such a newly developed tradable entity in the context of otherwise developed economies and financial markets is deemed particularly attractive.

The detailed analysis into the dynamics of Bitcoin prices this paper conducts is - to the best knowledge of the author - the first one to date.\textsuperscript{2} Applied are a number of both linear and non-linear GARCH models. The models under consideration allow for testing for the presence of features such as fat tails, asymmetric responses to positive and negative news, and the role of extreme price movements. These type of features are usually found in exchange rates, stock prices as well as commodity prices - thus, in financial markets considered similar to the Bitcoin market. Thus, in addition to a benchmark GARCH model, Glosten et al.’s (1993) TGARCH, Nelson’s (1991) EGARCH, and, finally, Chan and Maheu’s (2002) jump-GARCH model are applied.

The main findings that emerge from this empirical exercise can be summarised as follows: first, extreme price movements play a particularly strong role. This conclusion is based on the overall good performance of the standard GARCH model with student-\(t\) innovations as well as the combined jump-GARCH model. These models have in common that they are charac-

\textsuperscript{1}Data source: https://blockchain.info.
\textsuperscript{2}To be precise, the paper analyses the Bitcoin USD exchange rate. For ease of reading, this exchange rate is referred to as Bitcoin price.
terised by fat tails and, thus, are able to capture extreme price movements. Second, no evidence of leverage is found. Third, the role of extreme price movements is found to be larger than in the crude oil and the gold market. In a way, these results are anticipated: the Bitcoin market only recently emerged; thus the market is still in an immature state and the market participants are in a process of familiarising themselves with the market. In this type of environment individual events not surprisingly have a larger impact on prices than in more mature markets. However, these results gain additional importance as the Bitcoin market is characterised by a number of distinct market features: the total number of Bitcoins is fixed and the number of Bitcoins in circulation is known with certainty. An innovative way to interpret this is to say that Bitcoin shares these features with exhaustible resource commodities such as crude oil and gold. Various authors state that Bitcoin is not a currency but some sort of a speculative investment. In a way this type of conclusion is vague as it only rules out one possible interpretation without offering a clear direction. This paper’s emphasis of Bitcoin as exhaustible commodity resource remedies this as for those established pricing theories and a general a theoretical understanding exists.

There is a unique feature of Bitcoin which deserves emphasis. While the markets for crude oil and gold are characterised by considerable supply-side uncertainty - supply shocks, often sparked by political events, surprise discoveries and the sudden emergence of additional resources due to the emergence of new extraction technologies are frequent events in these markets - no uncertainty of these types exists on the supply-side of Bitcoin. Thus, it can be concluded that the observed price fluctuations are attributable
exclusively to demand side factors. This is an alternative way to phrase Ali et al.'s (2014) assertion that "digital currencies have meaning only to the extent that participants agree that they have meaning."

The extant empirical literature this paper contributes to can be summarised as follows: Hayes (2016) proposes a cost of production model for valuing Bitcoin; Ciaian et al.'s (2016) Bitcoin price formation model focuses on the role of market forces and Bitcoin attractiveness. The issue of Bitcoin volatility takes centre stage in various papers: Baek and Elbeck (2014) use the method of detrended ratios in order to study relative volatility as well as drivers of Bitcoin returns. They find that Bitcoin volatility is internally driven and conclude that the Bitcoin market is currently highly speculative. Cheah and Fry (2015) test for speculative bubbles in Bitcoin prices and find that they exhibit speculative bubbles. In addition, the authors state that the fundamental value of Bitcoin is zero. In a similar paper, Cheung et al. (2015) apply a recently proposed popular testing procedure in order to search for periodically collapsing bubbles. They find evidence of these type of bubbles in particular in the period between 2011 and 2013. Dyhrberg (2016a, 2016b) analyses the hedging capabilities of Bitcoin. Yelowitz and Wilson (2015), finally, use Google search data in order to shed light on the characteristics of users interested in Bitcoin. Their analysis shows that "computer programming enthusiasts" and criminals seem to be particularly interested in Bitcoin, while interest does not seem to be driven by political and investment motives.

The remainder of the paper is organised as follows: Section 2 provides a detailed descriptive analysis of the data, 3 outlines the empirical approaches
applied in this paper. Sections 4 and 5 present the main empirical results as well as a discussion of which; Section 6 offers some concluding remarks.

2 Data

It has already been mentioned above that Bitcoins are traded on various exchanges. This paper uses two Bitcoin price series from two different exchanges: first, Mt.Gox, until its shutdown the most liquid Bitcoin exchange, and second, Bitstamp. Bitstamp is among the exchanges with the highest market share. The periods of observation are 7/02/2011 - 2/24/2014 and 9/16/2011 - 8/14/2017, respectively; data frequency is daily, and log-returns of the prices are used.\(^3\) Figure 1 presents the data used in this paper in levels as well as in returns. These eye-catching price dynamics clearly deserve a closer investigation. Interest in Bitcoin from the general public began to increase when Bitcoin prices peaked at about 1,200 USD end of 2013 and beginning of 2014. Remarkable price movements, however, have been present even before that: in early stages of 2013, for instance, Bitcoin prices surged, reaching 200 USD for the first time. Subsequent to the hike witnessed in 2013/2014, prices seemed to have stabilised; at least for Bitcoin standards. Beginning of 2017, however, a new dramatic price hike occurred with prices reaching 4,000 USD. The plot of the Bitcoin price growth rates illustrates that volatility clusters are present throughout the period of observation. These clusters are even more pronounced between 2010 and 2013; thus prior to the most famous price hikes. During 2015 and 2016, volatility generally

\(^3\)Data source: www.bitcoincharts.com. The exchange BTC-e used to be among the most liquid exchanges; however also this market place has been closed due to legal issues.
Figure 1: Bitcoin prices - levels and returns

seems to have been slightly lower before picking up again in 2017.

The kernel density estimates as well as quantile-quantile plots presented in Figures 2 and 3 vividly illustrate that Bitcoin price returns are far from normally distributed. The empirical distributions are highly leptocurtic - more clustered around the mean and with heavier tails. This leptocurtosis is particularly pronounced in the Bitstamp market. The quantile-quantile plots confirm this finding. These plots, furthermore, indicate that extreme
Figure 2: Kernel density estimates. Left panel: Bitstamp, right panel: Mt.Gox.

price movements are very common in both markets.

3 Empirical methods

The price behaviour described in the previous section is analysed using a number of both linear and non-linear GARCH models. A standard GARCH(1,1) model serves as the benchmark:

\[
y_t = \mu + \sum_{i=1}^{l} \phi_i y_{t-i} + \epsilon_t \\
\epsilon_t = \sqrt{h_t} z_t \\
h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}
\]
Figure 3: Quantile-quantile plots. Left panel: Mt.Gox, right panel: Bitstamp

with $y_t$ denoting Bitcoin price returns.\footnote{The number of autoregressive parameters is selected using standard Information Criteria.} Gaussian as well as student-$t$ innovations are considered. In addition to testing the restriction $\alpha + \beta = 1$ (IGARCH, see Engle and Bollerslev, 1986) a number of extensions are used.\footnote{All extensions use Gaussian innovations only.} As a common feature of various financial market variables is an asymmetric response to negative and positive news, the TGARCH model proposed by Glosten et al. (1993) is useful. The conditional variance is then written as

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \psi \epsilon_{t-1}^2 I_{t-1}$$  \hspace{1cm} (4)$$

where $I_t = 1$ if $\epsilon_t < 0$. In order to test if the leverage effect is exponential, the EGARCH model proposed by Nelson (1991) is also used:
\[
\log(h_t) = \omega + \alpha \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}^2}} + \beta \log(h_{t-1}) + \kappa \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}^2}}
\]  

(5)

The presence of leverage effects can be tested using the following hypothesis: \( \kappa < 0.6 \)

A number of recent papers also point out that certain commodity prices are not only characterized by conditional heteroscedasticity but also by jumps: Gronwald (2012) as well as Lee et al (2012) find evidence of jumps in crude oil prices, Sanin et al (2015) in European carbon prices. These markets are generally considered "political" markets subject to various types of influences. The European carbon market, in addition, is also a newly established market. Jumps in commodity prices are generally considered reflecting reactions of prices to surprising news; see e.g. Jorion (1988). In order to analyse the role of extreme price movements in the Bitcoin market, the so-called autoregressive jump-intensity GARCH model proposed by Chan and Maheu (2002) is used. The benchmark model 3 is rewritten as follows:

\[
y_t = \mu + \sum_{i=1}^{I} \phi_i y_{t-i} + \sqrt{h_t} z_t + \sum_{k=1}^{n_r} X_{t,k}
\]

(6)

where \( h_t \) is still described by the GARCH(1,1) process \( h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \).

The last term in Equation 3 denotes the jump component. It is assumed that the (conditional) jump size \( X_{t,k} \) is normally distributed with mean \( \theta_t \).

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6This type of linear GARCH models is applied in a vast literature epitomized by papers such as Chkili et al. (2014), Trueck and Liang (2012), Wang and Wu (2012), Paolella and Taschini (2008) and Aloui et al. (2013). Markets under consideration in these papers are commodity markets, the European carbon market (EU ETS), and foreign exchange markets.
and variance $\eta_t^2$; $n_t$ describes the number of jumps that arrive between $t-1$ and $t$ and follows a Poisson distribution with $\lambda_t > 0$:

$$P(n_t = j|\Phi_{t-1}) = \frac{\lambda_t^j}{j!} e^{-\lambda_t}$$  \hspace{1cm} (7)

$\lambda_t$ is called jump-intensity. The model is estimated in two variants: a constant jump-intensity model with $\lambda_t = \lambda$, $\theta_t = \theta$, and $\eta_t^2 = \eta^2$ and a time-varying jump-intensity model. For the latter, $\lambda_t$ is assumed to follow the auto-regressive process

$$\lambda_t = \lambda_0 + \sum_{i=1}^{r} p_i \lambda_{t-i} + \sum_{i=1}^{s} \gamma_i \xi_{t-i}. \hspace{1cm} (8)$$

The application of the time-varying jump intensity model allows one to study how the influence of extreme price movements changes over time. According to Nimalendran (1994), finally, the total variance $\Sigma^2$ of a process can be decomposed in a jump-induced part and a diffusion-induced part:

$$\Sigma^2 = h_t + \lambda_t (\theta^2 + \eta^2). \hspace{1cm} (9)$$

This decomposition procedure allows one to compare statistical behaviour across different markets. Finally, calculating this measure using the time-varying jump intensity makes possible to study how the share of jump-induced variance changes over time. The following sections present and discuss the results.
4 Results

This section presents the results of the empirical analysis of Bitcoin prices. Table 1 compares the goodness-of-fit of the applied GARCH models; Table 2 presents the corresponding parameter estimates. Table 1 suggests that the performance of the TGARCH and EGARCH model is not significantly different compared to the benchmark GARCH model. Thus, leverage effects do not seem to play a considerable role. The parameter restriction implied in the IGARCH model leads to a considerable decrease in the goodness-of-fit; this issue is briefly discussed below. The best performing model is the GARCH with student-$t$ innovations, followed by the jump-GARCH models. The latter cannot outperform the former but still provide a considerable increase of the goodness-of-fit. The parameter estimates overall confirm these results: while the leverage parameters are not significant, all but one jump-parameter are statistically different from zero. Table 2, furthermore, indicates that the GARCH parameters of all models under consideration - the notable exception are the combined jump-GARCH models - are in sum larger than 1. This would indicate that the shocks in these GARCH models would be persistent. This finding can also be explained by the immaturity of the Bitcoin market.

As the performance of the jump-GARCH model is generally good and as this model has a number of interesting features, a more detailed discussion of these model estimates is useful. Figure 4 presents the estimated time-varying $\lambda$ coefficient as well as time-varying shares of variance induced by jumps. It is evident that higher jump-intensities are more frequent in 2011 -
2013 than in later years. The largest peaks occur during the fourth quarter of 2011, the third quarter of 2012, the second quarter 2013. In the beginning of 2014, Mt.Gox prices are marked by particularly large jumps related to the market turbulences prior to its shutdown. Bitstamp prices after 2014 seem to be considerably less sensitive to news. The variance decomposition, furthermore, shows that the share of variance induced by jumps fluctuates around 60%. Pronounced decreases occur a number of times; mostly to the same times large peaks of the jump-intensity occur. The share of jump-induced variance drops to about 15 – 30%. Although these findings at first glance seem to contradict each other, there is a simple explanation: in the aftermath of the extreme movements the volatility is generally higher, with a larger share of volatility captured by the GARCH component.\footnote{Gronwald (2012) finds a similar pattern for the crude oil market.}

In order to ease interpretation of these results, the same set of models is estimated using both crude oil and gold prices. Figure 5 presents the results for the jump models; detailed estimation results can be found in the Appendix. It is evident that the share of oil price variance induced by jumps
Table 2: Constant and time-varying jump-intensity models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mt.Gox</th>
<th>Bitstamp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH</td>
<td>GARCH</td>
</tr>
<tr>
<td>( \mu )</td>
<td>2.9E-03</td>
<td>2.5E-03</td>
</tr>
<tr>
<td></td>
<td>(0.0011)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>0.2963</td>
<td>0.2189</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.2663</td>
<td>0.2189</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.2663</td>
<td>0.2189</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.7456</td>
<td>0.7376</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.3425)</td>
<td>(0.2919)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.1944)</td>
<td>(0.3311)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

Note: p-values in parentheses. Number of endogenous lags as well as inclusion of constant is based on standard information criteria as well as significance of parameters. The \( \psi/\kappa \) row contains the leverage parameters.
Figure 4: Jump intensities and jump-induced variance
fluctuates around 40%. Thus, this measure is considerably lower than for the Bitcoin prices. Moreover, in the aftermath of extreme price movements associated with the OPEC collapse 1986, the Gulf War 1991, and the oil price record high of 2008, this share drops drastically to just 5 – 10% - also much lower than the values found for the Bitcoin market. After the oil price decline in 2014, the share is found be relatively low as well. This reflects that during this extraordinary period with a high degree of uncertainty, singular events play a smaller role. Shares of the jump-induced variance of about 60% as in the Bitcoin market are observed in the crude oil market in very early stages only - prior to 1986. In that period the crude oil market is considered very immature and, thus, extreme price movements play a considerably large role. The results for the gold market are overall similar: the jump component captures large gold price movements very well. In addition, the share of jump-induced variance fluctuates around 40%, with a few declines to about 10%.

5 Discussion

It has already been highlighted that the Bitcoin market is characterised by a number of unique features. This section further elaborates on this and also proposes innovative economic interpretations. First, the total number of Bitcoins is fixed - there are only 21 million units. Second, ”all of the quantities and growth rates of Bitcoins are known with certainty by the public” (Yermack, 2013) and every single trade of Bitcoins is recorded in a publicly available database (Dwyer, 2015). Third, it is ensured that the growth rate

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8This result is very similar to Gronwald (2012).
of Bitcoins remains constant over time: if Bitcoin mining becomes more
atractive, e.g. through higher Bitcoin prices, the complexity of the crypt-
tographic puzzles adjusts accordingly. These rules have been designed in
advance by the developers of Bitcoin. They will remain unchanged over
time and have been established without the intervention of any regulator
(see Boehme et al, 2015). For some authors these features are be problem-
atic from an economic perspective: Yermack (2013), for instance, states the
following: "In the case of a 'wild success’ of Bitcoins and the replacement

Figure 5: Comparison: Crude oil (left panel) and gold prices (right panel).

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of sovereign fiat currency it would not be possible to increase the supply of
Bitcoins in concert with economic growth.” In the same vein, Lo and Wang
(2014) conclude that “some features of bitcoin, as designed and executed to
date, have hampered its ability to perform the functions required of a fiat
money.” In addition to these general concerns, various authors believe that
Bitcoin cannot function as a currency because of the large volatility, see e.g.
Baur and Dimpfl (2017).

This paper does not dispute this; however, also puts forward a different
interpretation. It is just because of these unique market features that this
market is a fascinating object of study. As asserted above, the total number
of Bitcoins, the number of Bitcoins in circulation and the growth rate are
known with certainty. These features are reminiscent of those of exhaustible
resource commodities such as crude oil and gold.\textsuperscript{9} There is, however, an
important difference. Observers of the crude oil market are familiar with the
following: OPEC announcements regarding their future oil production rates
are usually followed with bated breath. Whether or not OPEC countries will
adjust production generally has considerable effects. In other words, current
availability of crude oil is uncertain. The same applies to crude oil reserves
as well as crude oil resources: new discoveries, the development of new
technologies, and the price of crude oil itself will have an effect on the overall

\textsuperscript{9}Ironically, the list of similarities is not exhausted yet: the Bitcoin production process
is commonly referred to as "mining". This process requires the usage of powerful com-
puting equipment; in other words some production or extraction technology is required.
In addition, generating Bitcoin is associated with so-called "mining cost". Due to tech-
nological progress in the area of computer hardware and to changes in the difficulty of the
cryptographic puzzles, these cost are even changing over time. However, it should also be
noted that, other than gold or crude oil, Bitcoin has no intrinsic value. In addition, gold is
considered a safe haven.
amount of crude oil that is available. It does not need further explanation that also these factors will affect the price of crude oil. Similar conditions exists on other commodity markets. The supply of Bitcoin, in contrast, is not uncertain; the price movements however are even more extreme. The implication of this observation is that the observed price fluctuations and, thus, also the identified price jumps, can only be caused by demand-side factors. This is nothing other than the resource economic way to phrase Ali et al.’s (2014) notion that ”digital currencies have meaning only to the extent that participants agree that they have meaning.”

6 Conclusions

Virtual currencies are a phenomenon that has emerged only recently and Bitcoin is the certainly most famous one - in terms of both economic relevance and also interest it received from the general public and academia alike. Center stage so far in the academic analysis takes the question whether Bitcoin is a currency or an asset and under which motivation economic agents get involved in Bitcoins. The preliminary conclusion is that Bitcoins are to be considered an asset or speculative investment rather than a currency. Yermack (2013), most prominently, argues that the fixed number of Bitcoins is a severe economic problem as the supply of money would not be able to be adjusted in concert with economic growth. A small but steadily increasing number of papers also studied Bitcoin prices empirically, mainly with the focus on Bitcoin volatility (see Baek and Elbeck, 2014) and bubble behaviour (see Cheung et al., 2015 and Cheah and Fry, 2015). These
papers find that speculative activity is a major driver of Bitcoin prices. Yelowitz and Wilson’s (2015) analysis of Google searches for Bitcoin shows that “computer programming enthusiasts and illegal activity drive interest in Bitcoin”. Dowd and Hutchinson (2015), finally, come to a very drastic conclusion: "Bitcoin will bite the dust”.

Regardless of whether or not this is going to happen, the Bitcoin market is a fascinating object of study. Bitcoin, in specific, and virtual currencies in general only recently emerged and are associated with the emergence of a new tradable entity and a new market place. The price dynamics observed in this new market can certainly be described as spectacular and it is noteworthy that Bitcoin itself has been developed without involvement of any regulatory authority or support from the academic front. Thus, as the following discussion of spectacular price movements and newly developed markets illustrates, this is a unique situation. Among the earliest representatives is certainly the Dutch tulip mania 1634-1637. Regardless of whether the observed price movements are a bubble or are justified by economic fundamentals (see Garber, 1990), the noteworthy feature is that in the 17th century a modern market economy has not developed yet and, likewise, economic knowledge of market participants has not been very developed either. A similar assessment holds for the Mississippi as well as the South Sea Bubbles. Spectacular price movements are certainly also present in the market for crude oil. This market, however, is well established and most of the market participants are professionals, often with economic background. An example for another recently developed market is the European Union Emission Trading Scheme, a market for tradeable pollution permits. This market
has been designed by politicians and lawyers and is based on economic reasoning. Nevertheless price movements are spectacular, but however can be largely explained by the design of the market (Hintermann, 2010; Gronwald and Hintermann, 2015). Bitcoin, in contrast, is a newly emerged tradable entity, the overall economic environment is advanced, Bitcoin has been designed without involvement of regulatory authorities, market participants can be assumed to have at least certain understanding of markets, and Bitcoin has some unique features. Various authors point out that Bitcoin cannot function as money or currency, respectively. Reference is made to either the observed volatility or the market features of fixed supply. However, these types of conclusions leave open what Bitcoin actually is. This paper emphasises the similarities Bitcoin has with exhaustible commodity resources. As these are relatively well understood, both empirically and theoretically, this interpretation paves the way for the future analysis of Bitcoin.

The eye-catching price movements observed in this market certainly justify a thorough analysis. Some existing research in this area dealt with issues such as price volatility and price fundamentals. This paper contributes to this literature by conducting the first extensive analysis into the price dynamics of Bitcoin. It applies a number of linear and non-linear GARCH models which allow one to analyse asymmetric responses to positive and negative news as well as extreme price movements. The importance of the latter, in addition, can be studied over time and across markets. This paper finds that Bitcoin price dynamics are particularly influenced by extreme price movements. This influence is found to be larger than in the markets for crude oil and gold. Among the explanations for this is certainly the
immaturity of the market. The unique market features discussed in this paper, however, also imply that there is no uncertainty on the supply-side and, thus, all extreme price movements can only be driven by demand side factors.

References


This appendix presents detailed estimation results for oil and gold prices. The model comparison presented in Table A1 shows that the results are overall similar to Bitcoin. The GARCH(1,1) with student-\( t \) innovations as well as the jump models generally perform better than the remaining models. However, the relative improvement in performance appears to be smaller. In addition, there seems to be some evidence of leverage effects; see Table A2. The asymmetric models slightly outperform the basic models but cannot compete with the best performing models.

### Table A1: Model performance

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Oil LogL</th>
<th>Oil AIC</th>
<th>Oil BIC</th>
<th>Gold LogL</th>
<th>Gold AIC</th>
<th>Gold BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(1,1) (normal)</td>
<td>19,200.69</td>
<td>-4.8184</td>
<td>-4.8149</td>
<td>17,678.03</td>
<td>-6.4457</td>
<td>-6.4409</td>
</tr>
<tr>
<td>GARCH(1,1) (student-( t ))</td>
<td>19,461.97</td>
<td>-4.8838</td>
<td>-4.8794</td>
<td>17,957.54</td>
<td>-6.5472</td>
<td>-6.5412</td>
</tr>
<tr>
<td>IGARCH</td>
<td>19,140.18</td>
<td>-4.8038</td>
<td>-4.8020</td>
<td>17,642.46</td>
<td>-6.4334</td>
<td>-6.4310</td>
</tr>
<tr>
<td>EGARCH</td>
<td>19,216.70</td>
<td>-4.8222</td>
<td>-4.8178</td>
<td>17,689.24</td>
<td>-6.4494</td>
<td>-6.4434</td>
</tr>
<tr>
<td>TGARCH</td>
<td>19,201.25</td>
<td>-4.8183</td>
<td>-4.8140</td>
<td>17,685.60</td>
<td>-6.4481</td>
<td>-6.4420</td>
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<tr>
<td>Constant JI</td>
<td>19,400.00</td>
<td>-4.8702</td>
<td>-4.8640</td>
<td>17,925.29</td>
<td>-6.5348</td>
<td>-6.5263</td>
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<tr>
<td>ARJI</td>
<td>19,438.87</td>
<td>-4.8788</td>
<td>-4.8709</td>
<td>17,932.84</td>
<td>-6.5356</td>
<td>-6.5247</td>
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</table>
Table A2: Constant and time-varying jump-intensity models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Oil</th>
<th>Gold</th>
<th>Oil</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH</td>
<td>GARCH</td>
<td>IGARCH</td>
<td>TGARCH</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.9E-04 (0.3134)</td>
<td>4.6E-04 (0.0201)</td>
<td>1.3E-04 (0.3967)</td>
<td>1.4E-04 (0.5082)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>5.8E-06 (0.0001)</td>
<td>5.1E-06 (0.0001)</td>
<td>5.8E-06 (0.0001)</td>
<td>5.1E-06 (0.0001)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.0908 (0.0001)</td>
<td>0.0685 (0.0001)</td>
<td>0.0618 (0.0001)</td>
<td>0.0853 (0.0001)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.9049 (0.0001)</td>
<td>0.9245 (0.0001)</td>
<td>0.9382 (0.0001)</td>
<td>0.9382 (0.0001)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>-</td>
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<tr>
<td>( \kappa )</td>
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<td>( \delta )</td>
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<tr>
<td>( \theta )</td>
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<tr>
<td>( \lambda )</td>
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<tr>
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<tr>
<td>( \gamma )</td>
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</tr>
</tbody>
</table>

Note: p-values in parentheses. Number of endogenous lags as well as inclusion of constant is based on standard information criteria as well as significance of parameters. The \( \psi \)/k row contains the leverage parameters.