Pollution in a globalized world: Is the decrease of debt in developing countries a solution?

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Abstract

This article analyzes the impacts of debt relief on production and pollution when countries are characterized by differences in their technology and by heterogeneous preferences, through the discount factors and the environmental sensitivities. We develop a two-country overlapping generations model with environmental externalities, public debts and perfect mobility of assets. GHG emissions arise from production, but agents may invest in private mitigation to abate pollution. The overall debt level remaining unchanged, we consider a decrease of the debt of the poor country balanced by an increase of the richer country's debt. We show that debt relief for polluting poor countries, characterized by a low productivity, makes it possible to engage these countries in the process of pollution abatement if they are sufficiently sensitive to environmental quality. Capital in both countries can even increase. We conclude that debt crisis in the richest countries should not compromise foreign aid.

JEL classification: F43, H23, Q56.

Keywords: Pollution; Abatement; Overlapping generations; Public debt; Capital market integration.

1 Introduction

The trend in CO2 emissions predict a global temperature increase of more than 3°C by the end of the century. The consequences will be severe: loss of agricultural yield, loss of biodiversity, sea level rise, climate migration, extreme weather events... are all phenomena that will increase. Poor countries will probably bear the brunt of the economic damages (Schelling, 1992; Mendelsohn, et al., 2006). The environmental challenges faced by poor countries are compounded by high

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levels of external economic vulnerability and public debt. They recurrently use public debt to absorb the impact of external shocks and natural disasters. In turn, higher levels of public debt constrain their capacity to address vulnerabilities.

The Kyoto Protocol (1997) and the Paris Aggreement (2016) failed to define strong and efficient commitments to reduce GHG emissions. Mitigation is costly, participation in a climate treaty is voluntary and compliance is difficult to enforce. How to design the best policy is still an open question and we are still searching for new instruments to accompany environmental policies. Treaties with deep interdependency between trade and environmental commitments could be a remedy (Nordhaus, 2015), but the rise of protectionism calls into question the effectiveness of this proposition.

The objective of this article is to participate in the debate on the alternative policies to be implemented to encourage all countries to engage in the fight against climate change. In a general equilibrium framework, we propose a mechanism linking public debt and pollution mitigation. We show that debt relief for polluting and poor countries makes it possible to engage these countries in the process of pollution abatement. The overall debt level remains unchanged, with budget support being provided by richer countries. This result is confirmed even when the aid donor country faces high level of debt, as are most Noth-American and Western European countries. It requires a sufficient sensitivity of environmental quality in the poor countries. Depending on the preferences of the different countries, this policy can even induce a higher level of capital in all countries.

The issues of policies to fight climate change and debt burden in a globalised world have been the subject of much works. Many fields of the economic literature have addressed this issue; our paper contributes to two strands, namely the interplay between trade, development and the environment, and the links between public debt and environmental quality.

Following the works of Baumol and Oates (1988), Grossman and Krueger (1995) and Coppeland and Taylor (2004), the trade literature concludes that free trade may generate economic benefits, but it may also exacerbate pollution. The consequences on economic growth and pollution result from the confrontation of three effects (technique, composition and scale), but without any empirical consensus on the respective size of each channel. Coppeland and Taylor (1995) find that free trade raises world pollution if incomes differ substantially across countries. International income transfers may not affect world pollution, but untied international transfers of income lower the recipient's pollution but raise the donor's. Coppeland and Taylor (1994) show in addition that an increase in the rich North's production possibilities increases pollution, while similar growth in the poor South lowers pollution, and unilateral transfers from North to South reduce worldwide pollution. We contribute to this literature by taking into account endogenous mitigation and debt transfer between rich and poor countries. Doing so, we find conditions such that pollution decreases while production increases, in both countries, conversely to above studies.

Development economic literature has focused on the consequences of international transfers, to see if it provides development support for indebted countries. While direct aid to poor countries are often criticized (see Burnside and Dollar (2000), Boone (1996), Dreher and Langlotz (2017), Rajan and Subramanian (2006), among the others), Cassimon et al. (2015) and Channing et al. (2016) conclude that the gains from debt cancellation promote health spending, public investment, and tax mobilization within the recipient countries. Regarding climate change issues, we give conditions such that untied debt cancellation promotes also environmental quality and economic growth world wild.

This literature has also questioned the characteristics of international transfers to protect the environment, to see whether donator countries should prioritize technology transfers oriented towards mitigation or adaptation. Sakamoto, et al. (2017) show that financial aid to enhance adaptation capacity of vulnerable countries is efficient. They find that North's assistance to South can facilitate pollution mitigation in both regions. Barañano and San Martín (2015) develop a dynamic equilibrium model in which public investments in both infrastructure and pollution abatement can be financed using international aid. They find that transfers linked to both infrastructure and pollution abatement may be the best welfare-enhancing alternative. In our article, we limit aid tranfers to untied debt cancellation while technology against GHG emission corresponds to mitigation. Our work differs from these contributions by highlighting the role of private agents in mitigation decisions, which depend on individual's preferences and net income.

The second part of the literature focuses on the interactions between public debt and the environment. Debt financing has been introduced in dynamic models with environmental concerns (Bovenberg and Heijdra (1998); Heijdra et al. (2006), Fernandez et al. (2010)). Debt policy only makes possible to redistribute welfare gains from future to existing generations: there is no debt financing of the pollution mitigation sectors. Fodha and Seegmuller (2012, 2014) and Fodha et al. (2018) analyze debt financing schemes for public and private mitigation. They show that in some cases, poverty-trap may exist, but efficient environmental tax reform may be designed, under specific conditions on abatement technologies and capital intensities. These models consider closed economy which is obviously an important limitation as capital markets are interconnected and globalized, as well as the climate.

Two papers consider environmental quality as a global externality. The first is the work of Bednar-Friedl et al. (2010). They model an overlapping generations economy of two industrialized countries, interconnected through free trade in commodities and in bonds emitted by governments. Countries differ in their levels of public debt per capita such that one country is a net creditor and the other one is a net debtor to the world economy. They show that when one country unilaterally reduces her cap on emissions, her output available for domestic and foreign consumption diminishes more than in the other country. The net welfare effect is negative in both countries. While, in this work, mitigation efforts are exogenous, we differ by taking into account endogenous mitigation expenditure. We study in details situations in which one country makes no effort to reduce its net emissions.

Second, Müller-Fürstenberger and Schumacher (2017) propose a dynamic model where all agents contribute to a global externality, but only those in a specific region suffer from it. They develop an overlapping generations model with two types of agents. In the non integrated economy, even though agents have the same technologies, if agents affected by the externality are sufficiently poor in terms of initial capital endowments, they may be stuck in this trap. Capital market integration eliminates the environmental poverty trap. This article, although close to our work, considers heterogeneity in the consequences of the climate change while we are interested in heterogeneity in preferences. Moreover, in addition to the integrated capital market assumption, we add public debt that influences savings and capital accumulation.

We develop an overlapping generations (OLG) model with pollution externalities and private mitigation. Production deteriorates the environmental quality, harming the welfare of future generations. We assume that there is perfect market for goods and capital stock, with perfect mobility between countries. Assets' supply is endogenous, determined by the savings choices of individuals. On the other hand, labour mobility is excluded, which exacerbates income differences between countries. Both countries pay interest on past debt by levying a tax on agents' income. In accordance with the empirical literature, we consider that countries lagging behind in technology tend not to engage in the fight against climate change, and public debt can be an obstacle. Then, we focus on the consequences of debt transfers on mitigation decision of agents, as well as on pollution and well-being.

Three important ingredients should be emphasized. First, debt is an obstacle to private abatement, because in countries where technology, and therefore income, is low, the environment may not be a priority. Lowering debt reduces taxation and increases net income, which in turn increases investment in private mitigation. Second, we do not consider debt conversion. Indeed, in the case of debt for climate swap for instance, the debt is not reduced, it is maintained, but rather than being repaid to the creditor country, it is used to protect the environment. Finally, we especially focus on debt transfers allowing poor countries to engage in environmental maintenance. We show that this may be inn accordance with an increase of caoital in all countries.

The rest of this paper is organized as follows. Section 2 presents the two-country OLG model with environmental externalities. Section 3 defines the intertemporal equilibrium and examines the properties of the steady states. Section 4 studies the situations where agents in the poor country have a weak environmental sensitivity. Section 5 presents the opposite configuration. Technical details are relegated to an Appendix.

2 The model

The world consists of two competitive economies indexed by $i \in \{D, F\}$. Within each country, a new generation of two-period lived agents is born at each period of time. Therefore, two generations alive in each period t: the workers and the retired people. In each country, the population size of the generation is constant and normalized to one. There is no mobility of labor between countries, whereas there is a perfect mobility of the assets and the final good.

2.1 Environmental quality

We consider the environment as a global public good, such as climate. We measure the evolution of this aggregate by an index, which deteriorates with pollution from global production and improves with private abatement. We assume that both the rates of pollutant emissions and the rates of abatement are equal among countries. In addition, we neglect natural absorption, so the environment is a stock whose accumulation varies with net pollution. Private mitigation corresponds, for example, to carbon sinks such as reforestation or investments in carbon capture and sequestration. Global environmental quality E_t evolves according to:

$$E_{t+1} = E_t - \theta(y_{Dt} + y_{Ft}) + \phi(a_{Dt} + a_{Ft}) \tag{1}$$

where y_{it} and a_{it} represent production and abatement of country i respectively. The pollution flow resulting from production is given by the emission factor $\theta > 0$ and efficiency of abatement is given by factor $\phi > 0$.

2.2 Firms

In each country, producers use capital and labor for the production of a unique final good, which is the numeraire. The technology used is Cobb-Douglas. Taking into account that labor is unit, the production function writes $y_{it} = A_i k_{it}^{\alpha}$, where k_{it} denotes capital, $A_i > 0$ the global productivity and $\alpha \in (0, 1)$ the capital share in income. Profit maximization gives:

$$w_{it} = (1 - \alpha) A_i k_{it}^{\alpha} \tag{2}$$

$$R_{it} = \alpha A_i k_{it}^{\alpha - 1} \tag{3}$$

where w_{it} the wage and R_{it} the return of capital in country i.[1]

2.3 Households

A generation born at period t derives utility form consumption when young c_{it} and old d_{it+1} and from environmental quality at both periods. Accordingly, the lifetime utility is given by:

$$\ln c_{it} + \beta_i \ln d_{it+1} + \delta_i \ln E_t + \gamma_i \ln E_{t+1} \tag{4}$$

where $\beta_i \in (0, 1)$ denotes the discount factor in country i, and $\delta_i \geqslant 0$ and $\gamma_i \geqslant 0$ the sensitivity to environmental quality when young and old respectively. We assume that these preference parameters are country specific, which means that $\beta_D \neq \beta_F$, $\delta_D \neq \delta_F$ and $\gamma_D \neq \gamma_F$. On the one hand, eterogeneity between β_i is empirically supported by the work of Wang, et al. (2016), which present results from a large-scale international survey on time preference, conducted in 53

[1] For simplification and taking into account the duration of a period, we assume full depreciation of capital after one period of use.

countries, and find that the waiting tendency is correlated with country specific characteristics, like innovation, environmental protection, and crediting rating. On the other hand, δ_i and γ_i aggregate both sensitivity and vulnerability to climate change which are also different among rich and poor countries (Schelling, 1992; Mendelsohn, et al., 2006).

When young, each agent inelastically supplies one unit of labor and receives real wage w_{it}. A lump-sum tax τ_{it} is levied on this income, which is shared between consumption c_{it}, savings s_{it} and private abatement a_{it}. Indeed, young people take care about the environmental quality they will face when old, but suffer from the past accumulation of pollution. Consumption when old d_{it+1} is entirely financed by the remunerated savings. Therefore, the two budget constraints faced by an agent born at period t write:

$$c_{it} + s_{it} + a_{it} = w_{it} - \tau_{it} \tag{5}$$

$$d_{it+1} = R_{it+1} s_{it} \tag{6}$$

A young agent maximizes her utility (4) taking into account the two budget constraints (5) and (6), the environmental quality (1) and the non-negativity of abatement $a_{it} \geqslant 0$. We obtain:

$$d_{it+1} = \beta_i R_{it+1} c_{it} \tag{7}$$

$$E_{t+1} \geqslant \gamma_i \phi c_{it}, \text{ with an equality when } a_{it} > 0 \tag{8}$$

We deduce that we have two main situations. Either $a_{it} > 0$ and:

$$s_{it} = \frac{\beta_i}{\gamma_i \phi} E_{t+1}, \quad c_{it} = \frac{E_{t+1}}{\gamma_i \phi}, \quad d_{it+1} = \frac{\beta_i}{\gamma_i \phi} R_{it+1} E_{t+1} \tag{9}$$

$$a_{it} = w_{it} - \tau_{it} - \frac{1+\beta_i}{\gamma_i \phi} E_{t+1} \tag{10}$$

or $a_{it} = 0$ and:

$$s_{it} = \frac{\beta_i}{1+\beta_i}(w_{it} - \tau_{it}), \quad c_{it} = \frac{1}{1+\beta_i}(w_{it} - \tau_{it}) \tag{11}$$

$$d_{it+1} = \frac{\beta_i}{1+\beta_i} R_{it+1}(w_{it} - \tau_{it}), \quad E_{t+1} > \frac{\gamma_i \phi}{1+\beta_i}(w_{it} - \tau_{it}) \tag{12}$$

When environmental quality is high enough with respect to the net income, an agent has no incentive to mitigate pollution. In contrast, if the net income is high enough and environmental quality too low, she engages in abatement activities. The latter case is all more likely as γ_i is high.

2.4 Government

The evolution of public debt in advanced and emerging countries and its consequences, especially concerning economic growth, is a major economic concern. Public debt issues are also included in international debates on the environment, in which the connection between public debt and the environment is recursively discussed. To keep analysis simple, we do not formalize public spending for environmental protection. Rather, we focus on interactions between public debt and the environment in an international context by assuming that final action to reduce global pollution is private. In this way, the government cannot *directly* use debt to finance future environmental quality and the direct link between public debt and environmental policy is ignored.

In each country, the government faces the following budget constraint:

$$b_{it+1} = R_{it} b_{it} - \tau_{it} \tag{13}$$

with the initial public debt level $b_{i0} \geqslant 0$ given. The government collects lump sum tax τ on workers and uses bond as debt instrument. Its expenditures include repayment of debt and interest payments.

3 Equilibrium with perfect mobility of the final good and assets

As international capital mobility is assumed to be perfect, foreign and domestic assets yield the same rate of return. Market clearing requires world savings equal to world investment:

$$R_{Dt} = R_{Ft} \tag{14}$$

$$s_{Dt} + s_{Ft} = k_{Dt+1} + b_{Dt+1} + k_{Ft+1} + b_{Ft+1} \tag{15}$$

And given output function, the law of motion for environmental quality is given by:

$$E_{t+1} = E_t - \theta(A_D k_{Dt}^{\alpha} + A_F k_{Ft}^{\alpha}) + \phi(a_{Dt} + a_{Ft}) \tag{16}$$

and $E_t > 0$. Using (3), equation (14) means that:

$$k_{Ft} = k_{Dt} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \tag{17}$$

We will investigate policies where debt in both countries are exogenous policy parameters, i.e. $b_{Dt} = b_D$ and $b_{Ft} = b_F$ for all t, while τ_{Dt} and τ_{Ft} are endogenous and satisfy the government budgets. In this way, public debt is a policy variable. We will in particular analyze the effect of an increase of b_D which allows to decrease b_F. Equation (13) becomes:

$$\tau_{it} = (\alpha A_i k_{it}^{\alpha-1} - 1) b_i \tag{18}$$

We consider a domestic economy richer than the foreign one. Numerous studies emphasize that the major part of the difference in incomes between rich and poor countries is due to differences in total factor productivity (see the survey by Caselli (2005) and the study by Hsieh and Klenow

(2010)). In addition, government debt data reveals that national debt levels are higher in rich countries than in emerging and developing economies (see IMF World Economic Outlook). To be consistent with these evidences we assume:

Assumption 1 $A_D > A_F$, $b_D > b_F$.

Note that debt over GDP is larger in country D if $b_D/y_D > b_F/y_F$. Using (17), this is equivalent to $b_D/A_D^{\frac{1}{1-\alpha}} > b_F/A_F^{\frac{1}{1-\alpha}}$. Therefore, the ranking of debt over GDP among countries also depend of the productivities in each country.

We focus on equilibria with $a_{Dt} > 0$ and either $a_{Ft} > 0$ or $a_{Ft} = 0$. Equilibria without any maintenance activities are excluded from our analysis while situations in which only one of the country do not invest in environmental protection are examined.

3.1 Equilibrium with $a_{Dt} > 0$ and $a_{Ft} = 0$

We focus here on an asymmetric equilibrium where the domestic households are engaged into maintenance, but foreigners do no maintenance. This means that:

$$\frac{\phi\gamma_D}{1+\beta_D}\left[(1-\alpha)A_D k_{Dt}^{\alpha} - \tau_{Dt}\right] > E_{t+1} > \frac{\phi\gamma_F}{1+\beta_F}\left[(1-\alpha)A_F k_{Dt}^{\alpha}\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}} - \tau_{Ft}\right] \tag{19}$$

Using (2), (11), (10) and (17), equilibrium on the asset market (15) satisfies:

$$\begin{aligned} k_{Dt+1}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] &- \frac{\beta_D}{\gamma_D\phi}E_{t+1} + b_{Dt+1} + b_{Ft+1} \\ &= \frac{\beta_F}{1+\beta_F}\left[(1-\alpha)A_F k_{Dt}^{\alpha}\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}} - \tau_{Ft}\right] \end{aligned} \tag{20}$$

Substituting (2), (10), (17) and $a_{Ft} = 0$, equation (16) becomes:

$$\left(1+\frac{1+\beta_D}{\gamma_D}\right)E_{t+1} = E_t - \phi\tau_{Dt} + A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha) - \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right] \tag{21}$$

An equilibrium with $a_{Dt} > 0$ and $a_{Ft} = 0$ satisfies equations (20) and (21), taking into account that inequalities (19) hold and the government budget constraint is given by (18).

3.2 Equilibrium with $a_{Dt} > 0$ and $a_{Ft} > 0$

Using inequality (12), we deduce that $a_{Dt} > 0$ and $a_{Ft} > 0$ require:

$$E_{t+1} < \phi \min\left\{\frac{\gamma_D}{1+\beta_D}\left[(1-\alpha)A_D k_{Dt}^{\alpha} - \tau_{Dt}\right]; \frac{\gamma_F}{1+\beta_F}\left[(1-\alpha)A_F k_{Dt}^{\alpha}\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}} - \tau_{Ft}\right]\right\} \tag{22}$$

Using (9) and (17), the equilibrium condition (15) rewrites:

$$\frac{1}{\phi} E_{t+1} \left(\frac{\beta_D}{\gamma_D} + \frac{\beta_F}{\gamma_F} \right) = k_{Dt+1} \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] + b_{Dt+1} + b_{Ft+1} \tag{23}$$

Now, using (2), (10) and (17), the dynamics of environmental quality (16) becomes:

$$\begin{aligned} &\left(1 + \frac{1+\beta_D}{\gamma_D} + \frac{1+\beta_F}{\gamma_F} \right) E_{t+1} = E_t - \phi(\tau_{Dt} + \tau_{Ft}) \\ &+ A_D k_{Dt}^{\alpha} \left[\phi(1-\alpha) - \theta \right] \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \end{aligned} \tag{24}$$

An equilibrium with $a_{Dt} > 0$ and $a_{Ft} > 0$ satisfies equations (23) and (24), taking into account the government budget constraint (18) and inequalities (22).

4 A foreign economy with low environmental sensitivity (γ_F arbitrarily small)

We consider first a foreign economy with green awareness lower than in the domestic economy, and even arbitrarily small and close to zero.

Assumption 2 *γ_F arbitrarily small, satisfying $\gamma_F < \gamma_D$, and close to 0.*

Substituting (18) in (19), the economy is in the asymmetric case $a_{Dt} > 0$ and $a_{Ft} = 0$ if the following inequalities are satisfied:

$$E_{t+1} < \frac{\phi \gamma_D}{1+\beta_D} \left[(1-\alpha) A_D k_{Dt}^{\alpha} - b_D \left(\alpha A_D k_{Dt}^{\alpha-1} - 1 \right) \right] \tag{25}$$

$$E_{t+1} > \frac{\phi \gamma_F}{1+\beta_F} \left[(1-\alpha) A_D k_{Dt}^{\alpha} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} - b_F \left(\alpha A_D k_{Dt}^{\alpha-1} - 1 \right) \right] \tag{26}$$

Under Assumption 2, we note that any steady state (k_D, E) will satisfy equation (26). Therefore, there will be no steady state with $a_F > 0$ when γ_F is small and close to zero. This explain that when we assume that Assumption 2 is satisfied, we restrict our attention to equilibria such that $a_{Dt} > 0$ and $a_{Ft} = 0$.

From equation (16), we see that a stationary equilibrium cannot be achieved when both countries do not engage in mitigation. Indeed, in this case, the net pollution is negative at each date, leading to a perpetual decrease in the quality of the environment.

4.1 Dynamics and steady states

Substituting (18) in equations (20) and (21), we obtain:

$$
\begin{aligned}
& k_{Dt+1}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]-\frac{\beta_D}{\gamma_D\phi}E_{t+1}+b_D+\frac{b_F}{1+\beta_F} \\
& =\frac{\beta_F}{1+\beta_F}\left[(1-\alpha)A_F k_{Dt}^{\alpha}\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}}-b_F\alpha A_D k_{Dt}^{\alpha-1}\right]
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
& \left(1+\frac{1+\beta_D}{\gamma_D}\right)E_{t+1}=E_t-\phi b_D\left[\alpha A_D k_{Dt}^{\alpha-1}-1\right] \\
& +A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)-\theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]
\end{aligned}
\tag{28}
$$

Substituting (28) into (27), we get:

$$
\begin{aligned}
& k_{Dt+1}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]=-\frac{b_D(1+\gamma_D)}{1+\beta_D+\gamma_D}-\frac{b_F}{1+\beta_F}+\frac{\beta_D}{1+\beta_D+\gamma_D}\frac{E_t}{\phi} \\
& +k_{Dt}^{\alpha}\left[(1-\alpha)A_F\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_F}{1+\beta_F}+A_D\left(1-\alpha-\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right)\frac{\beta_D}{1+\beta_D+\gamma_D}\right] \\
& -\alpha A_F k_{Dt}^{\alpha-1}\left[\frac{b_F\beta_F}{1+\beta_F}+\frac{b_D\beta_D}{1+\beta_D+\gamma_D}\right]
\end{aligned}
\tag{29}
$$

The dynamic system (28) and (29) explicitly gives (E_{t+1}, k_{Dt+1}) as functions of (E_t, k_{Dt}). Now, we use it to analyse the existence and the number of steady states, and their stability properties.

Using (28), we deduce that $E_{t+1} \geqslant E_t$ if and only if:

$$
\begin{aligned}
& E_t \leqslant \varphi(k_{Dt}) \equiv \\
& \left\{A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)-\theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]-\phi b_D\left[\alpha A_D k_{Dt}^{\alpha-1}-1\right]\right\}\frac{\gamma_D}{1+\beta_D}
\end{aligned}
\tag{30}
$$

To be able to ensure $E_t > 0$ even if there is under-accumulation of capital ($\alpha A_D k_{Dt}^{\alpha-1} > 1$), we assume:

Assumption 3 $\phi(1-\alpha) > \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)$.

This assumption is not so restrictive. Indeed, it can be rewritten as $\phi w_D > \theta(y_D + y_F)$: when the rich country spends all its income on mitigation, it has to absorb more than the total flow of pollution.

Using now (29), the inequality $k_{Dt+1} \gtrless k_{Dt}$ is equivalent to $E_t \gtrless \psi(k_{Dt})$, with:

$$\frac{\beta_D}{1+\beta_D+\gamma_D}\frac{1}{\phi}\psi(k_{Dt}) = k_{Dt}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] + \frac{b_D(1+\gamma_D)}{1+\beta_D+\gamma_D} + \frac{b_F}{1+\beta_F}$$
$$- k_{Dt}^{\alpha}\left[(1-\alpha)A_F\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_F}{1+\beta_F} + A_D\left(1-\alpha-\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right)\frac{\beta_D}{1+\beta_D+\gamma_D}\right]$$
$$+ \alpha A_F k_{Dt}^{\alpha-1}\left[\frac{b_F\beta_F}{1+\beta_F} + \frac{b_D\beta_D}{1+\beta_D+\gamma_D}\right] \tag{31}$$

Now, using (28), inequalities (25) and (26) are equivalent to:

$$E_t < A_D k_{Dt}^{\alpha}\left[\frac{\phi\gamma_D}{1+\beta_D}(1-\alpha) + \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right] - \left(\alpha A_D k_{Dt}^{\alpha-1} - 1\right)\frac{\phi\gamma_D b_D}{1+\beta_D} \equiv C_D(k_{Dt}) \tag{32}$$

$$E_t > -A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)\left(1-\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\frac{(1+\beta_D+\gamma_D)\gamma_F}{(1+\beta_F)\gamma_D}\right) - \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]$$
$$+ \left(\alpha A_D k_{Dt}^{\alpha-1} - 1\right)\phi\left[b_D - b_F\frac{(1+\beta_D+\gamma_D)\gamma_F}{(1+\beta_F)\gamma_D}\right] \equiv C_F(k_{Dt}) \tag{33}$$

where $C_D(k_{Dt})$ is increasing and concave and $C_F(k_{Dt})$ is decreasing and convex. Moreover, $C_D(k_{Dt}) > \varphi(k_{Dt})$ for all $k_{Dt} > 0$, which means that a steady state satisfies $a_D > 0$ whatever the value of k_D. As shown in Appendix 7.1, there is $\underline{k} > 0$ such that for all $k_{Dt} > \underline{k}$, we have $C_D(k_{Dt}) > \varphi(k_{Dt}) > C_F(k_{Dt})$. We also shown whether $\psi(k_{Dt}) \gtrless C_F(k_{Dt})$.

To complete the picture, a steady state is a solution $(E_t, k_{Dt}) = (E_{t+1}, k_{Dt+1}) = (E, k_D)$ satisfying equations (27) and (28):

$$\frac{\beta_D}{\gamma_D\phi}E = k_D\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]$$
$$+ b_D + \frac{b_F}{1+\beta_F} - \frac{\beta_F}{1+\beta_F}\left[(1-\alpha)A_F k_D^{\alpha}\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}} - b_F\alpha A_D k_D^{\alpha-1}\right] \tag{34}$$
$$\frac{1+\beta_D}{\gamma_D}E = -\phi b_D\left[\alpha A_D k_D^{\alpha-1} - 1\right] + A_D k_D^{\alpha}\left[\phi(1-\alpha) - \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right] \tag{35}$$

This allows us to deduce the number of steady states depending of the importance of the policy parameters b_D and b_F (see also Figure 1 for the case with two steady states):

Proposition 1 *Under Assumptions 1-3, there exist two steady states with $a_D > 0$ and $a_F = 0$, namely (k_j, E_j) with $j = \{l, h\}$ and $0 < k_l < k_h$, if public debt (at least in one country) is positive, but b_D and b_F are low enough. If public debt (in one or the two countries) is too large, there is no steady states with $a_D > 0$ and $a_F = 0$.*

If we further assume that:

$$\frac{\alpha}{1-\alpha} > \frac{\frac{\beta_D}{1+\beta_D} + \frac{\beta_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}} \tag{36}$$

the two existing steady states are characterized by under-accumulation.

Proof. See Appendix 7.2 ■

Proposition 1 shows that there are two steady states with $a_D > 0$ and $a_F = 0$ when the public debt in both countries is positive but not too high. Using the ingredients of this section and Appendix 7.1, we can also deduce the stability properties of the steady states and draw Figure 1. The steady state with the highest level of capital is stable, whereas the one with the lowest level of capital is a saddle in an environment with two predetermined variables. Therefore, it is generically unstable.

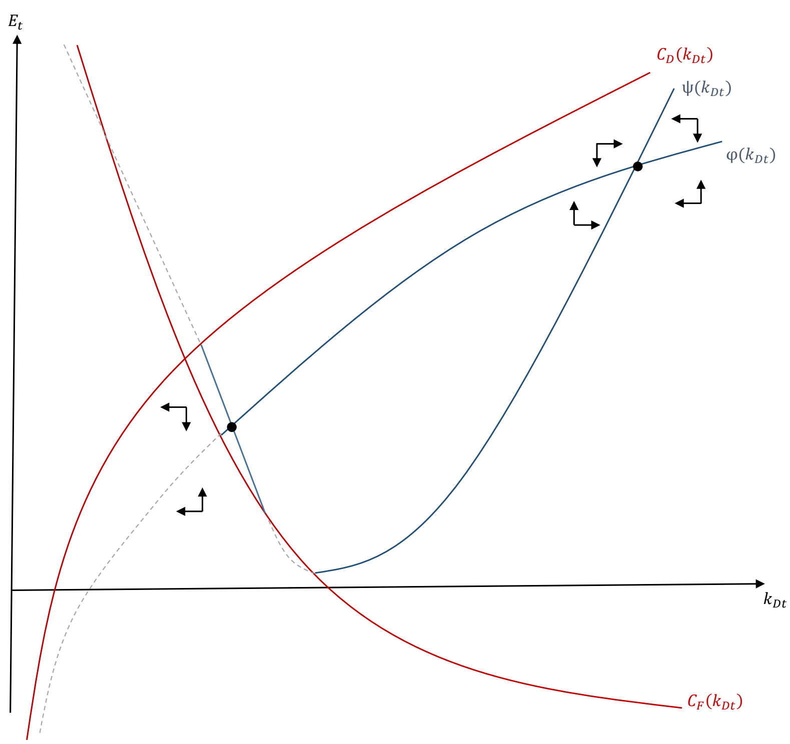

Figure 1 – *Global dynamics under γ_D arbitrarily small.*

As generations overlap, agents may over- or under-accumulate capital relatively to the Pareto optimum. An exogenous (policy) shock may thus have different implications according to the dynamic efficiency properties of the economy. However, the sufficient condition to have under-accumulation seems to be quite standard since it requires that capital income over labor income has to be higher than a weighted sum of the saving rates $\beta_i/(1+\beta_i)$.

4.2 Policy implications

This section explores how the welfare of consumers could be affected by changes of the level of public debt in both countries. Considering that Proposition 1 hold, we investigate various policies.

Proposition 2 *Under Assumptions 1-3 and inequality (36), we have:*

1. *Following an increase of b_D and/or b_F, k_l increases while k_h decreases;*
2. *If $\beta_D > \beta_F$, following an increase of b_D associated to a decrease of b_F of the same amount, k_l increases and k_h decreases. Following a decrease of b_D associated to an increase of b_F of the same amount, k_l decreases and k_h increases. If $\beta_D < \beta_F$, we get exactly the opposite.*

Proof. See Appendix 7.3 ■

Any increase of the public debt in one of the country has a crowding-out effect at the international level. Indeed, a larger level of debt, means that a larger share of global saving is devoted and less capital in the global economy. Since capital mobility is perfect, capital in the tow countries evolve in the same direction, which means that capital in the two countries reduces.

When debt transfer from domestic to foreign country entails a decrease in k_l and an increase in k_h, both economies can converge to a higher stock of capital and the size of the trap is reduced.

Then, we investigate the welfare effect of debt transfers, *i.e* an increase of b_D associated to a decrease of b_F of the same amount or reversely, in the case in which the economy achieves a stationary state with a high stock of capital k_h. The stationary agents' welfare -indirect utility- is given by the following expression:

$$V_i = (1+\beta_i)\ln(c_i) + \beta_i \ln(R_i) + (\delta_i + \gamma_i)\ln(E_h) + \beta_i \ln \beta_i \tag{37}$$

Debt policy affects welfare though its impact on the environment and on consumption spendings. Examining the effect of debt transfers on the environmental quality E we obtain these results:

Proposition 3 *Under Assumptions 1-3 and inequality (36):*

- *When the domestic economy is more patient that the foreign one, $\beta_D > \beta_F$, an increase of b_D associated to a decrease of b_F of the same amount reduces E_h. We get the reverse result following an increase of b_F associated to a decrease of b_D.*
- *When the domestic economy is less patient that the foreign one, $\beta_D < \beta_F$, an that*[2]

$$\mathcal{X} > \frac{(1+\beta_D)(1+\beta_F)}{b_D(\beta_F - \beta_D)} \tag{38}$$

[2]The expression for $\mathcal{X}$ is given in Appendix 7.3

an increase of b_D associated to a decrease of b_F of the same amount increases E_h. We get the reverse result following an increase of b_F associated to a decrease of b_D.

Proof. See Appendix 7.3 ■

A policy that consists in transferring an amount of debt from the patient to the less patient country leads economies to a state with a lower stock of capital. As a result, it reduces labor income and the environmental contribution of the domestic economy. Given the efficiency of abatement compare to the polluting intensity of production (Assumption 3), a fall in capital stock damages the environment. This is why a debt transfer from the domestic to the foreign economy damages environmental quality when $\beta_D > \beta_F$. We have the reverse mechanisms when the transfer is from the foreign to the domestic economy.

When the foreign economy is more patient ($\beta_D < \beta_F$) an additional condition is required to ensure that a debt transfer from the domestic to the foreign economy is efficient to improve the environment. Indeed, the domestic economy is the only contributor to environmental protection and a policy aiming at increase its debt level would entail direct negative income effect, with adverse consequences on abatement activities. The condition (38) presented in Proposition 3 implies that the elasticity of the capital stock to debt variation is sufficiently high to overtake the direct effect of debt variation on abatement spendings.

The consumption part of the welfare is affected by debt transfer through three effects: First, a direct debt effect, because any variation in the level of debt changes the tax on income. Second, an income effect because the variation of capital stock entails by the policy modifies the return on labor and on capital. Finally, a cost of debt effect because of the variation in the interest rate and hence in the country's debt burden. These effects can be competing.

Considering the impact of the policy on the environment and consumption spendings we have the following Proposition:

Proposition 4 *Under Assumptions 1-3, inequality (36), and* $\max\left\{\frac{\beta_D}{1+\beta_D};\frac{\beta_F}{1+\beta_F}\right\} < (1-\alpha)/\alpha$, *we have the following results:*

- *For $\beta_D > \beta_F$:*
  - *An increase of b_D associated to a decrease of b_F of the same amount reduces the welfare in the domestic country while the impact is ambiguous for the foreign one.*
  - *An decrease of b_D associated to an increase of b_F of the same amount increases the welfare in the domestic country while the impact is ambiguous for the foreign one.*
- *For $\beta_D < \beta_F$ and under condition (38):*
  - *An increase of b_D associated to a decrease of b_F of the same amount increases the welfare in the foreign country while the impact is ambiguous for the domestic one.*
  - *An decrease of b_D associated to an increase of b_F of the same amount decreases the welfare in the foreign country while the impact is ambiguous for the domestic one.*

Proof. See Appendix 7.3 ■

When the foreign economy is impatient relative to the domestic one, a debt transfer in its favor leads economies to converge to a lower stock of capital and to a worsen environmental quality. Nevertheless, if the debt reduction is sufficiently high, these negative effects could be compensated in the foreign country by the direct increase in consumption spendings. Reversely, when the foreign country increases its debt to reduce those of the domestic (patient) country, economies converge to a higher stock of capital and to a better environmental quality. Nevertheless, these positive effects can be accompanied by a fall of welfare in the foreign economy if they are overtaken by the negative direct effect of the debt increase on consumption spendings.

We get the opposite result if the foreign economy is relatively more patient and that the elasticity of the capital stock to debt is sufficiency high to ensure that capital stock variation following a debt transfer drives the variation of the environmental quality. More precisely, capital stock, environmental quality and welfare improve in the foreign economy if it benefits from a debt transfer while the impact remains unclear for the domestic economy because it suffers from an increase in its debt.

According to Proposition 1, when γ_F is arbitrarily small, the two steady states are always characterized by the same configuration, i.e domestic country contributes to environmental protection while the foreign economy does not. As long as green awareness of foreign agents is too low, a debt transfer from the domestic to the foreign economy is not an efficient policy tool to induce them to contribute to the environment. It reduces capital stock and the environmental quality. Reversely, a transfer from the foreign to the domestic economy is an efficient way to promote both capital accumulation and the environmental quality. However, acceptability and implementation of such a policy seems difficult because it requires to increase the fiscal burden of the less advanced economy, which could be harmful for its welfare.

5 A foreign economy with a significant environmental sensitivity (γ_F significant)

Households living in the foreign country are significantly affected by environmental quality. γ_F is no more arbitrarily small. We will highlight what types of differences it will imply with respect to the previous configuration with an arbitrarily small environmental sensitivity for the foreign economy. We especially emphasize some configurations where debt transfers between the two countries may induce significant changes on private maintenance. To be more specific, we assume:

Assumption 4 *γ_F is high enough, satisfying the following inequality:*

$$\frac{\frac{\gamma_F}{1+\beta_F}}{\frac{\gamma_D}{1+\beta_D}} > \max\left\{\left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}};\frac{b_D}{b_F}\right\}$$

By inspection of equation (32), we observe that the frontier defining whether a_{Dt} is positive or not $C_D(k_{Dt})$ is not affected by γ_F. Therefore, it has the same properties than in Section 4. Concerning $C_F(k_{Dt})$ defined in equation (33) and such that $a_{Ft} = 0$, we make restrictions such that the two terms into brackets are negative. Under this assumption, $C_F(k_{Dt})$ becomes an increasing and concave function.[3] We will see that this will have strong implications on the existence of steady states and on the implications of policies making some transfers of debt between the two countries.

5.1 Regime with $a_{Dt} > 0$ and $a_{Ft} = 0$

We start by focusing on equilibria with $a_{Dt} > 0$ and $a_{Ft} = 0$. By inspection of equations (30) and (31), we observe that $\varphi(k_{Dt})$ and $\psi(k_{Dt})$ are not affected by γ_F, meaning that they have the same properties than in Section 4. To locate these curves in the (k_{Dt}, E_t) plane and with respect to $C_D(k_{Dt})$ and $C_F(k_{Dt})$, we further assume:

Assumption 5

$$\frac{b_D}{b_F} < \frac{\phi(1-\alpha) - \theta\left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)}{\phi(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}$$

Note that this assumption implies that $\frac{b_D}{b_F} < \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}}$. This also means that the debt per GDP in country D, $b_D/A_D^{\frac{1}{1-\alpha}}$, is lower than in country F, $b_F/A_F^{\frac{1}{1-\alpha}}$. Moreover, when Assumption 5 holds, Assumption 4 simplifies to $\frac{\frac{\gamma_F}{1+\beta_F}}{\frac{\gamma_D}{1+\beta_D}} > \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}}$.

As shown in Appendix 7.4, where some technical details to construct the phase diagram when γ_F is significant are relegated, there exists k_1 and $k_0 (< k_1)$, such that $\varphi(k_{Dt}) > \max\{0, C_F(k_{Dt})\}$ for all $k_0 < k_{Dt} < k_1$. As a direct implication, if Assumption 5 is not satisfied, there is no steady state with $a_D > 0$ and $a_F = 0$. Indeed, when the debt per GDP is lower in the foreign economy, the fiscal cost of its debt burden is sufficiently low, which gives the incentive to contribute to environmental protection. We rule out such a configuration.[4] Moreover, since $\psi(k_{Dt})$ is inversely U-shaped, $\psi(k_{Dt})$ is higher than $C_F(k_{Dt})$ at least for k_{Dt} low or high enough.

[3] We will also have $\lim_{k_{Dt}\to 0} C_F(k_{Dt}) = -\infty$ and $\lim_{k_{Dt}\to +\infty} C_F(k_{Dt}) = +\infty$.

[4] Note that Assumption 5 implies $\max\left\{\left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}}; \frac{b_D}{b_F}\right\} \equiv \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}}$.

5.2 The regime with $a_{Dt} > 0$ and $a_{Ft} > 0$

Using (18), equations (23) and (24) rewrite:

$$\frac{1}{\phi}E_{t+1}\left(\frac{\beta_D}{\gamma_D}+\frac{\beta_F}{\gamma_F}\right)=k_{Dt+1}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]+b_D+b_F \tag{39}$$

$$\left(1+\frac{1+\beta_D}{\gamma_D}+\frac{1+\beta_F}{\gamma_F}\right)E_{t+1}=E_t-\phi(b_D+b_F)\left[\alpha A_D k_{Dt}^{\alpha-1}-1\right]$$
$$+A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)-\theta\right]\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] \tag{40}$$

Equation (40) gives the dynamics of environmental quality, whereas equation (39) gives a static link between E_t and k_{Dt}. Using (40), $E_{t+1} \geqslant E_t$ if and only if $E_t \leqslant \Omega(k_{Dt})$, with:

$$\left(\frac{1+\beta_D}{\gamma_D}+\frac{1+\beta_F}{\gamma_F}\right)\Omega(k_{Dt})=-\phi(b_D+b_F)\left[\alpha A_D k_{Dt}^{\alpha-1}-1\right]$$
$$+A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)-\theta\right]\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] \tag{41}$$

where $\Omega(k_{Dt})$ is strictly increasing and concave under Assumption 3. Substituting (40) into (39), $k_{Dt+1} \geqslant k_{Dt}$ is equivalent to $E_t \geqslant \Pi(k_{Dt})$, with:

$$\Pi(k_{Dt}) \equiv \phi\frac{\gamma_D\gamma_F+\gamma_F(1+\beta_D)+\gamma_D(1+\beta_F)}{\gamma_F\beta_D+\gamma_D\beta_F}\left[k_{Dt}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)+b_D+b_F\right]$$
$$+\phi(b_D+b_F)(\alpha A_D k_{Dt}^{\alpha-1}-1)-A_D k_{Dt}^{\alpha}\left[\phi(1-\alpha)-\theta\right]\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] \tag{42}$$

where $\Pi(k_{Dt})$ is a U-shaped function.

Of course, all dynamic trajectory (k_{Dt}, E_t) with $a_{Dt} > 0$ and $a_{Ft} > 0$ should satisfy $E_t < \min\{C_F(k_{Dt}), C_D(k_{Dt})\}$.

To complete the analysis, we examine if there exists a long term state in which both countries contribute to environmental protection. To this aim, we analyze if positive maintenance activities in the foreign and the domestic good are compatible with a stationary stock of capital and a constant value of environmental index. A steady state is defined by $E = \Omega(k_D)$ and

$$\frac{1}{\phi}E\left(\frac{\beta_D}{\gamma_D}+\frac{\beta_F}{\gamma_F}\right)=k_D\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]+b_D+b_F \tag{43}$$

with $E < \min\{C_F(k_D), C_D(k_D)\}$.

Proposition 5 *Under Assumptions 1 and 3-5,*

- *There is no steady state with $a_{Dt} > 0$ and $a_{Ft} > 0$ if $b_D + b_F$ is too large.*
- *There may exist two steady states with $a_D > 0$ and $a_F > 0$, namely (k_j^s, E_j^s) with $j = \{l, h\}$*

and $0 < k_l^s < k_h^s$*, if* $b_D + b_F > 0$ *is low enough. These steady states are admissible if* $E_i^s < \min\{C_F(k_j^s), C_D(k_j^s)\}$ *is satisfied and are characterized by under-accumulation if:*

$$\phi(1-\alpha) - \theta < \alpha \frac{(1+\beta_D)\gamma_F + (1+\beta_F)\gamma_D}{\gamma_F\beta_D + \gamma_D\beta_F} \tag{44}$$

Proof. See Appendix 7.5 ■

This sufficient condition to have under-accumulation is compatible with Assumption 3, for an appropriate level of ϕ, if $\alpha\frac{(1+\beta_D)\gamma_F+(1+\beta_F)\gamma_D}{\gamma_F\beta_D+\gamma_D\beta_F} > \theta(A_F/A_D)^{\frac{1}{1-\alpha}}$, which is for instance satisfied for θ low enough.

The construction of the phase diagram - see a representation on Figure 4 - and the dynamic analysis above show that the steady state with the highest level of capital is stable, whereas the other one is a saddle.

When Proposition 5 holds, we can investigate the effects of changes of public debt in both countries.

Proposition 6 *Under Assumptions 1 and 3-5, assume that case 2 of Proposition 5 holds. We have:*

1. *Following an increase of* b_D *and/or* b_F*,* k_l^s *increases while* k_h^s *decreases;*
2. *Any increase (decrease) of* b_D *associated to a decrease (increase) of* b_F *of the same amount has no effect on the steady states.*

Proof. See Appendix 7.7. ■

When both countries invest to improve environmental quality, a transfer of debt from the domestic to the foreign country, or the opposite, that let the total amount of debt unchanged is useless. By inspection of the equilibrium conditions (39) and (40), it even does not modify any dynamic trajectory characterized by $a_{Dt} > 0$ and $a_{Ft} > 0$. This happens because countries participate to a common international asset market where there is perfect mobility of assets.

5.3 The interplay between debt and the existence of steady states with positive maintenances

Taking into account that debt transfers have no effect in the regime where both countries are engaged in positive maintenance, we investigate now if such a policy can however promote the environmental maintenance of country F, moving from an equilibrium with $a_{Ft} = 0$ to an equilibrium with positive maintenance $a_{Ft} > 0$.

We investigate this question starting with a configuration where there exist two steady states with $a_D > 0$ and $a_F = 0$. Then, a reallocation of debt among countries will lead to a configuration where the steady state with low capital is characterized by $a_D > 0$ and $a_F = 0$, whereas the stable steady state with high capital is characterized by $a_D > 0$ and $a_F > 0$.

To make a difference between the two types of steady states, the constraint (26) is the relevant one. A steady state with $a_D > 0$ and $a_F = 0$ requires:

$$E > \frac{\phi\gamma_F}{1+\beta_F}\left[(1-\alpha)A_D k_D^\alpha \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - b_F\left(\alpha A_D k_D^{\alpha-1} - 1\right)\right] \tag{45}$$

Using (35), such a steady state should satisfy $\Theta(k_D) < 0$, with:

$$\begin{aligned}\Theta(k_D) \equiv A_D k_D^\alpha &\left[(1-\alpha)\left(\frac{\gamma_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1+\beta_D}\right) + \frac{\gamma_D}{1+\beta_D}\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right] \\ &+ (\alpha A_D k_D^{\alpha-1} - 1)\left(\frac{\gamma_D b_D}{1+\beta_D} - \frac{\gamma_F b_F}{1+\beta_F}\right)\end{aligned} \tag{46}$$

where $\Theta(k_D)$ is an increasing function under Assumption 4. Of course, a steady state with $a_D > 0$ and $a_F > 0$ requires the opposite inequality, i.e. $\Theta(k_D) > 0$.

At this stage, we note that, on the one hand, $\varphi(k_{Dt})$ and $\Omega(k_{Dt})$, that correspond to $E_{t+1} = E_t$, are both increasing and concave functions and cross the curve $C_F(k_{Dt})$ at the same point when a_{Ft} tends to 0. Moreover, $\varphi(k_{Dt})$ is above $C_F(k_{Dt})$ for $\mathrm{k}_{Dt} < k_1$. On the other hand, $\psi(k_{Dt})$ and $\Pi(k_{Dt})$, that correspond to $k_{Dt+1} = k_{Dt}$, are both convex U-shaped functions that cross the curve $C_F(k_{Dt})$ at the same points when a_{Ft} tends to 0. This means that there are at most two steady states taking into account the two regimes where $a_{Ft} = 0$ and $a_{Ft} > 0$.

To show that following a debt transfer, one can move from a situation where the steady state with high capital capital is characterized by $a_F = 0$ to one where both countries are doing maintenance, we focus on the configurations represented in Figures 2 and 3 (see Appendix 7.6 for details). To be more specific, using the notations of the proofs of Propositions 1 and 5 (see Appendices 7.2 and 7.5), we consider that $k_J > \overline{k}$, with k_J and $\overline{k}$ sufficiently close. Using (59), (60), (73) and (74), $k_J > \overline{k}$, which means also that $k_{JJ} > \widehat{k}$, is equivalent to:

$$\frac{\beta_D}{1+\beta_D}B_1 > \frac{\beta_F}{1+\beta_F}B_2 \tag{47}$$

with:

$$B_1 \equiv \frac{\gamma_F}{1+\beta_F}\left[\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\left(\phi(1-\alpha)-\theta+\frac{\theta}{\phi}\right)-(1-\alpha)\right] - \frac{\gamma_D}{1+\beta_D}\left[1-\alpha-\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]$$
$$B_2 \equiv \frac{\gamma_F}{1+\beta_F}(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1+\beta_D}\left[(\phi(1-\alpha)-\theta)\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)-(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]$$

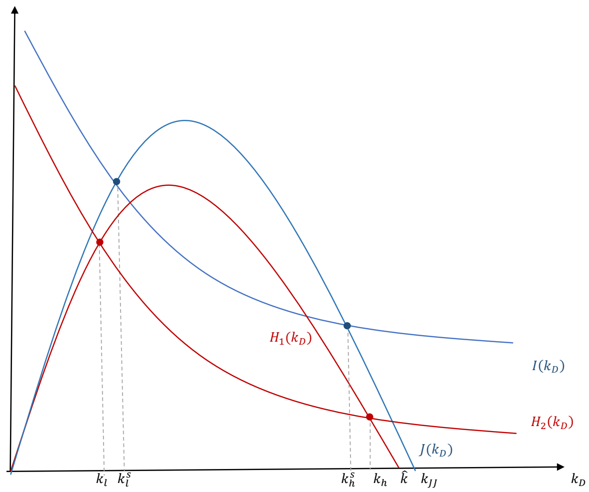

Figure 2 − *Steady state analysis with $\beta_F < \beta_D$*

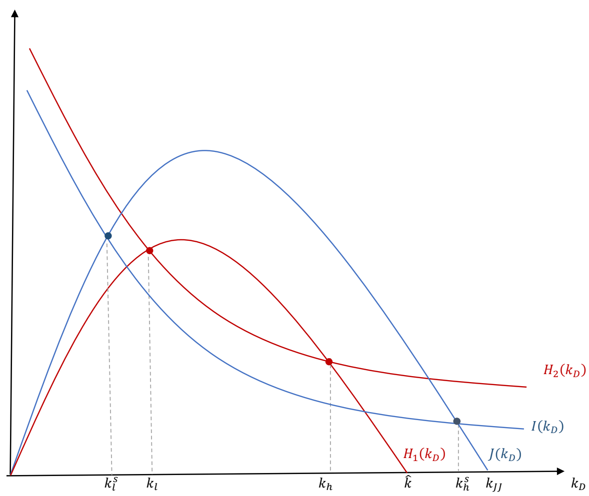

Figure 3 − *Steady state analysis with $\beta_F > \beta_D$*

Under the following assumption:

Assumption 6 $\phi > 2$

we can show that $B_1 > B_2$ and $B_1 > 0$ under Assumption 3. In this case, inequality (47) is satisfied if $\frac{\beta_D}{1+\beta_D} > \frac{\beta_F}{1+\beta_F} B_2/B_1$. Under this last condition, we focus on two configurations:

1. If $\Theta(k_{JJ}) < 0$,[5] there is no steady state with $a_D > 0$ and $a_F > 0$. Taking into account that Proposition 1 holds, that is b_D and b_F are not too large, the two steady states k_l and k_h with $a_D > 0$ and $a_F = 0$ exist. Using (46) and the expression of k_{JJ} given in Appendix 7.5, the

[5]It also implies that $\Theta(\hat{k}) < 0$ (as $k_{JJ} > \hat{k}$).

inequality $\Theta(k_{JJ}) < 0$ is equivalent to:

$$\frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_0 \tag{48}$$

with:

$$\Sigma_0 \equiv \frac{A_D\left(\frac{J_1}{J_2}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\alpha)\left(\frac{\gamma_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1+\beta_D}\right) + \frac{\gamma_D}{1+\beta_D}\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]}{\alpha A_D\frac{J_2}{J_1} - 1} > 0 \tag{49}$$

2. If $\Theta(\bar{k}) < 0$ and $\Theta(k_J) > 0$, we have $\Theta(k_l) < 0$ and $\Theta(k_h^s) > 0$. Since there are at most two steady states, we also have $\Theta(k_h) > 0$ and $\Theta(k_l^s) < 0$. Therefore, there exists one steady state k_l with $a_D > 0$ and $a_F = 0$ and one steady state k_h^s with $a_D > 0$ and $a_F > 0$. The two inequalities $\Theta(\bar{k}) < 0$ and $\Gamma(k_J) > 0$ are equivalent to:

$$\Sigma_2 > \frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_1 \tag{50}$$

with:

$$\Sigma_1 \equiv \frac{A_D\left(\alpha\frac{G_1}{F_1}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\alpha)\left(\frac{\gamma_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1+\beta_D}\right) + \frac{\gamma_D}{1+\beta_D}\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]}{A_D\frac{F_1}{G_1} - 1} > 0 \tag{51}$$

$$\Sigma_2 \equiv \frac{A_D\alpha\left(\frac{J_1}{J_2}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\alpha)\left(\frac{\gamma_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1+\beta_D}\right) + \frac{\gamma_D}{1+\beta_D}\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]}{A_D\frac{J_2}{J_1} - 1} > 0 \tag{52}$$

and $\Sigma_0 > \Sigma_2 > \Sigma_1$. We deduce the following proposition (see also Figure 4)[6]:

Proposition 7 *Under Assumptions 1, 3-6, $\frac{\beta_D}{1+\beta_D} > \frac{\beta_F}{1+\beta_F} B_2/B_1$ and inequality (44), we have the following:*

1. *If $\frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_0$, there exist two steady states with $a_D > 0$ and $a_F = 0$, k_l and k_h;*
2. *If $\Sigma_2 > \frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_1$, there exists one steady state k_l with $a_D > 0$ and $a_F = 0$ and one steady state k_h^s with $a_D > 0$ and $a_F > 0$.*

The stability properties of the steady states presented in Proposition 7 are deduced from the global dynamics analysis depicted in Figures 4 and 5.

Proposition 7 allows us to study whether a debt transfer from one country to the other one, keeping the global level of debt $b_D + b_F$ constant promote the environmental maintenance in country F. In the following proposition, we exploit the fact that a decrease of b_F exactly compensated by an increase of b_D implies a decrease of $\frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D}$ under Assumption 4.

[6] We are not interested in situations where the maintenance of country D at the lowest steady state becomes positive, because we think that it is more relevant to analyse the policy implications on the stable steady state, which is characterized by the highest level of capital.

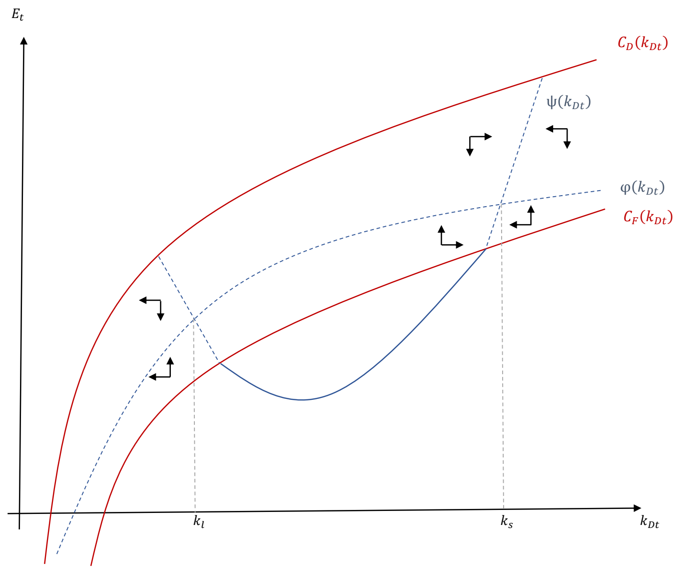

Figure 4 – *Global dynamics for* $\frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_0$

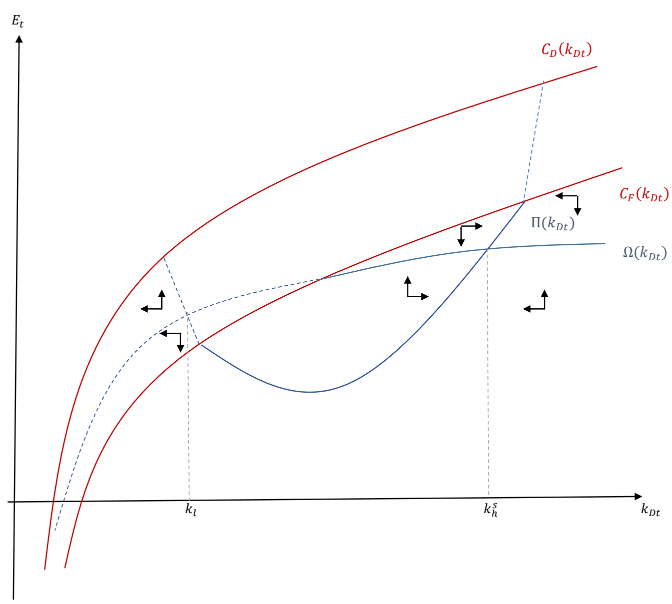

Figure 5 – *Global dynamics for* $\Sigma_2 > \frac{\gamma_F b_F}{1+\beta_F} - \frac{\gamma_D b_D}{1+\beta_D} > \Sigma_1$

Proposition 8 *Under Assumptions 1, 3-6,* $\frac{\beta_D}{1+\beta_D} > \frac{\beta_F}{1+\beta_F} B_2/B_1$ *and inequality (44), a reallocation of public debt from country F to country D may allow to move from a configuration where the two steady states are characterized by* $a_D > 0$ *and* $a_F = 0$ *to a configuration where the steady state with the lowest level of capital is characterized by* $a_D > 0$ *and* $a_F = 0$ *and the steady state with the highest level of capital by* $a_D > 0$ *and* $a_F > 0$.

Of course, we will have the opposite result following a reallocation of public debt from country D to country F.

This proposition shows that following a decrease of debt in country F exactly compensated by an increase of debt in country D, the long-run stable steady state moves from a configuration where maintenance is constrained in country F to a configuration where it is no more. What is interesting is that we can further deduce some implications of the effect of such a policy on the level of capital at the stable steady state. Using also Proposition 2, we see that when $\beta_D > \beta_F$, such a policy reduces the level of capital at the constrained high steady state. When the transfer of debt becomes high enough, the stable steady state is characterized by positive maintenance and is no more affected by the debt transfer between countries that we consider. In contrast, when $\beta_D < \beta_F$, the level of capital at the constrained high steady state increases. Again, when the debt transfer is sufficiently high, the stable steady state becomes unconstrained and independent of the debt transfer considered.

We deduce that such a policy of transferring debt from country F to country D is relevant in terms of the level of capital, and also environmental quality, when country D is the most impatient one (β_D lower than β_F). The debt transfer from one country to the other one implies a reallocation of income from country D to country F, since taxes increases in the first country, whereas it decreases in the other one. Taking into account the equilibrium on the international asset market (14) and (15), we know that capitals in both countries move in the same direction because the returns are equal. This means that capital will increase if global saving raises. This is possible if the redistribution of income between countries is in favor of the population that saves more.

6 Conclusion

We show that debt relief has indirect effects on climate, but direct effects on production and capital in both countries. Starting from the failure of environmental policies to effectively fight GHG emissions, our results suggest the use of macroeconomic policy instruments. Indeed, we show that debt relief for polluting countries, with low productivity, makes it possible to engage these countries in the process of pollution abatement. The overall debt level remains unchanged. Development aid is provided by high productivity countries. We highlight that, even if indebted, advanced countries should help the least developed countries to better fight GHG emissions. Finally, we show that the efficiency of this policy strongly depends on the preferences in each country. Not only the sensitivity to environmental quality plays a key role, but also the discount factor. Indeed, this last one is a determinant of the saving rate. Their heterogeneity among countries may induce a positive effect of net income of redistribution on capital and environmental quality.

Our model does not take into account Debt for nature swaps (Deacon and Murphy,1997; Cassimon et al., 2011, 2014). Indeed, in our article, debt relief is undertaken without any counterpart. This is a limit to our results. The literature on economic development underlines the inefficiencies of this type of aid as soon as donor country monitoring is not put in place. The risk of inefficiency of this policy is all the higher as the country faces problems of quality governance, or extreme poverty.

7 Appendix

7.1 Phase diagram when γ_F is arbitrarily small and close to 0

We easily see from (30), that $\varphi(k_{Dt})$ is strictly increasing ($\varphi(k_{Dt})' > 0$) and concave ($\varphi(k_{Dt})'' < 0$), with $\lim_{k_{Dt}\to 0}\varphi(k_{Dt}) = -\infty$ and $\lim_{k_{Dt}\to +\infty}\varphi(k_{Dt}) = +\infty$.

Using equation (31), we have $\lim_{k_{Dt}\to 0}\psi(k_{Dt}) = +\infty$ and $\lim_{k_{Dt}\to +\infty}\psi(k_{Dt}) = +\infty$, with $\lim_{k_{Dt}\to +\infty}\psi(k_{Dt})/\varphi(k_{Dt}) = +\infty$. We can also show that $\psi''(k_{Dt}) > 0$. Since $\lim_{k_{Dt}\to 0}\psi'(k_{Dt}) = -\infty$ and $\lim_{k_{Dt}\to +\infty}\psi'(k_{Dt}) > 0$, $\psi(k_{Dt})$ is a convex function, decreasing for low values of k_{Dt} and increasing for high values of k_{Dt}.

By direct inspection of (30) and (32), we easily see that $C_D(k_{Dt})$ is increasing and concave, with $\lim_{k_{Dt}\to 0} C_D(k_{Dt}) = -\infty$, $\lim_{k_{Dt}\to +\infty} C_D(k_{Dt}) = +\infty$ and $C_D(k_{Dt}) > \varphi(k_{Dt})$ for all $k_{Dt} > 0$.

By direct inspection of equation (33), under Assumptions 1-3, the two terms into brackets are positive. It implies that $C_F(k_{Dt})$ is decreasing and convex, with $\lim_{k_{Dt}\to 0} C_F(k_{Dt}) = +\infty$ and $\lim_{k_{Dt}\to +\infty} C_F(k_{Dt}) = -\infty$

Now, we examine the conditions such that $\varphi(k_{Dt}) > C_F(k_{Dt})$:

Lemma 1 *Under Assumptions 1-3, there exists $\overline{k} > 0$ such that $\varphi(k_{Dt}) > \max\{0; C_F(k_{Dt})\}$ for all $k_{Dt} > \overline{k}$.*

Proof. Using (30) and (33), $\varphi(k_{Dt}) \geqslant C_F(k_{Dt})$ is equivalent to $F(k_{Dt}) \leqslant \widetilde{F}$, with:

$$F(k_{Dt}) \equiv \alpha k_{Dt}^{-1} - \frac{1}{A_D} k_{Dt}^{-\alpha} \tag{53}$$

$$\widetilde{F} \equiv \frac{\left[\phi(1-\alpha) - \theta\left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]\frac{\gamma_D}{1+\beta_D} - \phi(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\frac{\gamma_F}{1+\beta_F}}{\phi\left(\frac{b_D\gamma_D}{1+\beta_D} - \frac{b_F\gamma_F}{1+\beta_F}\right)} > 0 \tag{54}$$

We can show that $F(k_{Dt})$ decreases from $+\infty$ to $\underline{F} \equiv (\alpha-1)A_D^{-\frac{1}{1-\alpha}} (< 0)$ when k_{Dt} increases from 0 to $\widehat{k} \equiv A_D^{\frac{1}{1-\alpha}}$ and increases to 0 when k_{Dt} increases from $\widehat{k}$ to $+\infty$. Using these properties, there is a unique $k_1 > 0$ such that $F(k_{Dt}) \leqslant \widetilde{F}$ for all $k_t \geqslant k_1$.

Moreover, $\varphi(k_{Dt}) > 0$ is equivalent to $F(k_{Dt}) < F_0$, with:

$$F_0 \equiv \frac{\phi(1-\alpha) - \theta\left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)}{\phi b_D} \tag{55}$$

there is a unique $k_0 > 0$ such that $F(k_{Dt}) < F_0$ for all $k_t > k_0$. We deduce the lemma by noting $\underline{k} = \max\{k_1; k_0\}$. ■

For all $k_{Dt} > k_1$, we have $C_D(k_{Dt}) > \varphi(k_{Dt}) > C_F(k_{Dt})$. To construct a phase digram and be able to have a picture of global dynamics, we also need to analyse whether $\psi(k_{Dt}) \gtrless C_F(k_{Dt})$. Using (31) and (33), this inequality is equivalent to $\Gamma(k_{Dt}) \gtrless 0$, with:

$$\begin{aligned}
\Gamma(k_{Dt}) \equiv k_{Dt}\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] - k_{Dt}^{\alpha}(1-\alpha)A_D\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\beta_F}{1+\beta_F}+\frac{\beta_D}{1+\beta_F}\frac{\gamma_F}{\gamma_D}\right)\\
-\alpha k_{Dt}^{\alpha-1}\left[A_D\left(\frac{\beta_D b_D}{1+\beta_D+\gamma_D}-\frac{\beta_D\gamma_F b_F}{(1+\beta_F)\gamma_D}\right)-A_F\left(\frac{\beta_F b_F}{1+\beta_F}+\frac{\beta_D b_D}{1+\beta_D+\gamma_D}\right)\right]\\
+b_D+\frac{b_F}{1+\beta_F}\left[1-\frac{\gamma_F\beta_D}{\gamma_D}\right] \qquad (56)
\end{aligned}$$

We deduce that $\lim_{k_{Dt}\to+\infty}\Gamma(k_{Dt}) = +\infty$, which means that there is a value of k above which $\psi(k_{Dt}) \geqslant C_F(k_{Dt})$.

7.2 Proof of Proposition 1

Using (34) and (35) and substituting E, a steady state solves $H_1(k_D) = H_2(k_D)$,[7] where:

$$H_1(k_D) = G_1 k_D^{\alpha} - F_1 k_D \qquad (57)$$

$$H_2(k_D) = G_2 k_D^{\alpha-1} + F_2 \qquad (58)$$

with:

$$F_1 = 1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} > 0 \qquad (59)$$

$$G_1 = A_F\frac{\beta_F}{1+\beta_F}(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{\alpha}{1-\alpha}} + A_D\frac{\beta_D}{1+\beta_D}\left[1-\alpha-\frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right] > 0 \qquad (60)$$

$$F_2 = \frac{b_D}{1+\beta_D}+\frac{b_F}{1+\beta_F} > 0 \qquad (61)$$

$$G_2 = \alpha A_D\left(\frac{b_D\beta_D}{1+\beta_D}+\frac{b_F\beta_F}{1+\beta_F}\right) > 0 \qquad (62)$$

See also Figures 2 and 3 for representations of these two curves.

When $b_D = b_F = 0$, we have $F_2 = G_2 = 0$. Steady states are solutions solving $H_1(k_D) = 0$. There are two solutions, $k_D = 0$ and $k_D = (G_1/F_1)^{\frac{1}{1-\alpha}} \equiv \widehat{k}$. [8] We note that $H_1(k_D)$ is an inverse U-shaped strictly concave function which attains its maximum value for $k_D = (\alpha G_1/F_1)^{\frac{1}{1-\alpha}} \equiv \overline{k}$.

[7] It is also a solution (k_D, E) defined by $E = \varphi(k_D) = \psi(k_D)$. Since $\varphi(k_D)$ is concave and $\psi(k_D)$ is convex and as we will show, there are generically two steady states or no steady state.

[8] We recover the result presented in (25), i.e without any debt there is one unique non-trivial steady state.

This maximum value is equal to:

$$H_1(\overline{k}) = \alpha^{\frac{\alpha}{1-\alpha}} G_1^{\frac{1}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}} (1-\alpha) \equiv \overline{H}_1 \tag{63}$$

By direct inspection of (58), we also see that, for a strictly positive b_D and/or b_F, $H_2(k_D)$ is strictly decreasing and convex, with $lim_{k_D \to 0} H_2(k_D) = +\infty$ and $lim_{k_D \to +\infty} H_2(k_D) = F_2$. In addition, we have $H_2(\overline{k}) = F_2 + G_2 F_1/(\alpha G_1)$. See also Figures 2 and 3. Then, $H_2(\overline{k}) \leqslant H_1(\overline{k})$ if:

$$\alpha F_2 G_1 + G_2 F_1 \leqslant (1-\alpha)\alpha^{\frac{1}{1-\alpha}} G_1^{\frac{2-\alpha}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}} \tag{64}$$

Using (61) and (62), we easily deduce that this last inequality is satisfied if b_D and b_F are low enough. In this case, there are two stationary solutions to the equation $H_1(k_D) = H_2(k_D)$. Since a steady state satisfies (35), inequality (25) is fulfilled. We also note that since (34) and (35) are independent of the parameter γ_F, a steady state (k_D, E) does not depend on γ_F. Therefore, inequality (26) evaluated at a steady state is satisfied for γ_F small enough. More precisely, for a small γ_F, each steady state satisfies the condition to be in an asymmetric regime, *i.e* $\varphi(k_D) > C_F(k_D)$.

From (64), if the following sufficient condition holds

$$F_2 > (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} G_1^{\frac{1}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}} \tag{65}$$

we have $H_2(k_D) > H_1(k_D)$ for all $k_D > 0$ and there is no steady state. This last inequality is satisfied if b_D and/or b_F are sufficiently high.

We further note that since both steady states are lower than $\widehat{k} = (G_1/F_1)^{\frac{1}{1-\alpha}}$, they are characterized by under-accumulation if $\widehat{k} < (\alpha A_D)^{\frac{1}{1-\alpha}}$. Using (59) and (60), it is equivalent to:

$$\frac{\alpha}{1-\alpha} > \frac{\frac{\beta_D}{1+\beta_D} + \frac{\beta_F}{1+\beta_F}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}$$

7.3 Proof of Propositions 2-3

We examine in this subsection this impact of debt variation on the steady state capital stocks k_D and hence on the stationary environmental quality index when domestic economy converges to the high level of capital k_h. Using (57)-(62), we obtain:

$$\frac{dk_D}{db_i} = \frac{\frac{\beta_i}{1+\beta_i}\alpha A_D k_D^{\alpha-1} + \frac{1}{1+\beta_i}}{H_1'(k_D) - H_2'(k_D)} \text{ , for } i = \{D, F\} \tag{66}$$

We deduce the first result of the proposition taking into account that $H_1'(k_D) > H_2'(k_D)$ when $k_D = k_l$ and $H_1'(k_D) < H_2'(k_D)$ when $k_D = k_h$. To get the second result, we consider the following variation: $db_D = -db_F = db$. Using (66), we get:

$$\frac{dk_D}{db} = \frac{(\alpha A_D k_D^{\alpha-1} - 1)(\frac{\beta_D}{1+\beta_D} - \frac{\beta_F}{1+\beta_F})}{H_1'(k_D) - H_2'(k_D)}$$

which allows us to conclude the proof of Proposition 2 taking alternatively $db > 0$ or $db < 0$.

Then, we analyze how the environmental quality that prevails on the high steady state, *i.e* E_h, evolves with a debt transfer. Using (35) we have

$$\frac{1+\beta_D}{\gamma_D}\frac{dE_h}{db} = -\phi(\alpha A_D k_h^{\alpha-1} - 1) + \phi b_D A_D \alpha(1-\alpha) k_h^{\alpha-2}\frac{dk_h}{db} + \alpha A_D k_h^{\alpha-1}\frac{dk_h}{db}\left[\phi(1-\alpha) - \theta\left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]$$

When $\beta_D > \beta_F$, taking $db > 0$ E_h decreases while taking $db < 0$ E_h increases.

We rewrite the previous expression to examine the case $\beta_F > \beta_D$:

$$\frac{1+\beta_D}{\gamma_D}\frac{dE_h}{db} = \phi + \phi A_D \alpha k_h^{\alpha-1}\left[(1-\alpha)\epsilon_{k_h/b} - 1\right] + \alpha A_D k_h^{\alpha-1}\frac{dk_h}{db}\left[\phi(1-\alpha) - \theta\left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]$$

with $\epsilon_{k_h/b} \equiv \frac{dk_h}{db}\frac{b_D}{k_h}$. Under Assumptions 1-3, $\epsilon_{k_h/b}$ is decreasing with k_h and we have $\frac{dk_h}{db} > 0$ if inequality (36) is satisfied and $\beta_F > \beta_D$. As a result, the condition $\epsilon_{k_h/b} > 1/(1-\alpha)$ guarantees $\frac{dE_h}{db} > 0$. As the elasticity $\epsilon_{k_h/b}$ is a decreasing function of k_h and that $k_h < \hat{k}$, a sufficient condition is to have

$$\epsilon_{\hat{k}/b} \equiv \frac{d\hat{k}}{db}\frac{b_D}{\hat{k}} > 1/(1-\alpha)$$

Using (57)-(62) and (66), this condition corresponds to

$$\frac{b_D(1+\beta_D)(1+\beta_F)}{\beta_F - \beta_D} < \frac{\alpha A_D F_1 - G_1}{G_1^{\frac{2-\alpha}{1-\alpha}} F_1^{\frac{-\alpha}{1-\alpha}} - G_2 F_1} \equiv \mathcal{X}$$

and Proposition 3 follows.

We then examine the effect of a debt transfer on the agent's welfare in both economies. Using agent's consumption choices, its indirect utility function along the high steady state is given by:

$$V_i = (1+\beta_i)\ln(c_i) + \beta_i \ln(R_i) + (\delta_i + \gamma_i)\ln(E_h) + \beta_i \ln \beta_i$$

We decompose the welfare into a consumption (V_{Ci}) and an environmental component (V_{Ei}).

$$V_i = \underbrace{(1+\beta_i)\ln(c_i) + \beta_i \ln(R_i)}_{V_{Ci}} + \underbrace{(\delta_i + \gamma_i)\ln(E_h)}_{V_{Ei}} + \beta_i \ln \beta_i \tag{67}$$

The environmental part of the welfare evolves as the environment. We have $Sign\frac{dV_{Ei}}{db} = \frac{dE_h}{db}$.

Then, we examine how the consumption part of the welfare changes with a debt transfer.

$$Sign\frac{dV_{Ci}}{db} = (1+\beta_i)\frac{dc_i}{db}R_i + \beta_i\frac{dR_i}{db}c_i$$

Given (14), we have

$$\frac{dR_F}{db} = \frac{dR_D}{db} = A_D\alpha(\alpha-1)k_h^{\alpha-2}\frac{dk_h}{db}$$

Using (11), (9) and (35) we have

$$(1+\beta_D)\frac{dc_D}{db} = \alpha A_D k_h^{\alpha-1}\left[(1-\alpha) - \frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]\frac{dk_h}{db} - (R_D-1) + b_D\alpha(1-\alpha)A_D k_D^{\alpha-2}\frac{dk_h}{db}$$

$$(1+\beta_F)\frac{dc_F}{db} = \alpha A_D k_h^{\alpha-1}(1-\alpha)\frac{dk_h}{db}\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - (R_F-1) + b_F\alpha(1-\alpha)A_D k_h^{\alpha-2}\frac{dk_h}{db}$$

Finally, we have

$$\begin{aligned} Sign\tfrac{dV_{CD}}{db} =& \underbrace{\left(1-\frac{(1-\alpha)\beta_D}{\alpha(1+\beta_D)}\right)\left[(1-\alpha) - \frac{\theta}{\phi}\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)\right]\frac{dk_h}{db}R_D^2}_{\text{income effect}} \\ &+ \underbrace{\frac{b_D R_D(1-\alpha)}{k_h}\left(R_D + \frac{(R_D-1)\beta_D}{1+\beta_D}\right)\frac{dk_h}{db}}_{\text{debt burden effect}} - \underbrace{(R_D-1)R_D}_{\text{direct debt effect}} \end{aligned}$$

$$\begin{aligned} Sign\tfrac{dV_{CF}}{db} =& \left(1-\tfrac{(1-\alpha)\beta_F}{\alpha(1+\beta_F)}\right)\tfrac{dk_h}{db}\left(\tfrac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}(1-\alpha)R_F^2 \\ &+ \tfrac{b_F R_F(1-\alpha)}{k_h}\left(R_F + \tfrac{(R_F-1)\beta_F}{1+\beta_F}\right)\tfrac{dk_h}{db} + (R_F-1)R_F \end{aligned}$$

Proposition 4 follows.

7.4 Phase diagram when γ_F is significant

Lemma 2 *Under Assumptions 1, 3 and 4, there exists $k_1 > 0$ such that $C_F(k_{Dt}) < \varphi(k_{Dt})$ for all $k_{Dt} < k_1$. In addition, there exists $k_0(< k_1)$ such that $\varphi(k_{Dt}) > 0$ for all $k_{Dt} > k_0$ if:*

$$\frac{b_D}{b_F} < \frac{\phi(1-\alpha) - \theta\left(1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right)}{\phi(1-\alpha)\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}} \tag{68}$$

Proof. Since $\frac{b_F\gamma_F}{1+\beta_F} > \frac{b_D\gamma_D}{1+\beta_D}$, $\varphi(k_{Dt}) > C_F(k_{Dt})$ is equivalent to $F(k_{Dt}) > \widetilde{F}$, where $F(k_{Dt})$ is given by (53) and $\widetilde{F}$ by (54). Under Assumption 4, $\widetilde{F} > 0$ which means that there exists a unique $k_1 > 0$ such that $F(k_{Dt}) > \widetilde{F}$ for all $k_{Dt} < k_1$.

Moreover, $\varphi(k_{Dt}) > 0$ is equivalent to $F(k_{Dt}) < F_0$, with $F_0 > 0$ given by (55). $F_0 > \widetilde{F}$ is ensured by inequality (68). In this case, there exists k_0 lower than k_1 such that $F(k_{Dt}) < F_0$ for all $k_{Dt} > k_0$. ■

We still have that $\psi(k_{Dt}) \geqslant C_F(k_{Dt})$ is equivalent to $\Gamma(k_{Dt}) \geqslant 0$, where $\Gamma(k_{Dt})$ is given by (56). However, under Assumption 4, we deduce that $\lim_{k_{Dt}\to 0}\Gamma(k_{Dt}) = +\infty$, $\lim_{k_{Dt}\to+\infty}\Gamma(k_{Dt}) = +\infty$ and $\Gamma(k_{Dt})$ is convex:

$$\Gamma''(k_{Dt}) = k_{Dt}^{\alpha-2}\alpha(1-\alpha)^2 A_D\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\beta_F}{1+\beta_F}+\frac{\beta_D}{1+\beta_F}\frac{\gamma_F}{\gamma_D}\right)$$
$$-\alpha(\alpha-1)(\alpha-2)k_{Dt}^{\alpha-3}\left[A_D\left(\frac{\beta_D b_D}{1+\beta_D+\gamma_D}-\frac{\beta_D\gamma_F b_F}{(1+\beta_F)\gamma_D}\right)-A_F\left(\frac{\beta_F b_F}{1+\beta_F}+\frac{\beta_D b_D}{1+\beta_D+\gamma_D}\right)\right]\geqslant 0$$

This means that $\psi(k_{Dt})$ is higher than $C_F(k_{Dt})$ at least for k_{Dt} low or high enough.

Under Assumption 3, we observe, using (41), that $\Omega(k_{Dt})$ is strictly increasing ($\Omega(k_{Dt})' > 0$) and concave ($\Omega(k_{Dt})'' < 0$), with $\lim_{k_{Dt}\to 0}\Omega(k_{Dt}) = -\infty$ and $\lim_{k_{Dt}\to+\infty}\Omega(k_{Dt}) = +\infty$.

By inspection of equation (42), we have $\lim_{k_{Dt}\to 0}\Pi(k_{Dt}) = +\infty$ and $\lim_{k_{Dt}\to+\infty}\Pi(k_{Dt}) = +\infty$, with $\lim_{k_{Dt}\to+\infty}\Pi(k_{Dt})/\Omega(k_{Dt}) = +\infty$. We can also show that $\Pi''(k_{Dt}) > 0$. Since $\lim_{k_{Dt}\to 0}\Pi'(k_{Dt}) = -\infty$ and $\lim_{k_{Dt}\to+\infty}\Pi'(k_{Dt}) > 0$, $\Pi(k_{Dt})$ is a convex function, decreasing for low values of k_{Dt} and increasing for high values of k_{Dt}.

We can further study whether $C_F(k_{Dt}) \geqslant \Omega(k_{Dt})$. After some computations, we can show that when $\frac{b_F\gamma_F}{1+\beta_F} > \frac{b_D\gamma_D}{1+\beta_D}$, this inequality is equivalent to $F(k_{Dt}) \leqslant \widetilde{F}$. Using Lemma , we immediately deduce that this is satisfied for $k_{Dt} \geqslant k_1$. Using (53) and (54), we also deduce that k_1 increases with $\frac{b_F\gamma_F}{1+\beta_F} - \frac{b_D\gamma_D}{1+\beta_D}$.

7.5 Proof of Proposition 5

Using $E = \Omega(k_D)$ with (43), a steady state is a solution k_D to the equation $I(k_D) = J(k_D)$, with:

$$I(k_D) = I_1 + I_2 k_D^{\alpha-1} \tag{69}$$
$$J(k_D) = J_1 k_D^{\alpha} - J_2 k_D \tag{70}$$

and

$$I_1 = (b_D+b_F)\frac{\gamma_F+\gamma_D}{(1+\beta_D)\gamma_F+(1+\beta_F)\gamma_D} > 0 \tag{71}$$
$$I_2 = (b_D+b_F)\frac{\gamma_F\beta_D+\gamma_D\beta_F}{(1+\beta_D)\gamma_F+(1+\beta_F)\gamma_D}\alpha A_D > 0 \tag{72}$$
$$J_1 = \frac{(\gamma_F\beta_D+\gamma_D\beta_F)A_D\left[\phi(1-\alpha)-\theta\right]\left[1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right]}{(1+\beta_D)\gamma_F+(1+\beta_F)\gamma_D} > 0 \tag{73}$$
$$J_2 = 1+\left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \equiv F_1 > 0 \tag{74}$$

See Figures 2 and 3 for representations of these two curves. The analysis for the existence and multiplicity of steady states is quite similar to the case with $a_{Dt} > 0$ and $a_{Ft} = 0$. Indeed, $J(k_D)$ is an inverse U-shaped strictly concave function which attains its maximum value for $k_D = (\alpha J_1/J_2)^{\frac{1}{1-\alpha}} \equiv k_J$, with:

$$J(k_J) = \alpha^{\frac{\alpha}{1-\alpha}} J_1^{\frac{1}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} (1-\alpha) \tag{75}$$

If $b_D + b_F = 0$, we have $I_1 = I_2 = 0$. Steady states are solutions solving $J(k_D) = 0$. There are two solutions, $k_D = 0$ and $k_D = (J_1/J_2)^{\frac{1}{1-\alpha}} \equiv k_{JJ}$.

If $b_D + b_F > 0$, we see that $I(k_D)$ is strictly decreasing and convex, with $lim_{k_D \to 0} I(k_D) = +\infty$ and $lim_{k_D \to +\infty} I(k_D) = I_1$. We have $I(k_J) \leqslant J(k_J)$ iff:

$$\alpha J_1 I_1 + J_2 I_2 \leqslant (1-\alpha)\alpha^{\frac{1}{1-\alpha}} J_1^{\frac{2-\alpha}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} \tag{76}$$

Using (71)-(74), we note that J_1 and J_2 do not depend on b_D and b_F, while I_1 and I_2 both linearly increase with $b_D + b_F$. Therefore, inequality (76) is satisfied if $b_D + b_F$ is low enough. In this case, there are two stationary solutions to the equation $H_1(k_D) = H_2(k_D)$. The steady states are admissible if $E < \min\{C_F(k_D), C_D(k_D)\}$ is satisfied for each steady state.

On the contrary, if:

$$I_1 > \alpha^{\frac{\alpha}{1-\alpha}} J_1^{\frac{1}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} (1-\alpha) \tag{77}$$

we have $I(k_D) > J(k_D)$ for all $k_D > 0$. This last inequality is satisfied if $b_D + b_F$ is sufficiently high. In this case, there is no steady state.

In the case in which there exist two steady states, we know that their associated levels of capital are lower than $k_{JJ} = (J_1/J_2)^{\frac{1}{1-\alpha}}$. Thus, they are characterized by under-accumulation if $k_{JJ} < (\alpha A_D)^{\frac{1}{1-\alpha}}$. Using (73) and (74), this condition can be written as inequality (44) and Proposition 5 follows.

7.6 Main ingredients to understand Figures 2 and 3

We need to compare the curves that define steady states with $a_F = 0$ and steady states with $a_F > 0$. Using (57)-(62) and (69)-(74), we observe first that $F_1 = J_2$ and $J_1 > G_1$ when $\frac{\beta_D}{1+\beta_D} > \frac{\beta_F}{1+\beta_F} B_2/B_1$. This means that $J(k_D) > H_1(k_D)$.

Second, $I(k_D) > H_2(k_D)$ is equivalent to:

$$\frac{\beta_D - \beta_F}{(1+\beta_D)\gamma_F + (1+\beta_F)\gamma_D} \left(\frac{b_F \gamma_F}{1+\beta_F} - \frac{b_D \gamma_D}{1+\beta_D} \right) \left(\alpha A_D k_D^{\alpha-1} - 1 \right) \geqslant 0$$

Under Assumption 4 and inequality (44), which implies that $\alpha A_D k_D^{\alpha-1} \geqslant 1$ for all $k_D \leqslant k_{JJ}$, $I(k_D) > H_2(k_D)$ if and only if $\beta_D > \beta_F$, whereas $I(k_D) < H_2(k_D)$ if and only if $\beta_D < \beta_F$.

7.7 Proof of Proposition 6

Using (69)-(74), we obtain:

$$\frac{dk_D}{db_i} = \frac{I(k_D)}{(b_D + b_F)(I'(k_D) - J'(k_D))} \text{ , for } i = \{D, F\} \tag{78}$$

We deduce the first result of the proposition taking into account that $I'(k_D) > J'(k_D)$ when $k_D = k_l^s$ and $I'(k_D) < J'(k_D)$ when $k_D = k_h^s$. The second result is obvious since all policies keeping $b_D + b_F$ invariant do not alter the equilibrium conditions.

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