Incentives under list proportional representation*

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Abstract

We develop a novel moral hazard model of elections under closed- and open-list PR. Parties compete for legislative seats on the basis of their electoral output, which is a CES function of the effort choices of their candidates. Voters are well informed about these effort choices, or not. We show that, If effort choices are substitutable within parties, the cost of effort function is not too convex and voters are not well informed about the choices of individual politicians, closed lists generate higher electoral outputs than open lists. Our findings are robust to many extensions of the basic model, such as allowing for more than two parties, for ideological differences within the electorate, or for candidates to also care about their party winning control of the executive office.

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1 Introduction

Proportional representation (PR henceforth) is the democratic world's most frequently used electoral rule. As of 2015, 94 out of the 147 democracies rely on PR for their legislative elections (as reported in Cruz, Keefer and Scartascini (2016)). In all PR systems, voters in a district elect more than one legislator. Parties thus offer lists of candidates to the electorate. Of the 94 democracies relying on PR, roughly 25 use open or flexible lists; the rest relies more heavily on closed lists. Under closed-list PR, political parties propose an ordered list of candidates to voters. The ballot structure allows the electorate to vote only for parties. Voters cannot express their preferences for individual candidates. Each party's seat share in parliament is proportional to its vote share. The legislative seats a party won are allocated to its list candidates, following the order on that list. Thus, with closed lists, voters do not influence which candidates within each party are elected into parliament. Democracies that rely fully on closed-list PR include Argentina, Costa Rica, Guatemala, Israel, Spain and Turkey.

Under open-list PR, the ballot structure allows voters to cast a preference vote for an individual candidate on one of the party lists. The total number of preference votes received by all candidates on a party's list determines the party's vote share and its number of seats in parliament. Contrary to what happens with closed lists, the intraparty allocation of legislative seats to candidates is determined by the number of preference votes each candidate received, as seats go to the candidates having received the highest number of such votes.[1] Thus, with open lists, voters have a direct influence on which candidates within each party are elected into parliament. Such lists are used for example in Austria, Brazil, Finland, Greece, Indonesia and Japan.

Under PR, electoral success requires cooperation between list members but, with open lists, competition between party members also becomes important. Persson, Tabellini and Trebbi (2003, p.961) offer a crisp summary of the issues at stake: 'Politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. Under proportional representation, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [if] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance". Intuition then suggests that open lists should provide candidates

[1]Flexible PR systems are a mixture of these two extreme cases, possibly offering voters the right to cast more than one vote, as in Belgium, or to cast these votes both for a party and a candidate.

better incentives than closed-list PR. However, because of its modelling complexity, the analysis of incentives under various PR systems has received relatively little attention in the theoretical literature.

To analyze these questions, we develop a model of elections under closed- and open-list PR. We model the election as a contest between political parties. Parties are teams of candidates that compete for multiple and indivisible prizes, the legislature's seats. Candidates contribute to their party's success by exerting costly effort to improve their party's platform, their party's electoral output (as in Caillaud and Tirole (2002)). An important parameter in the model is the degree of convexity of the marginal cost function. A low degree of convexity corresponds to elections in which the main issues are standard and candidates and their parties have a long track record and a lot of experience dealing with these issues. Candidates can 'simply' repeat party arguments on the issue. To the contrary, if the election is centered around a novel and complex issue, the intensity and depth of the work and research required to be able to craft, propose and defend a platform is much higher, and thus the cost of the required efforts increases more quickly.

The contest takes place at two levels, between and within parties. Between parties, under both systems, a team contest allocates legislative seats on the basis of parties' electoral outputs. We use a constant elasticity of substitution (CES) function to aggregate the efforts of the different candidates on a list into the party's electoral output. A CES function allows to vary the degree of complementarity between the efforts of candidates on the list and to analyze its impact on the incentives to provide effort. A party's electoral output will exhibit complementarity between its list candidates when the underlying policy is multi-dimensional or multi-disciplinary. In such a case the party needs to rely on the efforts of many of its specialists to be able to craft, propose and defend a high quality platform.

We introduce a new binomial-Tullock mechanism to allocate the seats as a function of parties' output. The probability that a party wins a given legislative seat follows a Tullock (1980) contest success function based on the parties' electoral outputs. A party's probability of winning a certain number of seats then follows a binomial distribution with, as key parameters, the total number of seats in the legislature and the Tullock probability based on party electoral outputs.

Within parties, candidates' incentives vary with the ballot structure. With closed lists, candidates exert effort only to increase their party's electoral success. As a candidate's chance to get a seat depends on his position on the list, he will thus exert effort to increase the number of seats won by the party to a level where he is allocated a seat.

Under an open list, candidates exert effort not only to increase the number of seats won by their party, but also to attract preference votes, as these preference votes improve a candidate's position in the ranking that determines how seats are allocated. To model the mapping from preference votes to seat attribution within parties under open-list PR, we adapt the contest model of Clark and Riis (1996).

We parametrize the sensitivity of the outcome of the intraparty contest to individual effort with a noise parameter. A noisy contest corresponds to the case of poorly informed voters. Voters' information may depend on the size of electoral districts or the type and the level of the election. For example, Carey and Hix (2011 p. 385) argue that voters in " []...] a lower magnitude multimember district say, with magnitude of two to six" should have a relatively clear preference ordering over the candidates or lists [...]. By contrast, in a high magnitude multimember district "say, with magnitude above 10" [...] voters are unlikely to have clear preference ranking over all the options [...]."[2] Similarly, there is evidence that voters in second-tier elections are not too well informed. For example, Hobolt and Wittrock (2011, p. 39) conclude their study about voting behavior in elections for the European Parliament by stating that "voters are likely to base their EP vote choices on sincere preferences relating to the dominant dimension of contestation in national politics" (emphasis added).[3] With no information about individual efforts, voters randomize their preference votes and all candidates get the same chance to go to parliament. As all candidates are on an equal footing, we label open-list PR with uninformed voters the egalitarian rule.

Turning to our findings, we first confirm the literature's previous finding that PR is associated with a strong free-rider problem. Indeed, all candidates contribute to the list's success by exerting costly effort while the benefits are shared by all candidates. This free-riding problem is particularly acute when lists are closed. Preference votes under open lists mitigate the issue as candidates exert effort to also improve their ranking.

However, the two ballot structures create another important difference. With open lists, all candidates face the same incentives and they all exert the same level of effort. If lists are closed, the distribution of efforts is bell shaped as a function of the candidates' positions on the list: candidates at the top and bottom of the list exert little effort, while candidates around the median list position are those exerting highest effort.

[2]More generally, Miller (1956) showed that individuals find it difficult to process and compare alternatives containing more that five or seven components.

[3]Ferraz and Finan (2008) also show that voters' information has a first-order importance in mayoral elections in Brazil.

We compare outcomes under open and closed lists to derive our main result. We find that open lists have an advantage at dealing with the free-riding problem, especially when voters are well-informed about candidates individual efforts and as a consequence when preference votes are very responsive to effort. However, we show that closed-list PR can also lead to a higher electoral output than open-list PR. This is possible when the complementarity between the efforts of candidates on the same list are weak and when the cost of effort is not too convex. Then, high effort by a few list members is enough to generate higher team output. This result has clear implications in terms of constitutional design as well as in terms of the incentives of parties to adopt internal rules that lead to incentives for their candidates that resemble closed-list or open-list systems.

We then turn to various extensions and robustness checks. In all of them, the parameter values that determines the superiority of a ballot structure are the same as in the main result. We first derive the set of monotonic allocation rules that maximize a party's electoral output when voters are ill-informed about candidates' efforts. A monotonic rule is a mechanism that allocates seats to candidates such that a candidate's probability of winning a seat is increasing in the number of seats their party won. We find that the two allocation rules that maximize a party's electoral output are either the closed list or the egalitarian rule. We then consider extensions to account for several important regularities observed in elections under PR. First, we allow more than two parties; we then analyze biased contests (in which one party is advantaged because of voters' ideology); finally, we solve the model when candidates also care about their party winning a majority of seats to gain control of the executive and discuss how possible intraparty struggles for the allocation of the rents linked to executive control impact effort provision incentives.

The rest of the paper is structured as follows. The next section reviews the related literature. Section 3 introduces the model. Section 4 derives our main findings. Section 5 and 6 present the extensions and robustness checks. The last section concludes and discusses possible future research. All proofs can be found in the Appendix.

2 Related Literature

We contribute to the recent strand of research on the incentive effects of electoral rules, following in particular the modelling philosophy of Caillaud and Tirole (2002) and Castanheira, Crutzen and Sahuguet (2010). In these models, like in ours, candidates spend costly effort to improve electoral platforms.[4] The received wisdom in both economics and political science about the incentive effects of PR is quite negative, especially PR with closed-lists; see Persson, Tabellini and Trebbi (2003) for a crisp summary of this view. Our results show that, under some conditions, closed lists can provide the right incentive structure to candidates and yield better outcomes than open lists.[5] Crutzen and Sahuguet (2018) also offer some cautionary tales about the negative views regarding closed-list PR in a model where the comparison is between closed-list PR and plurality rule.

The binomial-Tullock mechanism we employ to model electoral competition between parties is novel and is essentially a generalization of the contest success function of Tullock (1980) to multiple indivisible prizes. The shape of the map from a party's vote share to its seat share that is generated by the binomial-Tullock mechanism is strikingly similar to the one Buisseret, Folke, Prato and Rickne (2018) estimate using data on the full universe of Swedish politicians to this date, comforting this modelling choice of ours. A classical predecessor to our mechanism is the probabilistic voting model developed by Enelow and Hinich (1982) and Lindbeck and Weibull (1987) and used recently by Galasso and Naniccini (2015) in their analysis of candidate selection issues under closed-list PR. In Galasso and Naniccini (2015), the probability of winning a prize is independent of the number of prizes a team has already won, whereas in our model the teams' probability of winning an extra prize decreases with the number of prizes already won, a feature that we believe is desirable, if anything simply because it is arguably more realistic.

In our model, candidates can only exert effort to improve their party's platform. In restricting the strategy set of candidates in such a way, we are clearly stacking the deck against closed-list PR and in favor of open-list PR. Indeed, our modeling choices eliminate by construction a central issue associated with open lists: they generate a tension between party and candidate strategies. A candidate, to gain preference votes may take actions that are not in line with what the party wants that candidate to do. For example, open-list PR generates strong incentives for candidates to cater to subsets of the voting population to ensure they receive enough individual votes to get elected – see for example Myerson (1993, 1999) and Ames (1995a, b, 2002). This force is shut down in our model as the same candidates' effort is used to determine party's electoral success and preference

[4]Given that political corruption can be interpreted as the opposite of effort in our model, empirical counterparts to our theoretical study include Persson, Tabellini and Trebbi (2003), Kunicova and Rose-Ackermann (2005) Golden and Chang (2001), Tavits (2007) and Schleiter and Voznaya (2014).

[5]A few other recent papers compare the various types of PR systems (closed, open, flexible), but they all focus on selection matters; see Galasso and Naniccini (2015), Buisseret and Prato (2018a), Buisseretet al. (2018) and Hangartner, Nelson and Tukiainen (2018).

votes. Intuition suggests that the 'incentives to cultivate favored minorities', to borrow from the title of Myerson (1993), are strong when the electoral issue is a complex and multi-dimensional one – because it is easier for a candidate to generate their own favored minority –, and thus when a party's electoral output exhibits strong complementarity between its list members. Previous contributions have highlighted that incentives to generate personal votes imply that with open lists candidates run more personal campaigns (Bowler and Farrell, 2011; Zittel, 2015), and – once elected – will do more constituency service (Heitshusen, Young and Wood, 2005), introduce particularized legislation (Crisp et al., 2004), break from the party ranks more often (Carey, 2009; Sieberer, 2006) and have and maintain stronger local connections (Shugart, Valdini and Suominen, 2005; Tavits, 2010). Such incentives to focus on narrow, personalized campaigns and policies can even lead to deadlocks in the political system; see for example Ames (1995b and 2002) and Myerson (1993, 1999). In closed-list systems, by contrast, candidates are more likely to concentrate exclusively on presenting to voters the coherent policy packages that their party pledges to pursue in office (Carey and Shugart, 1995). Also, our model highlights that voters need to be well informed about individual candidates for an open-list PR system to generate proper incentives.

Other models focus on the incentive effects of different electoral systems to offer different policy bundles to voters; see for example Persson and Tabellini (1999, 2000, 2003), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002).[6] Persson and Tabellini (1999, 2000, 2003) and Milesi-Ferretti, Perotti and Rostagno (2002) are positive contributions that aim to rationalize cross-country economic policy differences using differences in political institutions. Lizzeri and Persico (2001, 2005) focus more on analyzing the conditions under which different political systems and institutions lead to superior outcomes in terms of policy mix.

Our paper also contributes to the literature on contests.[7] It offers a bridge between two different strands of the literature: team contests and contests for multiple prizes. In team contests, several teams compete in order to win one prize, which may be of a public or private nature, or a mix of both. This strand of the literature focuses on incentives within teams and more specifically on the

[6] Buisseret and Prato (2018b) focus on the selection of politicians and compare electoral outcomes under single- and multi-member electoral districts and pin down conditions such that multi-member electoral systems are better than single-member ones at balancing the interests of voters and parties. As single-member districts are typical of plurality rule and PR always comes with multi-member districts, we can interpret their results as pointing out that under certain conditions selection under PR generates higher voter welfare than that under plurality rule.

[7] The literature on contests is too vast to be reviewed here. We refer to Corchón (2007), Konrad (2009) and Vojnovic (2015) for extensive reviews.

sharing rule that splits the single available (private part of the) prize across the winning team's members, so as to maximise team output. Important contributions include Nitzan (1991), Lee (1995), Esteban and Ray (2001), Ueda (2002), Baik and Lee (2001), Nitzan and Ueda (2011), Baik and Lee (2012) and Balart, Flamand and Troumpounis (2016).[8] Another strand of this literature uses the all-pay auction model to analyse competition between teams; see for instance Fu, Lu and Pan (2015), Barbieri, Malueg and Topolyan (2014), Barbieri and Malueg (2016) and Eliaz and Wu (2017). We contribute to this literature by extending it to the case of multiple indivisible prizes. We also contribute to the group size paradox literature by showing that, depending on which optimal intrateam allocation rule teams rely on, the group size paradox may be present or not.

We also contribute to the literature on contests for multiple prizes.[9] Most of this literature focuses on contests among individuals who can win at most one prize, Clark and Riis (1996) being a very prominent contribution. The intrateam allocation rules we consider are not contests, as the allocation does not depend on individual efforts.[10] The tournament literature (see for instance Nalebuff and Stiglitz (1983)) also considers the case of multiple prizes. One major advantage of our approach is its analytical tractability. Papers using the all-pay auction model have also considered the issue of allocating multiple prizes in contests. Moldovanu and Sela (2001) consider a contest among individuals in which the contest designer can decide on both the number and value of the prizes on offer. As their contest is among individuals, the issue of the intrateam prize allocation rule is absent, by construction. Their analysis also focuses mostly on the convexity of the cost of effort function. We have a contest model between teams and the team output exhibits different degrees of complementarity and substitutability between individual efforts, even though the number and value of the prizes is exogenously given. The focus of our paper is thus on the intrateam prize allocation rule. We pin down the optimal intraparty allocation rule as a function not only of the convexity of the individual cost of effort function, but also of the degree of complementarity between efforts in the team production function.

Our paper also contributes to the literature on incentives in teams, and in particular to the literature that links incentives and discrimination or non-equal treatment of ex-ante identical team members. Winter (2004) analyses whether agents who are identical in their qualifications should receive asymmetric rewards to improve incentives and efficiency. Winter (2004) relies on an O-Ring technology where all agents must succeed in their task for the team to be successful. In contrast, as

[8]For a recent survey on sharing rules in collective rent seeking, see Flamand and Troumpounis (2015).

[9]For a recent survey of the literature on contests with multiple prizes, see Sisak (2009).

[10]Crutzen and Sahuguet (2018) analyse a model in which the allocation of prizes within teams is itself a contest.

we use a CES production function, we parametrise complementarity in a more continuous way. Also, Winter (2004) models effort as a binary choice and thus cannot discuss the role of the convexity of the cost function. We also allow for a continuum of efforts to analyse the effect of the convexity of the cost function. Bose, Pal and Sappington (2010) study the unequal treatment of identical agents in teams. In their model, complementarity in team production leads to higher output when effort decisions are taken sequentially as opposed to simultaneously. They find that, within each team, it is optimal to treat differently the two team members in the presence of strategic complementarity. Our production function is a CES, and all team members choose their efforts simultaneously. In that case, complementarity calls for equal treatment. At least for the application we focus on, the use of a sequential efforts does not appear to be a natural assumption.

Closer to our setup is Ray, Baland and Dagnelie (2007) who also use a CES function to model team production. However, their model focuses on one team only and production can be shared continuously. They find that unequal sharing rules are efficient when efforts are substitutes. Our findings suggest that their result extends to the case of team contests for multiple indivisible prizes. Our analysis also goes further than Ray et al. (2007), as we derive the optimal monotonic allocation rules under contractible and non-contractible efforts, when efforts are complements as well as substitutes.

3 Proportional representation as a contest between teams

3.1 Politicians' efforts and parties output

Two parties are competing in an election for n legislative seats, n odd. Each party is composed of n identical politicians who can win at most one seat. All politicians have the same effort productivity, which we normalize to unity, and they all face the same cost of effort function, which is increasing and convex:

$$C(e_{ij}) = e_{ij}^{\beta}/\beta, \text{ with } \beta > 1, \tag{1}$$

where $e_{ij} \geq 0$ is effort by politician i in party j to improve his party's electoral output and thus its chances of winning seats. Each seat has value V. Party j's electoral output is denoted by E_j. We assume that the production function aggregating individual efforts exhibits constant elasticity of substitution:

$$E_j = \left[\sum_{i=1}^{n} (e_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \text{ with } \sigma \in [0, 1) \tag{2}$$

When $\sigma = 0$, individual efforts are perfect substitutes and a party's electoral output is the sum of the efforts of its list candidates. The complementarity of efforts increases with σ.

3.2 Allocation of seats between parties

The allocation of seats between parties depends on their electoral output. The distribution of seats is random and follows a Binomial-Tullock distribution. This distribution generalizes the Tullock contest (1980) to multiple prizes. As in a Tullock contest, the probability that party j wins a particular seat is given by the contest success function $p_j = \frac{E_j}{E_1+E_2}$. Seats are then awarded to party j using independent draws from a Bernoulli distribution with parameter p_j.[11] The probability that party j wins k seats follows a binomial distribution and is given by, letting $C_k^n = \frac{n!}{(n-k)!k!}$:

$$P_j(k) = C_k^n \left(p_j\right)^k \left(1 - p_j\right)^{n-k}. \tag{3}$$

To illustrate this, figure 1 plots the probability that a party wins x seats or more in the symmetric equilibrium in which each party expects to win half the seats (we set $n = 31$; this curve is just the opposite of the CDF of our binomial-Tullock). As the figure shows, it is very likely that a party wins the first few seats, but it is increasingly unlikely that a party wins a substantial majority of seats, let alone the whole legislature. The inverted S-shape generated by the binomial distribution is in line with the empirical distribution of seats won by a party, as documented by Buisseret et al. (2018) in their analysis of party nomination strategies in Sweden (see their figure 1).

[11] In section 6, we generalise the formula for p_j to allow for aggregate noise or different mappings from a party's electoral output to the number of seats it wins. Our results are not affected by these alternative specifications.

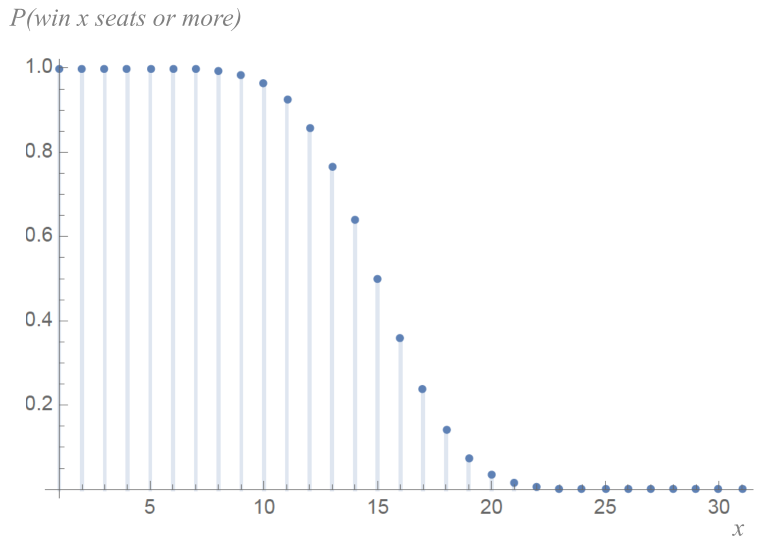

Figure 1: Binomial-Tullock probabilities

3.3 Allocation of seats within parties

Under closed-list PR, the allocation of seats follows a rigid list order designed by the party prior to the election. Candidates on the list are thus treated differently, with candidates higher on the list being advantaged through a higher probability of winning a seat. Even though individual effort does not affect the allocation of seats within the party, politicians still exert effort to increase their party's electoral success and indirectly increase their chance of winning a seat: given the other parties' outputs, higher effort by a candidate increases their party's output, which in turn increases $p_j = \frac{E_j}{E_1+E_2}$ and thus the expected number of seats a party is expected to win.

If lists are open, all candidates are put on an equal footing prior to the election. Indeed, parties still run in the election with a list of candidates but voters cast their ballot not only for the party but also for the candidate(s) they prefer. These preference votes influence the allocation of seats in a party. A higher number of preference votes maps into a higher chance of being offered one of the seats the party won. And higher individual effort increases the number of preference votes a candidate receives, all else equal. We now present in more details these two systems.

3.4 Closed lists

Under a closed list, the n list members are ordered on a list that determines the order of the allocation of seats won by the party. Thus, the politician in mth position on the list wins a seat if their party wins at least m seats. List member in mth position on the list of party j solves:

$$Max_{e_{mj}}\left(V\sum_{k=m}^{n}P_j(k)-\frac{e_{mj}^{\beta}}{\beta}\right) \tag{4}$$

Notice that the summation goes from m to n and not from 1 to n, as the list member in the mth position on the list only gets a seat when his party wins at least m seats.

3.5 Open lists

Under an open list, like under a closed list, voters decide to vote for one of the two parties as a function of electoral outputs E_1 and E_2. On top of this, voters decide to cast their ballot for a specific candidate of the party they wish to support on the basis of the effort choices of that party's candidates. That is, within each party, the seats a party won are attributed on the basis of the preference votes each of the candidates received. We model this competition for seats within a party as a contest between n candidates for $m \leq n$ prizes of value V. To do this, we adapt the model of Clark and Riis (1996) to our setting. Denote with e_i the effort of politician i. The probability that i ends up among the m candidates with the highest number of preference votes, and thus among the m first candidates to be attributed a seat by their party, is given by:

$$Q_i(m)=q_1+\sum_{j=2}^{m}q_j\left(\prod_{s=1}^{j-1}(1-q_s)\right), \tag{5}$$

where q_j is the probability that i ends up on position m on the list. This probability is given by a standard Tullock ratio contest success function based on individual effort choices among the candidates who have not yet been given a seat:

$$q_j=\frac{e_i^r}{e_i^r+\sum_{k\neq i}e_k^r}, \#k=n-m. \tag{6}$$

One can interpret (5) as the result of a sequential process. The contestants make one contribution that is valid for the entire intraparty sequential contest. The winner of the first prize is decided using the Tullock contest function with the contributions of all the n contestants. The winner and his contribution are then erased and the winner of the second seat the party won is decided using the Tullock contest function with the contributions of the remaining $n-1$ contestants. This process continues until all seats a party won have been awarded.

In (6), r parametrizes the noise in the preference voting mapping. In the limit, when $r = 0$, noise determines the intraparty outcome fully, for example because voters do not have (enough) information about the choices of individual candidates to cast an informed vote and thus simply randomize across all candidates. In such case, all candidates have the same probability of winning a legislative seat. Then $Q_i(k) = k/n, \forall i$. The intraparty seat allocation rule becomes fully egalitarian: it treats all party candidates equally. We label this case the egalitarian rule.[12]

Overall, candidate i in party j chooses e_{ij} to maximize:

$$V \sum_{m=1}^{n} P_j(m) Q_i(m) - \frac{e_{ij}^{\beta}}{\beta}, \tag{7}$$

where $Q_i(m) = q_1 + \sum_{j=2}^{m} q_j \left(\prod_{s=1}^{j-1} (1 - q_s) \right)$ and $P_j(m) = C_m^n P_j^m (1 - P_j)^{n-m}$.

Under the egalitarian rule, when $r = 0$, for any number of seats k a party won, all politicians of that party have the same probability k/n of winning a seat. Then, politician i in team j chooses his level of effort to solve:

$$Max_{e_{ij}} \left(V \sum_{k=1}^{n} P_j(k) \frac{k}{n} - \frac{e_{ij}^{\beta}}{\beta} \right) \tag{8}$$

4 Equilibrium efforts

4.1 Closed lists

Each candidate maximizes their objective function, equation (4), with respect to their individual effort choice. Even though this problem is individual-specific as it depends on the position on the list a candidate is in, the equilibrium is symmetric across parties. We then have:

Proposition 1 *Under the closed-list, in a symmetric Nash equilibrium, a party's electoral output E_{CL}^* is given by:*

$$E_{CL}^* = \left[\sum_{k=1}^{n} \left(k C_k^n \left(\frac{1}{2} \right)^{n-1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \right]^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left(\frac{V}{4} \right)^{\frac{1}{\beta}} \tag{9}$$

Individual effort e_m^* of list member in mth position is given by:

$$e_m^* = \left(\frac{V \left(m C_m^n \left(\frac{1}{2} \right)^{n+1} \right)^{\frac{\beta}{\beta+\sigma-1}}}{\sum_{k=1}^{n} \left(k C_k^n \left(\frac{1}{2} \right)^{n+1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}} \right)^{1/\beta} \tag{10}$$

Proof. See appendix. ■

[12] We are thankful to an anonymous referee for suggesting this interpretation of the egalitarian rule.

Given that the distribution of mC_m^n is symmetric and unimodal at the median value of m, individual effort is bell-shaped with the candidate in the median list position exerting the highest amount of effort. Figure 2 illustrates this finding. This property of the distribution of efforts within parties turns out to be crucial for our result.

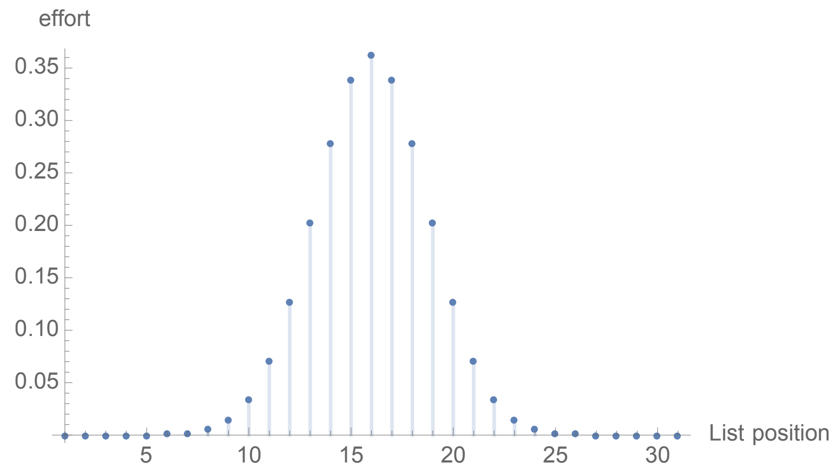

Figure 2: Distribution of efforts under closed-list PR

4.2 Open lists

With open lists, the game is completely symmetric: all candidates maximize the same objective function (7). We then have:

Proposition 2 *Under the open-list, in a symmetric Nash equilibrium, a party's electoral output E^*_{OL} is given by:*

$$E^*_{OL} = n^{\frac{1}{1-\sigma}} \left[\frac{V}{4n} + rV \sum_{m=1}^{n} C_m^n \left(\frac{1}{2}\right)^n \left(1 - \frac{m}{n}\right) \left(\sum_{j=1}^{m} \frac{1}{n-j+1} \right) \right]^{1/\beta} \tag{11}$$

Individual effort e^*_{OL} is given by:

$$e^*_{OL} = \left[\frac{V}{4n} + rV \sum_{m=1}^{n} C_m^n \left(\frac{1}{2}\right)^n \left(1 - \frac{m}{n}\right) \left(\sum_{j=1}^{m} \frac{1}{n-j+1} \right) \right]^{1/\beta} \tag{12}$$

Proof. See appendix. ■

The first term within the square brackets corresponds to the incentives to work for the party to increase the number of seats won, the second term corresponds to the incentives to get preferences votes to improve one's position on the list. As r goes to 0, the second term vanishes and effort converges to the effort under the egalitarian allocation rule, namely $\left(\frac{V}{4n}\right)^{1/\beta}$. Individual effort increases in the value of a seat V and decreases with the number of available seats n, while a party's electoral output increases in both V and n.

4.3 Comparing closed lists and open lists

4.4 Effort comparison

The two systems differ in two main dimensions. The first difference comes from preference votes. With open lists, individual effort influences both the party's electoral success and the candidate's intraparty ranking (in terms of preference votes). Competing for preference votes within one's party is a source of incentives that does not exist with closed lists.

The second difference comes for the homogeneity (or heterogeneity) of incentives. With open lists, all politicians face the same incentives and exert the same effort in equilibrium. With closed lists, incentives and individual equilibrium efforts vary with a candidate's position on the list. In particular, the politicians located around the median position face the largest marginal benefit of effort. To the contrary, candidates at the top and at the bottom of the list have little incentive to exert effort.

The received wisdom in the political economy and political science literature typically views open lists as superior to closed lists. That is mainly because preference votes under open lists mitigate the free-riding issue. The heterogeneity of incentives also creates very weak incentives for many candidates on the list (those near the top and near the bottom). However, if incentives are indeed weak at the top and bottom of the list, they are much stronger for politicians in the middle of the list.

To compare the parties' electoral outputs under the two ballot structures, we first shut down preference votes – setting r to zero under open lists – and compare a closed list to the egalitarian rule. The comparison boils down to the effect of the heterogeneity of incentives on party output. We show that the convexity of the cost function (β) and the degree of complementarity (σ), play a central role. When the cost of effort is not too convex and efforts are not strong complements, heterogeneous incentives lead to higher party output, and a closed list dominates the egalitarian rule. By continuity, when voters are not too well informed about the individual efforts of candidates (r is close enough to 0), closed lists lead to higher party outputs than open lists. We summarize our findings in the following proposition.

Theorem 3 *Closed lists lead to higher party output than open lists PR,* $E^*_{CL} \geq E^*_{OL}$ *when* $\beta \leq 2 - 2\sigma$ *and* r *is small enough. Open lists lead to higher party output than closed lists,* $E^*_{OL} \geq E^*_{CL}$ *when* $\beta \geq 2 - 2\sigma$ *or when* $\beta \leq 2 - 2\sigma$ *and* r *is large enough. The two types of ballot yield the same party's output* $E^*_{OL} = E^*_{CL}$ *when* $\beta = 2 - 2\sigma$ *and* $r = 0$.

Proof. See appendix. ■

The intuition behind this result is as follows. As we saw above, individual incentives are uniform under the egalitarian rule and bell-shaped under closed lists. Suppose that the cost of effort function is close to being linear (β is close to 1). If individual efforts are highly substitutable (σ close to 0), electoral output is close to equal to the sum of efforts, and this sum is what matters, not so much the level of the different individual efforts. In this case, inducing differences in efforts can be optimal. When efforts are complementary ($\sigma > 1/2$), inducing differences in individual efforts is suboptimal as a party's electoral output depends more heavily on the lowest effort decisions. What about the convexity of the cost of effort function? Suppose for simplicity that $\sigma = 0$. When this function is very convex ($\beta > 2$), the marginal cost is also convex. Then, asymmetric incentives are bad for party performance. Indeed, starting from equal marginal benefits of effort, increasing the marginal benefit of one politician and decreasing the benefit of another will have a positive effect if the marginal cost increases more slowly for the individual with stronger incentives than for the one with weaker incentives. However, when the marginal cost of effort is convex, this is simply not possible.

For an example on low complementarity between party candidates, consider an election centered around a 'standard' electoral issue. Then candidates expose and defend the 'conventional' party line that was crafted by the party specialists who worked on the issue. Once the party platform is finalized, most other candidates can simply stand behind their party's official position. To the contrary, if the central electoral issue requires parties to make use of a portfolio of skills and competencies to develop their platform, then the efforts put in by their candidates become much more complementary to each other, as the decision to put low effort into the contest by a single candidate can 'destroy' all the efforts of their party fellows.

An increase in r increases the incentives to exert effort under an open list but not under a closed list. When preference votes are very responsive to individual effort, exerting effort is less about contributing to the public good and more about the individual benefit of improving one's rank in the allocation of seats. In what follows, we set r to zero and focus on the comparison between a closed list system and the egalitarian rule.

4.5 Choice of electoral system

The design of better electoral systems is at the center of the public debate. Many countries, including France, have thought about moving towards PR but agreement over which type of PR to adopt is more difficult. Theorem 3 has direct implications in terms of constitutional design. A benevolent constitution would pick the optimal electoral system in terms of society welfare. In the model, candidates' effort and electoral output are interpreted as a means towards improving the quality of the party's political platform. Effort benefits voters and a good electoral system is a system that maximizes the party's electoral outputs. In that case, the conditions of theorem 3 should guide the design of the optimal system. If voters are well informed about candidates efforts, then an open-list system could be superior to a closed list system. If voters are poorly informed about politicians' effort, an open-list system loses its main advantage. In that case, the convexity of the effort case and the complementarity of candidates effort become the driving forces behind the choice of which electoral system to adopt.

In the model, effort by candidates is not only effort spent to mobilize and attract voters around the party platform during the campaign. Effort also helps to create an appropriate and high quality party platform. Even with closed lists, party candidates have several months between the moment the list is announced and the election. This gives them time to improve the electoral platform that they will defend during the campaign in the weeks before election day. For example, in Belgium, the names of the leading candidates of all mainstream parties were announced around Christmas 2018, for the May 2019 legislative elections (together with those for the European Parliament). This is roughly 5-6 months ahead of election day. Once parties announced their leading candidates, these moved immediately to the preparation of their party programme, which they release a few weeks before election day. All the leading figures of the party help craft the programme, with a special role for the leading candidate's garde rapprochee, a set of politicians who typically ends up in the leading positions of the party list (within party list turnover between elections is quite high, especially when there is a change of leadership). In this preparation phase, intraparty work to craft and finalize the party programme is at its peak. Nevertheless, if we were to interpret effort as advertising resources spent during the campaign to convince voters but that are not welfare enhancing in and by themselves, the constitution should then aim at minimizing such efforts. In that case, the condition of theorem 3 would still apply but its implications in terms of choice of electoral system would have to be reversed.

If it is natural to consider the choice between closed-list and open-list at the level of constitutional design, the party leadership also has its say in practice. A striking example is Colombia. After a reform in 2003, each party can present and chose the type of ballot to use in any district: parties can opt for closed or open lists, and the choice can differ across districts; see Shugart, Moreno and Fajardo (2006) and Hangartner, Ruiz and Tukiainen (2018) for more on this case.

Less extreme examples can be found in many countries in which parties adopt strategies and practices that appear to be a strategic reaction to some aspects of the electoral environment they are embedded in. For instance, Italy before the Mani Pulite scandal of 1992 was officially using open-list PR.[13] However, in practice, many parties were using methods resembling a closed-list system. Katz (1985) argues that the Communist party was closely controlling the list of candidates and would give instructions to their partisans about preference votes. The final outcome would look like that under a closed list, as the party decided the ranking of candidates and voters implemented it.

The opposite situation also happens in countries with a closed-list system. Some parties organize primary elections to decide the names and positions of list candidates. Even if the set of voters in the primary and in the general election is not exactly the same, organizing a primary clearly shifts a closed-list system towards an open-list system. For instance, in the last Israeli elections, several parties, including Likud, Labor, the Jewish Home, and Meretz had systems in which the leadership and most candidates on their lists were first elected in primary elections. Similarly, some small Turkish parties use primary elections to set up their list of candidates.

To study such strategies, we add a stage to the model. In stage one, the party leadership chooses between the egalitarian rule (open list with no preference votes) and a closed list. In the second stage, after observing the choice of both parties, candidates choose how much effort to exert. In the appendix, we solve for equilibrium efforts in the subgame in which parties choose different rules. We show that a party's electoral success follows the same condition as that of Theorem 3. This means that in the subgame perfect equilibrium of the two-stage game, the condition driving the parties' choice of the allocation rule is the same as in Theorem 3.

Proposition 4 *In the subgame perfect equilibrium of the two-stage game, parties choose a closed list if $\beta \leq 2 - 2\sigma$ and the egalitarian rule if $\beta \geq 2 - 2\sigma$.*

[13] Between 1946 and 1993, parties stood in front of voters with lists of candidates in each of Italy's 32 constituencies. Voters could give as many as four preference votes to the candidates of their favorite party. Seats won by a party went to the candidates with the highest number of preference votes.

Proof. See appendix. ∎

Thus, for given values of β and σ, parties have a dominant strategy (strictly dominant when $\beta \neq 2-2\sigma$) in the choice of allocation rule. Note that this proposition does not require that parties have the same values of parameters β and σ. As the proof makes apparent, the choice of system that maximizes effort does not depend on the output of the other party.

To wrap up, theorem 3 and proposition 4 show that the convexity of the marginal cost of effort, the complementarity of the team production function and the noisiness of the election drive the choice of the allocation rule. The noisiness of the election plays a very intuitive role. When the marginal cost of effort is convex, giving powerful incentives to a few individuals is not productive, as even these individuals are not going to exert much effort. With convex marginal costs, it is thus more efficient to give all politicians within a party the same incentives and treat them in an equal, symmetric way. When the marginal cost is not too convex or even concave, it is efficient to provide powerful incentives to few individuals who will exert very high levels of effort. The degree of complementarity plays a similar role. When efforts are substitutes, there is no cost in generating very different effort levels within the team. When efforts are complementary, it is better to induce similar effort levels, so that the egalitarian rule is optimal.

The above findings suggest that when efforts are not strong complements within each party and/or the individual cost of effort is not too convex, the use of closed lists can be optimal. In particular, it gives better incentives than a system which treats all candidates in an ex-ante fair and egalitarian way. Thus, theorem 3 and proposition 4 provide an argument for the use of closed lists PR. In the next section, we go further and show that closed lists and the egalitarian rule are actually the two incentive mechanisms that maximize party outputs if we impose a weak monotonicity constraint.

5 Mechanism design

For simplicity, we assume in this section that efforts are perfect substitutes, $\sigma = 0$.[14] An allocation rule can be represented as a $n * (n+1)$ matrix of weights $[\lambda_{ik}]$. Entry λ_{ik} corresponds to the probability that politician i gets a seat when his party has won k seats. Probabilities need to be between 0 and 1, and the number of seats distributed cannot be larger than the number of seats

[14]The results also hold for $\sigma > 0$, but the algebra is cumbersome. The condition for proposition 4 would then be in terms of $\beta - 2\sigma$ (as in theorem 3).

won by the party. These feasibility constraints thus require $0 \leq \lambda_{ik} \leq 1$ and $\sum_{i=1}^{n} \lambda_{ik} = k$. We focus on monotonic rules. Under a monotonic rule, the probability that a candidate wins a seat is (weakly) increasing in the number of seats won by their party, that is $\lambda_{ik} \leq \lambda_{ik+1}$ for any i and k.

The egalitarian allocation rule can be represented as a matrix in which each column has equal entries $\lambda_{ik} = k/n$. The closed-list can be represented as a matrix with $\lambda_{ik} = 0$ if $i > k$ and $\lambda_{ik} = 1$ if $i \leq k$.

Proposition 5 *When $\beta > 2$, the egalitarian rule maximizes party output among all monotonic rules. When $\beta < 2$, closed lists maximize party outputs. When $\beta = 2$, both rules maximize party outputs.*

Proof. See appendix. ■

The intuition behind the result is simple. The intraparty prize allocation rule determines individual incentives and thus effort choices. When $\beta > 2$, it is optimal to equalize incentives across politicians within a party, while when $\beta < 2$, it is optimal to make incentives as strong as possible for some politicians. The maximization problem is similar to that of the optimal allocation of risk. With risk-averse individuals, the allocation is as egalitarian as possible, while with risk-loving agents, it is optimal to make the allocation as unequal as possible.

If we remove the monotonicity constraint, an allocation rule can give negative incentives to some politicians. Negative incentives appear when an individual faces a higher probability of getting a seat when his team gets fewer seats. Yet, the effect of these negative incentives is limited, as effort cannot be negative. Also, negative incentives free up incentive tokens that can be redistributed to other politicians. The combination of the zero lower bound on effort and the possibility of redistributing incentives may then generate a higher electoral output than under the optimal monotonic rule.

We illustrate in the appendix how this redistribution of incentives can indeed generate higher electoral output with two examples. As the optimal rule can be non-monotonic, the politicians' labels should no longer be given any ranking interpretation. Indeed, a politician with a lower rank is not necessarily treated more favorably by a non-monotonic rule. In the context of elections, these non-monotonic rules are more of a theoretical curiosity than a realistic mechanism. However, in other contexts where team contests for multiple prizes exist, using non-monotonic incentive schemes could be of interest.

6 Extensions and robustness checks

6.1 More than two parties, ideology, and noise

We now extend our model in some important directions. All formal derivations can be found in Appendix B. We first allow for $K > 2$ parties as it is common to see more than two parties competing in elections under PR. With more than two parties, the distribution of efforts under closed lists becomes right-skewed. Indeed, in a symmetric equilibrium, each politician expects his party to win n/K prizes. Thus, depending on the number of parties, the relevant median list member, who exerts highest effort, is around position n/K on the party list. With two parties, the member exerting highest effort was in the middle of the list.

Second, we extend the model to allow for biased contests. In real world elections, some parties enjoy an ex-ante ideological advantage over their competitors. We can view such an advantage as a bias in favour of one of the parties in the contest. Suppose party 1 is advantaged over party 2. We assume that the probability that party 1 wins a seat given outputs E_1 and E_2 as $\lambda E_1 / (\lambda E_1 + E_2)$ with $\lambda > 1$. The distribution of seats is now given by:

$$P_1(k) = C_k^n \left(\frac{\lambda E_1}{\lambda E_1 + E_2}\right)^k \left(1 - \frac{\lambda E_1}{\lambda E_1 + E_2}\right)^{n-k}.$$

Finally, we can use $\frac{E_i^\gamma}{E_1^\gamma + E_2^\gamma}$ or $\frac{\gamma}{2} + (1-\gamma)\frac{E_i}{E_1+E_2}$ as the probability that a party wins a given seat, where γ parametrises the responsiveness of success to effort.

These three extensions do not affect our main result and Theorem 3 still applies in those cases.

6.2 Access to government and internal party struggles

Politicians care not only about getting a seat (as we assumed so far) but also want their party to win a majority of seats to gain control of the executive office. Assume that candidates enjoy an additional payoff M when their party gets at least a majority of the legislative seats.

Under the egalitarian rule, candidates choose their effort to maximise:

$$V \sum_{k=1}^{n} P_j(k) \frac{k}{n} + M \sum_{k=\frac{n+1}{2}}^{n} P_j(k) - \frac{e^\beta}{\beta}$$

where $\sum_{k=\frac{n+1}{2}}^{n} P_j(k)$ is the probability that the candidate's party wins at least a majority of the seats.

With a closed list, the candidate in position m on the list of party j chooses e_m to maximize:

$$V \sum_{k=m}^{n} P_j(k) + M \sum_{k=\frac{n+1}{2}}^{n} P_j(k) - \frac{e_m^{\beta}}{\beta}$$

In the appendix, we solve for the equilibrium efforts and show that theorem 3 goes through unchanged.

We can also consider the case in which candidates have to compete within their party to secure (some of) the benefits of their party winning control of the executive (in the spirit of Katz and Tokatlidu, 1996).[15] We do not need to model explicitly this post-election contest. From an ex-ante perspective, internal struggles decrease the value of winning the executive office, as this ex-post contest leads to rent dissipation. If we parametrize rent dissipation by some parameter $\lambda < 1$ the benefits of winning office are now given by λM. The more costly this struggle is, the lower value of λ, the less value candidates attach to their party winning a majority of seats. In the limit, the prospect of such an intraparty struggle can totally nullify the incentive benefits of the presence of the executive office. If the intensity of the post-election struggles does not depend on the ballot structure (λ is the same with open lists and closed lists), then the comparison between the ballot structures of theorem 3 remains valid. The intensity of the post-election struggles could also change with the ballot structure. However, it is not clear which system leads to more rent dissipation. If the closed list clearly identifies the candidates in line for the top executive positions, an open list system could also give legitimacy to candidates that gathered the most preference votes.

6.3 Contractible efforts

We now allow parties to make the allocation of seats directly depend on the efforts of the candidates. When efforts are contractible, allocation rules can rely not only on the incentives generated by the number of prizes won by the party, but also directly on the effort exerted by each candidates. Under the egalitarian rule, the contract between the party and each candidate specifies that, provided the candidate exerts at least e^* (that is pinned down by a candidate's participation constraint), his chance of getting one of the m seats won by the party is equal to m over the number of candidates who honored their party contract. In equilibrium, this chance is thus m/n. Any deviation below e^* makes that probability go to zero.

Under the closed list, the contract still assigns to each politician a rank in the list, but now also specifies a minimal effort level associated with each rank. If a politician exerts the specified effort

[15] We are grateful to the Editor for suggesting this extension.

(or more), they get a seat if the party wins a number of seats equal to at least their rank. If they exert less effort or if the party wins fewer seats than their rank, they get no seat.[16] Then, we have:

Proposition 6 *When efforts are contractible, the egalitarian allocation rule leads to higher electoral output than the closed-list for all values of parameters β and σ. Thus open lists dominate closed lists for any value of $r \geq 0$.*

Proof. See appendix. ■

When efforts are observable and contractible, the level of effort is determined by the participation constraint. The contract imposes an effort level that drives each politician's utility to their outside option. The cost of effort enters directly in the participation constraint, whereas when effort is not contractible, the marginal cost is what matters. Comparing electoral outputs across the two rules, we find that the egalitarian rule generates higher output than the closed list when $\beta \geq 1 - \sigma$, which is always satisfied. Thus, with contractible effort, it is always optimal to treat all politicians equally. This last finding suggests that the imperfect contractibility of effort is a necessary condition for the optimality of closed lists.

Whereas the extent to which parties (and voters) are informed about the decisions of politicians is an empirical question, it seems natural to assume that reality is in between the two extremes of perfectly observable and perfectly unobservable efforts.

7 Conclusion

We studied the incentive effects of the two main ballot structures in proportional representation elections: open and closed lists. We showed that the amount of information voters have about candidate choices, the convexity of the marginal cost of effort and the degree of complementarity between efforts within parties drive which ballot structure is best for incentives. Voters' information about candidates' effort increases the responsiveness of preference votes to effort and favors the open list system. A convex marginal cost of effort and the presence of complementarity in candidates' efforts makes homogeneous incentives more effective than heterogeneous ones. In that case, an open list dominates a closed one, even when voters are poorly informed. When the cost of effort is not too convex, efforts are substitutes and voters are poorly informed, a closed list may dominate an open one.

[16]For the contract to be credible, the part must have a list of at least n+1 candidates, with these extra candidates only winnning a seat if some of the other candidates did not exert the required effort.

Our model is tractable and is amenable to many extensions and applications. For instance, Crutzen and Sahuguet (2018) adapt the present set-up to compare politicians' effort under majoritarian and proportional electoral systems (in particular British-style first-past-the-post and Israeli-style closed-list PR) when political parties play an active role in the selection of their candidates. They find that the received wisdom that suggests that majoritarian systems provide incentives more efficiently than PR may need revisiting when parties are active players in the selection process.

The main limitation of the current paper is the symmetry imposed in the model. Politicians have the same effort productivity (or cost of effort), and candidates are competing for prizes of identical value (a seat in parliament). Konishi, Sahuguet, Crutzen and Flamand (2018) extend the present model to allow for candidates with heterogeneous abilities and allow for additional payoffs linked to minister positions. The paper then studies how parties should constitute their lists and where on the list they should put their most able candidates. Another important puzzle they address is: why are a party's lead candidates teaming up with the other candidates on the electoral list when it is anyway common knowledge that most of these candidates will take on jobs and positions after the election that are not compatible with them sitting in parliament?

Finally, if our model is well suited for the analysis of elections under PR, it could also be used in other contexts where contests between teams that allocate multiple indivisible prizes are important. For instance, our model delivers stark predictions that could be empirically tested in the context of the internal organization of firms. In some industries, complementarity among workers are much more important than in others. In such sectors, compensation contracts should treat workers more equally than in other sectors. Some public finance problems, especially in federal democracies, can also be modelled as team contests for multiple indivisible prizes. Examples include the construction of schools, hospitals and military bases for which several local governments (municipalities) belonging to larger subnational entities (regions, states) may compete.

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Appendix A (proofs)

Proof of proposition 1

We start by proving two useful lemmas.

Lemma 1: $\frac{dE_j}{de_{ij}} = \left(\frac{E_j}{e_{ij}}\right)^{\sigma}$

Proof:

Using the definition of E_j, $E_j = \left(\sum_{i=1}^{n} (e_{ij})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, we get:

$$\begin{aligned}
\frac{dE_j}{de_{ij}} &= \frac{1}{1-\sigma}(1-\sigma)(e_{ij})^{-\sigma}\left(\sum_{i=1}^{n}(e_{ij})^{1-\sigma}\right)^{\frac{1}{1-\sigma}-1} \\
&= (e_{ij})^{-\sigma}\left(\sum_{i=1}^{n}(e_{ij})^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \\
&= \left(\frac{E_j}{e_{ij}}\right)^{\sigma}
\end{aligned}$$

$$e_m^* = \left(E_{CL}^{*\sigma-1} mC_m^n \left(\frac{1}{2}\right)^{n+1} V\right)^{\frac{1}{\beta+\sigma-1}} \tag{13}$$

$$= \left(\frac{V}{\sum_{k=1}^{n}\left(kC_k^n \left(\frac{1}{2}\right)^{n+1}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}}\right)^{1/\beta} \left(mC_m^n\left(\frac{1}{2}\right)^{n+1}\right)^{\frac{1}{\beta+\sigma-1}} \tag{14}$$

■

Lemma 2

$\sum_{k=m}^{n} C_k^n \left(kp^{k-1}(1-p)^{n-k} - (n-k)(1-p)^{n-k-1}p^k\right) = mC_m^n p^{m-1}(1-p)^{n-m}$

Proof:

We show that terms in the sum cancel. Consider the second term within the summation sign: $(n-k)C_k^n(1-p)^{n-k-1}p^k$. Using the identity $(n-k)C_k^n = (k+1)C_{k+1}^n$ we can write it as $(n-k)C_k^n(1-p)^{n-k-1}p^k = (k+1)C_{k+1}^n(1-p)^{n-k-1}p^k$, which corresponds exactly to the first term within the summation sign for the index $k+1$. These two terms cancel leaving only the first and last term of the sum. The first term is $mC_m^n p^{m-1}(1-p)^{n-m}$. The last term is equal to zero.

■

Under closed-list PR, the politician on the mth position on the list maximizes:

$$\sum_{k=m}^{n} C_k^n \left(\frac{E_j}{E_1+E_2}\right)^k \left(1-\frac{E_j}{E_1+E_2}\right)^{n-k} V - \frac{e_{mj}^{\beta}}{\beta}$$

Denoting $p_j = \frac{E_j}{E_1+E_2}$, the first-order condition is:

$$V\sum_{k=m}^{n}\frac{dE_j}{de_{ij}}\frac{E_i}{(E_1+E_2)^2}C_k^n\left(k\left(p_j\right)^{k-1}\left(1-p_j\right)^{n-k}-(n-k)\left(p_j\right)^k\left(1-p_j\right)^{n-k-1}\right)-\left(e_{mj}\right)^{\beta-1}=0$$

At a symmetric equilibrium, using Lemmas 1 and 2, the first-order condition simplifies to:

$$\left(\frac{E}{e_m}\right)^{\sigma}\frac{m}{E}C_m^n\left(\frac{1}{2}\right)^{n+1}V-e_m^{\beta-1}=0$$

Thus

$$\begin{aligned}
(e_m)^{\beta+\sigma-1} &= E^{\sigma-1}mC_m^n\left(\frac{1}{2}\right)^{n+1}V \\
&\Longleftrightarrow \\
e_m &= \left(E^{\sigma-1}mC_m^n\left(\frac{1}{2}\right)^{n+1}V\right)^{\frac{1}{\beta+\sigma-1}}
\end{aligned}$$

A party's electoral output is thus:

$$\begin{aligned}
E &= \left\{\sum_{k=1}^{n}\left(E^{\sigma-1}kC_k^n\left(\frac{1}{2}\right)^{n+1}V\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{1}{1-\sigma}} \\
&\Longleftrightarrow \\
E^{\frac{\beta}{\beta+\sigma-1}} &= \left\{\sum_{k=1}^{n}\left(kC_k^n\left(\frac{1}{2}\right)^{n+1}V\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{1}{1-\sigma}} \\
&\Longleftrightarrow \\
E &= \left(\frac{V}{4}\right)^{\frac{1}{\beta}}\left\{\sum_{k=1}^{n}\left(kC_k^n\left(\frac{1}{2}\right)^{n-1}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.
\end{aligned}$$

■

Proof of proposition 2

The first-order condition to the problem faced by candidate i (in party L, say) is:

$$\left[\sum_{m=1}^{n}\frac{\partial P_L(l)}{\partial e_i}Q_i(m)+\sum_{m=1}^{n}P_L(m)\frac{\partial Q_i(m)}{\partial e_i}\right]V=e_i^{\beta-1}.$$

As all candidates face the same problem, the equilibrium is symmetric within and across parties. As there are n candidates competing for one of the list slots, the equilibrium probability of being offered slot m on the list is $Q^*(m) = \frac{m}{n}$. We also have:

$$\frac{dP_L(m)}{de_i} = \left(\frac{E_1}{e_i}\right)^\sigma \frac{E_2}{(E_1+E_2)^2} C_m^n (2m-n)\left(\frac{1}{2}\right)^{n-1} = n^{\frac{\sigma}{1-\sigma}} \frac{C_m^n}{4E}(2m-n)\left(\frac{1}{2}\right)^{n+1}$$

Finally:

$$\frac{dQ_i(m)}{de_{iL}} = \frac{r}{e^*}\left(1-\frac{m}{n}\right)\sum_{j=1}^{m}\frac{1}{n-j+1}$$

and thus in the symmetric equilibrium, the FOC boils down to:

$$\begin{aligned}(e^*)^{\beta-1} &= V\sum_{m=1}^{n} n^{\sigma/1-\sigma}\frac{C_m^n}{n^{1/1-\sigma}e^*}(2m-n)\left(\frac{1}{2}\right)^{n+1}\frac{m}{n} + Vr\sum_{m=1}^{n} C_m^n\left(\frac{1}{2}\right)^n\left[\frac{1}{e^*}\left(1-\frac{m}{n}\right)\sum_{j=1}^{m}\frac{1}{n-j+1}\right] \\ &= \frac{V}{n^2}\sum_{m=1}^{n}\frac{C_m^n}{e^*}(2m-n)\left(\frac{1}{2}\right)^{n+1} m + Vr\sum_{m=1}^{n} C_m^n\left(\frac{1}{2}\right)^n\left[\frac{1}{e^*}\left(1-\frac{m}{n}\right)\sum_{j=1}^{m}\frac{1}{n-j+1}\right]\end{aligned}$$

Recall that $(1+x)^n = \sum_{m=0}^{n} C_m^n x^m$

Taking the derivative with respect to x leads to

$$n(1+x)^{n-1} = \sum_{m=0}^{n} mC_m^n x^{m-1}$$

and doing it again yields:

$$n(n-1)(1+x)^{n-2} = \sum_{m=0}^{n} m(m-1)C_m^n x^{m-2}$$

Setting $x = 1$, in these formulas yield:

$$\sum_{m=0}^{n} mC_m^n = \sum_{m=1}^{n} mC_m^n = n2^{n-1}$$

Similarly,

$$\begin{aligned}\sum_{m=0}^{n} m^2C_m^n = \sum_{m=1}^{n} m^2C_m^n &= n(n-1)2^{n-2} + \sum_{m=1}^{n} mC_m^n \\ &= n(n-1)2^{n-2} + n2^{n-1} \\ &= \left(n^2+n\right)2^{n-2}\end{aligned}$$

Thus:

$$\begin{aligned}\sum_{m=1}^{n} C_m^n(2m-n)\left(\frac{1}{2}\right)^{n+1} m &= \left(\frac{1}{2}\right)^{n+1}\left(2\sum_{m=1}^{n} m^2C_m^n - n\sum_{m=1}^{n} mC_m^n\right) \\ &= \left(\frac{1}{2}\right)^{n+1}\left[2\left(n^2+n\right)2^{n-2} - n^2 2^{n-1}\right] \\ &= \frac{n}{4}.\end{aligned}$$

Equilibrium effort e^* is therefore given by:

$$e^* = \left(\frac{V}{4n} + rV \sum_{m=1}^{n} C_m^n \left(\frac{1}{2}\right)^n \left(1 - \frac{m}{n}\right) \left(\sum_{j=1}^{m} \frac{1}{n-j+1} \right) \right)^{1/\beta}.$$

■

Proof of theorem 3

Set r to zero. Comparing efforts under both allocation rules, we see that the egalitarian rule dominates the closed-list when:

$$n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} > \left\{ \sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{2}\right)^{n-1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}$$

We can rewrite this inequality as:

$$\left\{ \frac{\sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{2}\right)^{n-1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}}{n} \right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} < 1$$

which simplifies as:

$$\sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{2}\right)^{n-1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} < n.$$

Note that $\sum_{k=1}^{n} kC_k^n \left(\frac{1}{2}\right)^{n-1} = n$. Jensen's inequality realtes the concavity or convexity of the function $g(x) = x^{\frac{1-\sigma}{\beta+\sigma-1}}$ to whether the inequality is satisfied or not.

We have that $\sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{2}\right)^{n-1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \leq n$ when $\frac{1-\sigma}{\beta+\sigma-1} \leq 1$. This last inequality simplifies to $\beta \geq 2 - 2\sigma$.

To prove the last statement of theorem 3, just notice that equilibrium effort under open list increases with r.

■

Proof of proposition 4 (choice of rule by party)

Given the choice of allocation rule by one party, the best response of the other party is to choose the allocation rule that maximizes party output the number of seats won. We need to show that the condition $\beta \geq 2 - 2\sigma$ also determines the ranking of outputs between the egalitarian rule and the closed-list

We first consider the egalitarian allocation rule. The first order condition of party 1's ith politician under the egalitarian allocation rule is:

$$V\left(\frac{E_1}{e_{i1}}\right)^{\sigma}\frac{E_2}{(E_1+E_2)^2}-(e_{i1})^{\beta-1}=0.$$

This yields, denoting $p_1=\frac{E_1}{E_1+E_2}$:

$$Vn^{\frac{\sigma}{1-\sigma}}\frac{p_1\left(1-p_1\right)}{E_1}=(e_{i1})^{\beta-1}.$$

Thus:

$$e_1=\left(Vn^{\frac{\sigma}{1-\sigma}}\frac{p_1\left(1-p_1\right)}{E_1}\right)^{\frac{1}{\beta-1}}.$$

Therefore

$$\begin{aligned} E_1 &= \left\{\sum_{k=1}^{n}\left(\left(Vn^{\frac{\sigma}{1-\sigma}}\frac{p_1\left(1-p_1\right)}{E_1}\right)^{\frac{1}{\beta-1}}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} \\ &= n^{\frac{1}{1-\sigma}}\left(\frac{p_1\left(1-p_1\right)}{E_1}V\right)^{\frac{1}{\beta-1}}n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}. \end{aligned}$$

which implies

$$E_1=\left(p_1\left(1-p_1\right)V\right)^{\frac{1}{\beta}}n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.$$

We now turn to the closed-list. Under this rule, the first order condition for the mth politician on the list of party 1 is:

$$V\left(\tfrac{E_1}{e_{m1}}\right)^{\sigma}\tfrac{E_2}{(E_1+E_2)^2}\sum_{k=m}^{n}C_k^n\left(kp_1^{k-1}\left(1-p_1\right)^{n-k}-(n-k)\,p_1^k\left(1-p_1\right)^{n-k-1}\right)=(e_{m1})^{\beta-1}.$$

Using Lemma 2 and exploiting the fact that $\frac{E_2}{(E_1+E_2)^2}=\frac{p_1(1-p_1)}{E_1}$, this simplifies to:

$$e_{m1}=\left(VE_1^{\sigma}C_m^n\frac{m}{E_1}p_1^m\left(1-p_1\right)^{n-m+1}\right)^{\frac{1}{\beta+\sigma-1}}.$$

And thus

$$
\begin{aligned}
E_1 &= \left\{ \sum_{k=1}^{n} \left[\left(V E_1^{\sigma} C_k^n \frac{k}{E_1} p_1^k \left(1-p_1\right)^{n-k+1} \right)^{\frac{1}{\beta+\sigma-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
&\iff \\
E_1^{\frac{\beta}{\beta+\sigma-1}} &= \left\{ \sum_{k=1}^{n} \left(k C_k^n p_1^k \left(1-p_1\right)^{n-k+1} V \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \right\}^{\frac{1}{1-\sigma}} \\
&\iff \\
E_1 &= \left\{ \sum_{k=1}^{n} \left(k C_k^n p_1^{k-1} \left(1-p_1\right)^{n-k} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left(p_1 \left(1-p_1\right) V\right)^{\frac{1}{\beta}} .
\end{aligned}
$$

We need to compare the expected number of seats won under both allocation rules, given the other party's allocation rule. We know that if both parties choose the same allocation rule, individual efforts are symmetric and thus party electoral outputs are equal and both parties win on average $n/2$ seats. We therefore need to compare party electoral outputs when parties choose different allocation rules.

From the calculation above, if party 1 uses the egalitarian rule and party 2 uses the closed-list, denoting $p_j = E_1/(E_j + E_2)$, we get:

$$
\begin{aligned}
E_1 &= \left[p_1 \left(1-p_1\right) V\right]^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}, \\
E_2 &= \left\{ \sum_{k=1}^{n} \left[k C_k^n p_2^{k-1} \left(1-p_2\right)^{n-k} \right]^{\frac{1-\sigma}{\beta+\sigma-1}} \right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left[p_2 \left(1-p_2\right) V\right]^{\frac{1}{\beta}}
\end{aligned}
$$

Dividing the above two party outputs and exploiting the fact that $p_1 = 1 - p_2$ which implies that $\left(p_1 \left(1-p_1\right) V\right)^{\frac{1}{\beta}} = \left(p_2 \left(1-p_2\right) V\right)^{\frac{1}{\beta}}$, we get:

$$
\frac{E_2}{E_1} = \left(\frac{\sum_{k=1}^{n} \left[k C_k^n p_2^{k-1} \left(1-p_2\right)^{n-k} \right]^{\frac{1-\sigma}{\beta+\sigma-1}}}{n} \right)^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} .
$$

We need to pin down the conditions such that $\frac{E_2}{E_1}$ is greater or smaller than 1. Remark now that $\sum_{k=1}^{n} \left(k C_k^n p_2^{k-1} \left(1-p_2\right)^{n-k} \right) = n$. Then we use the argument based on Jensen's inequality used in the proof of Proposition 3 to show that the condition $\beta > 2 - 2\sigma$ determines which party has the highest output. This implies in turn that it is a dominant strategy for both parties to choose the egalitarian allocation rule if and only if $\beta > 2 - 2\sigma$. If $\beta = 2 - 2\sigma$ both rules yield the same payoff. ■

Proof of proposition 5 (mechanism design)

To simplify the exposition, we only present the case of perfect substitutes ($\sigma = 0$). The extension to the case of complements is straightforward, albeit algebraically more tedious.

Politician i in party j maximizes $V \sum_{k=1}^{n} \lambda_{ik} P_j(k) - \frac{e_{ij}^{\beta}}{\beta}$. In a Nash equilibrium, the first order condition implies that the optimal effort of i is given by:

$$e_i = \max \left\{ \left(V \sum_{k=1}^{n} \frac{dE}{de_{ij}} \frac{E_i}{(E_1+E_2)^2} \lambda_{ik} C_k^n \left(k P_j^{k-1} (1-P_j)^{n-k} - (n-k) P_j^k (1-P_j)^{n-k-1} \right) \right)^{\frac{1}{\beta-1}}, 0 \right\} \tag{15}$$

At a symmetric Nash equilibrium, the above boils down to:

$$e_i = \max \left\{ \left(V \sum_{k=1}^{n} \frac{dE}{de_i} \frac{1}{4E} \lambda_{ik} C_k^n \left(\frac{1}{2} \right)^{n-1} (2k-n) \right)^{\frac{1}{\beta-1}}, 0 \right\} \tag{16}$$

Simplifying and forgetting for now the non-negativity constraint on effort, we get:

$$E = \left\{ \sum_i \left(\frac{V}{2^{n+1}} \sum_{k=1}^{n} \lambda_{ik} C_k^n (2k-n) \right)^{\frac{1}{\beta-1}} \right\}^{\frac{\beta-1}{\beta}} \tag{17}$$

Denoting $\Delta_{ik} = \lambda_{i(n-k)} - \lambda_{ik}$ and exploiting the fact that $\lambda_{i0} = 0$, we can rewrite a party's electoral output as:

$$E = \left\{ \sum_i \left(\frac{V}{2^{n+1}} \sum_{k=0}^{\lfloor n/2 \rfloor} \Delta_{ik} C_k^n (n-2k) \right)^{\frac{1}{\beta-1}} \right\}^{\frac{\beta-1}{\beta}} \tag{18}$$

The constraints take two forms. First, there is the seat budget constraint: $0 \leq \sum_i \Delta_{ik} \leq 1$. Second, individual probabilities must be between 0 and 1, implying that: $-1 \leq \Delta_{ik} \leq 1$.

Thus, a party's output is maximized when $\sum_i \left[\sum_{k=0}^{\lfloor n/2 \rfloor} \Delta_{ik} C_k^n (n-2k) \right]^{\frac{1}{\beta-1}}$ is maximized.

There are two cases to consider depending on the value of β.

Case 1: $\beta \geq 2$ (concave objective function)

When $\beta \geq 2$, we have that $\frac{1}{\beta-1} \leq 1$ and party electoral output is the sum over politicians of a concave function of their individual efforts. The first order conditions to the problem are thus both necessary and sufficient to pin down the solution.

The party's objective is given by

$$E^{\frac{\beta}{\beta-1}} = \sum_i \left(\frac{V}{2^{n+1}} \sum_{k=0}^{\lfloor n/2 \rfloor} \max\left(\Delta_{ik} C_k^n (n-2k), 0\right) \right)^{\frac{1}{\beta-1}}$$

and the budget constraint implies $\sum_{i=1}^{n} \Delta_{ik} = n - k$.

Note first that the monotonicity constraint imposes that $\Delta_{ik} \geq 0$ for any i and k. The first order conditions to this maximization problem (forgetting for now about the non-negativity constraints) yield that the partial derivatives $\frac{\partial E^{\frac{\beta}{\beta-1}}}{\partial \Delta_{ik}}$ and $\frac{\partial E^{\frac{\beta}{\beta-1}}}{\partial \Delta j}$ are equal to μ, the Lagrange multiplier associated with the budget constraint, for any i and j. To meet this condition, the coefficients Δ_{ik} must be equal across candidates. The optimal Δ_{ik} that solve the unconstrained problem are also such that $0 \leq \Delta_{ik} \leq 1$. The solution of the unconstrained problem is thus also a solution to the optimization problem with the constraint. Thus, the egalitarian rule is the optimal rule when $\beta \geq 2$.

Case 2: $\beta \leq 2$ **(convex objective function)**

Assume that in column k there exists λ_{ik} with $0 < \lambda_{ik} < 1$. By construction, this means that there exists λ_{jk} with $j \neq i$, such that $0 < \lambda_{jk} < 1$. Now, suppose without loss of generality that $\sum_{k=1}^{n} \lambda_{ik} C_k^n (2k - n)$ is greater than $\sum_{k=1}^{n} \lambda_{jk} C_k^n (2k - n)$. Given that the objective function is convex in $\sum_{k=1}^{n} \lambda_{mk} C_k^n (2k - n)$, increasing the larger of the two terms above and decreasing the smaller one increases the sum. The only way the allocation rule cannot be improved upon is when it is deterministic with all the entries λ_{mk} being either 0 or 1.

Applying the monotonicity requirement implies that the list rule is the optimal rule. Indeed, for $k = 1$, a deterministic rule gives the prize to one candidate. To be monotonic, the rule also needs to allocate a seat to that candidate when more seats are won, when $k > 1$. This means that this candidate has the top spot on the list. The reasoning is similar for all following seats and thus confirms that the optimal monotonic rule is the closed-list.

■

7.1 Non-monotonic rules

Example 7 *Optimal rules with four seats*

With four seats, the optimal allocation rule maximises $\sum_{i=1}^{4} \max\left(1 + 2(\lambda_{i3} - \lambda_{i1})^{\frac{1}{\beta-1}}, 0\right)$. When $\beta = 2$, the optimal allocation rule maximises $\sum_{i=1}^{4} \max(1 + 2(\lambda_{i3} - \lambda_{i1}), 0)$. Under the closed-list and the egalitarian allocation rule, $\sum_{i=1}^{4}(\lambda_{i3}-\lambda_{i1})=2$. This means that $\sum_{i=1}^{4}\left[(1+2(\lambda_{i3}-\lambda_{i1})\right]=8$.

Once we remove the monotonicity constraint, we can set some $(\lambda_{i3}-\lambda_{i1})$ to be negative, generating negative incentives for politician i who, as a consequence, chooses not to exert effort. Yet, this also frees incentive tokens that can be strategically redistributed to the most responsive politician(s). Exploiting this redistribution, an optimal rule is: $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

This is not the only optimal rule, as optimality puts no constraints on the value the different λ_{i2} can take on.

The optimal allocation rule is non-monotonic in the sense that politician 1 wins a seat if the party wins one seat only but does not win a seat if the party wins three seats. With that rule, $\sum_{i=1}^{4}\max\left(1+2\left(\lambda_{i3}-\lambda_{i1}\right),0\right)=9>8$. Also, observe that politicians 2 to 4 are all treated equally (the weights in columns 2 and 4 do not matter for incentives): the entries in columns 1 and 3 are the same for these players.

This rule clearly remains optimal when $\beta\leq 2$ since it creates even more unequal incentives than the closed-list. For $\beta>2$, there is a trade-off between relying on the egalitarian allocation rule and an optimal non-monotonic rule. To pin down the value of β below which it is optimal to depart from the egalitarian rule, one need only compare the party's output under the egalitarian rule, $\sum_{i=1}^{4}(1+2*1/2)^{\frac{1}{\beta-1}}=4*2^{\frac{1}{\beta-1}}$, with that under the non-monotonic list, and $3*(1+2*1)^{\frac{1}{\beta-1}}=3*3^{\frac{1}{\beta-1}}$, where the effort of only three politicians matter as under the non-monotonic rule politician 1 is inactive. Then, simple algebra implies that the egalitarian rule is suboptimal whenever $4*2^{\frac{1}{\beta-1}}<3*3^{\frac{1}{\beta-1}}\iff\beta<\frac{\ln(3/2)+\ln(4/3)}{\ln(4/3)}\simeq 2.4$.

Example 8 *Optimal rule with five seats*

With more seats, the optimal non-monotonic rule can take different forms depending on the value of β.

The optimal allocation rule maximises $\sum_{i=1}^{4}\max\left[5+15\left(\lambda_{i4}-\lambda_{i1}\right)+10\left(\lambda_{i3}-\lambda_{i2}\right),0\right]^{\frac{1}{\beta-1}}$.

As before, when $\beta\leq 2$ the function above is a convex function of $\lambda_{i4}-\lambda_{i1}$ and $\lambda_{i3}-\lambda_{i2}$, implying that it is optimal to make incentives as heterogeneous as possible. Also, as all columns of the rule matrix but the first and the last enter in the equation of party effort, the optimal non-monotonic rule is this time unique. It is easy to check that it is given by $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$.

Thus, when $\beta \leq 2$, it is optimal to treat differently politicians 1 and 2 and treat equally politicians 3 to 5.

When $\beta > 2$, identifying which intra-party allocation rule is optimal is a bit more involved. Using numerical analysis to pin down the critical values for β, it appears that there are two cases to consider: $\beta \geq 3$ and $\beta \in (2, 3)$.

When $\beta \geq 3$, the egalitarian rule is optimal: the convexity of the individual cost of effort is too strong for the benefits of generating negative incentives to compensate for their costs.

When $\beta \in (2, 3)$, the optimal rule gives equal incentives to four politicians and negative incentives to one: $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1/4 & 3/4 & 1 & 1 \\ 0 & 0 & 1/4 & 3/4 & 1 & 1 \\ 0 & 0 & 1/4 & 3/4 & 1 & 1 \\ 0 & 0 & 1/4 & 3/4 & 1 & 1 \end{bmatrix}$.

The last example shows that, under non-monotonicity and with a sufficient number of seats, the space of parameter values for β can be partitioned more finely to fine tune incentives than when the allocation rules are constrained to be monotonic.

8 Appendix B (extensions)

Number of parties

The derivation of equilibrium individual effort and party output is similar to that in the proof of proposition 2 replacing $p_j = 1/2$ by $p_j = 1/K$. The proof of proposition 3 applies directly with $p = E_1 / \sum_{j=1}^{K} E_j$.

■

Biased contest

The proof of proposition 3 applies directly with $p = \lambda E_1 / (\lambda E_1 + E_2)$ given that:

$$\frac{\partial \frac{\lambda E_1}{\lambda E_1 + E_2}}{\partial e_{i1}} = \frac{\lambda E_2}{(\lambda E_1 + E_2)^2} = \frac{p(1-p)}{E_1}$$

$\blacksquare$

Contest technology

Winning a majority

In what follows, we exploit lemmas 1 and 2 and the fact that at a symmetric Nash equilibrium, $\left(\frac{E_j}{e_{ij}}\right)^{\sigma} = n^{\frac{\sigma}{1-\sigma}}$.

The first order condition to the politician's problem under the egalitarian rule is given by:

$$\begin{aligned} e^{*\beta-1} &= \frac{V}{4E} n^{\frac{\sigma}{1-\sigma}} + \frac{M(n+1)}{E} n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2} \\ &\Longleftrightarrow \\ e^{*} &= \left(\frac{V}{4E} n^{\frac{\sigma}{1-\sigma}} + \frac{M(n+1)}{E} n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right)^{\frac{1}{\beta-1}} \end{aligned}$$

The electoral output of each party in the symmetric equilibrium is thus:

$$\begin{aligned} E &= \left\{\sum_{k=1}^{n}\left[\left(\frac{V}{4E} n^{\frac{\sigma}{1-\sigma}} + \frac{M(n+1)}{E} n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right)^{\frac{1}{\beta-1}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} \\ &\Longleftrightarrow \\ E^{\frac{\beta}{\beta-1}} &= \left\{\sum_{k=1}^{n}\left[\left(\frac{V}{4} n^{\frac{\sigma}{1-\sigma}} + M(n+1) n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right)^{\frac{1}{\beta-1}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} \\ &\Longleftrightarrow \\ E &= \left\{\sum_{k=1}^{n}\left[\left(\frac{V}{4} n^{\frac{\sigma}{1-\sigma}} + M(n+1) n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right)^{\frac{1}{\beta-1}}\right]^{1-\sigma}\right\}^{\frac{\beta-1}{\beta(1-\sigma)}} \\ &\Longleftrightarrow \\ E &= \left\{n\left[\left(\frac{V}{4} n^{\frac{\sigma}{1-\sigma}} + M(n+1) n^{\frac{\sigma}{1-\sigma}} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right)^{\frac{1}{\beta-1}}\right]^{1-\sigma}\right\}^{\frac{\beta-1}{\beta(1-\sigma)}} \end{aligned}$$

and thus we have that under the egalitarian rule, the output of a party is equal to:

$$E^*_E = \left\{\frac{V}{4} + M(n+1) C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+2}\right\}^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}$$

The first order condition to the politician's problem under the closed-list is given by:

$$(e^*_m)^{\beta-1} = V\left(\frac{E}{e^*_m}\right)^{\sigma} \frac{m}{E} C^n_m \left(\frac{1}{2}\right)^{n+1} + M\left(\frac{E}{e^*_m}\right)^{\sigma} \frac{n+1}{2E} C^n_{\frac{n+1}{2}} \left(\frac{1}{2}\right)^{n+1}$$

which implies that politician m's optimal choice is given by

$$e_m^* = \left[V E^{\sigma} \frac{m}{E} C_m^n \left(\frac{1}{2}\right)^{n+1} + M E^{\sigma} \frac{n+1}{2E} C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+1} \right]^{\frac{1}{\beta+\sigma-1}}$$

The electoral output of each party in the symmetric equilibrium is thus:

$$\begin{aligned} E &= \left\{ \sum_{k=1}^{n} \left[\left(V E^{\sigma-1} k C_k^n \left(\frac{1}{2}\right)^{n+1} + M E^{\sigma-1} \frac{n+1}{2} C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+1} \right)^{\frac{1}{\beta+\sigma-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\ &\Longleftrightarrow \\ E^{\frac{\beta}{\beta+\sigma-1}} &= \left\{ \sum_{k=1}^{n} \left[\left(V k C_k^n \left(\frac{1}{2}\right)^{n+1} + M \frac{n+1}{2} C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+1} \right)^{\frac{1}{\beta+\sigma-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \end{aligned}$$

and thus we have that with the closed-list, the output of a party is equal to:

$$E_L^* = \left\{ \sum_{k=1}^{n} \left[\left(V k C_k^n \left(\frac{1}{2}\right)^{n+1} + M (n+1) C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+2} \right)^{\frac{1}{\beta+\sigma-1}} \right]^{1-\sigma} \right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}$$

To compare E_E^* and E_L^*, we take their ratio:

$$\begin{aligned} \frac{E_L^*}{E_E^*} &= \frac{\sum_{k=1}^{n} \left(V k C_k^n \left(\frac{1}{2}\right)^{n+1} + M (n+1) C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+2} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}}{\left(\frac{V}{4} + M (n+1) C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+2} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} n} \\ &= \frac{\sum_{k=1}^{n} \left(V k C_k^n \left(\frac{1}{2}\right)^{n+1} + \tilde{M} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}}{n \left(\frac{V}{4} + \tilde{M} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}}. \end{aligned}$$

where $\tilde{M} = M (n+1) C_{\frac{n+1}{2}}^n \left(\frac{1}{2}\right)^{n+2}$.

If $\frac{1-\sigma}{\beta+\sigma-1} = 1$, then $\sum_{k=1}^{n} \left(V k C_k^n \left(\frac{1}{2}\right)^{n+1} + \tilde{M} \right) = n \left(\frac{V}{4} + \tilde{M} \right)$ and $\frac{E_L^*}{E_E^*}$ is equal to one, implying that both intra-party seat allocation rules generate the same equilibrium party output. Then, applying Jensen's inequality as in proposition 3 implies that a party's output is higher under the closed-list than under the egalitarian rule if and only if $\frac{1-\sigma}{\beta+\sigma-1} > 1 \iff \beta < 2 - 2\sigma$.

■

Contractible effort

In a symmetric equilibrium, when both parties use the egalitarian rule, individual efforts are given by the participation constraint:

$$e^{\beta}/\beta = V/2.$$

Individual effort is given by:

$$e = (\beta V/2)^{1/\beta}.$$

This leads to party electoral output being equal to

$$\begin{aligned} E &= \left(\sum e^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\ &= n^{\frac{1}{1-\sigma}} e \\ &= n^{\frac{1}{1-\sigma}} \left(\frac{\beta V}{2}\right)^{1/\beta}. \end{aligned}$$

Turning to the case where both parties use the closed-list, at a symmetric equilibrium, individual effort of the politician in mth position is given by the participation constraint:

$$\begin{aligned} (e_m)^{\beta}/\beta &= \sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n V \\ &\Longleftrightarrow \\ e_m &= \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n \beta V\right)^{1/\beta} \end{aligned}$$

Party output is thus equal to:

$$\begin{aligned} E &= \left(\sum_{m=1}^{n} e_m^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\ &= \left\{\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n \beta V\right)^{\frac{1-\sigma}{\beta}}\right\}^{\frac{1}{1-\sigma}} \\ &= \left\{\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^{n-1}\right)^{\frac{1-\sigma}{\beta}}\right\}^{\frac{1}{1-\sigma}} \left(\frac{\beta V}{2}\right)^{1/\beta}. \end{aligned}$$

The argument of proposition 3 applies but now the condition for the egalitarian rule to lead to higher party output is that $(1-\sigma)/\beta < 1$ that is $\beta > 1-\sigma$ which is always the case.

■