Social Cost of Carbon under stochastic tipping points: when does risk play a role?

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Abstract

Carbon dioxide emissions impose a social cost on economies, owing to the damages they will cause in the future. In particular, climate change may trigger tipping points in the climate or economic system. Tipping points induce higher expected damages and risk that damages may be catastrophic, both of which increase the Social Cost of Carbon. However, the respective contributions of higher expected damages and risk have not been disentangled. In this article, we develop a methodology to compare how much expected damages explain the Social Cost of Carbon, compared to the risky nature of a stochastic tipping point. We analyze the conditions under which approaches relying on expected damages underestimate the Social Cost of Carbon in the presence of tipping points. We find that it takes productivity shocks higher than 10%, for risk aversion to play a role, which is on the high end of the range of damages commonly assumed in Integrated Assessment Models. Deterministic approaches are suitable to estimate SCC for lower shocks.

Keywords: Climate change; Tipping points; Expected utility; Integrated Assessment Models; Risk; Social Cost of Carbon

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1. Introduction

There is a consensus that climate change will induce damages in the future, although the range of possible levels for these damages is uncertain. Some consider climate change to be worrysome because damages will be high, others because there is a small chance they could be catastrophic. In the former case, optimal climate policy arises from a simple intertemporal cost-benefit analysis, while in the latter case, emissions reductions result from a precautionary approach as an insurance against the risk of disastrous impacts.

This tension between two potential sources for the harmfulness of climate change can be found in the categorization of the "Reasons for Concerns" (RFC) by the Intergovernmental Panel on Climate Change. How much do "Aggregate impacts" (RFC 4) play a role, compared to the "Risk of large-scale singular events" (RFC 5), such as the breakdown of the thermohaline circulation? The latest assessment of the severity of each Reason for Concern (O'Neill et al., 2017) shows that additional risk due to climate change jumps from moderate to high around the same temperature for both of these Reasons for Concern, suggesting that they contribute by the same magnitude to making climate change worrisome.

However, the balance between both Concerns is not a done deal among climate economists. For instance, Pindyck (2013) and Weitzman (2009) argue that catastrophic outcomes should be the primary driver of climate mitigation. Broome (2010), wondering "whether the most important thing about climate change is the harm it is likely to cause or alternatively the utter catastrophe that it may possibly – though very improbably – cause", gave an opposing view. One the one hand, previous literature emphasizes the impact of the level of expected damages on optimal emissions (Weitzman, 2012; Pizer, 2003; Dumas and Ha-Duong, 2005; Ackerman and Stanton, 2012; Wouter Botzen and van den Bergh, 2012), with some arguing that damage estimates should be revised upward (Dietz and Stern, 2015; Weitzman, 2012)). Indeed, estimates of climate damages are subject to numerous uncertainties and limitations (see Diaz and Moore (2017) for a recent review). On the other hand, other authors put forward this uncertainty as a reason to dismiss the use of deterministic damage functions to represent the impacts of climate change (Pindyck, 2013). Thus, damage functions have been criticized both for the difficulty to determine the best-guess expected damages, and that to model the risk of catastrophic outcomes.

This question of "level" versus "risk" is particularly salient in the case of non-marginal or abrupt changes referred to as "tipping points" (Lenton et al., 2008; Alley et al., 2003; Steffen et al., 2018). Examples of such phenomena include the shutdown of thermohaline circulation, the melting of the Arctic sea-ice or the die back of the Amazonian rainforest, but it could also come from the limited ability of social and economic systems to cope with climate conditions past some threshold.

Tipping points stand in sharp contrast with deterministic damage functions, which have been traditionally used in Integrated Assessment Models. In his initial calibration of the DICE model, Nordhaus included a 30% increase in the damage estimates as a way to account for the expected damages from catastrophes (Nordhaus, 1994). Conversely, explicit modeling of tipping points found a significant effect on optimal policy (Mastrandrea and Schneider, 2001; Keller et al., 2004; Lemoine and Traeger, 2014b; Lontzek et al., 2012). But since tipping points raise expected damages, this effect could be due to this increase rather than the stochastic nature of the tipping points. Both methodologies of dealing with tipping points, either by a change of the damage function, or explicit modeling of tipping points have not been compared. When integrating tipping points into a model, authors have not separated changes due to risk aversion to the realization of the tipping point from those coming from a mere increase of expected damages. They fail to disentangle whether tipping points matter from a level or risk perspective, and whether a deterministic approach using expected damages is a good proxy for calculations of the SCC.

Ackerman et al. (2010) and Dietz (2011) have performed similar exercises on the "fat tail" of climate sensitivity, comparing SCC with risk on parameters for climate sensitivity with that when using the expected value for climate sensitivity, but they relied on Monte Carlo procedure to capture risk, which only gives the expected value of the Social Cost of Carbon (i.e. the present social value of an additional emission of carbon, hereafter SCC), not the shadow price of carbon along an optimal path facing risk. A comparison of SCC under parametric uncertainty finds that methods relying on expected damages tend over-estimate optimal emissions (Crost and Traeger, 2013). This kind of analysis has not been performed under the possibility of stochastic tipping points.

Since the damage function is the least-grounded aspect of Integrated Assessment Models, and it has a strong impact on the SCC, it is essential to build rigorous methodologies that compare how different representations of damages affect the SCC (Guivarch and Pottier, 2018; Pottier et al., 2015). For instance, whether the possibility of tipping points changes SCC solely because they raise the estimates for expected damages, or because they impose a risk that society is not willing to take.

In this article, we analyze the respective contribution of level (expected damages) and risk in the case of a stochastic tipping point inducing a productivity shock. We use an Integrated Assessment Model to calculate the SCC under two settings: one with a stochastic tipping point, and one with a modified deterministic damage function to capture the expected damages of the tipping point. That way, we are able to highlight how much expected damages drive the SCC, and under which conditions deterministic approaches lead to underestimate the SCC. We analyze the influence of preferences of the decision maker and the nature of the catastrophe on our results.

We find that explicit modeling of the tipping point and our approach relying on expected damages lead to similar values for the SCC, suggesting that expected damages explain most of the value for the SCC. This results holds as long as we stay within the range of productivity shocks usually considered in the literature. However, under both rhigh productivity shocks and high risk aversion, precaution to avoid the tipping point drives abatement, so that using a deterministic method underestimates the SCC, and becomes ill-suited to compute its value.

Our findings offer a possible explanation for the so-called *risk aversion puzzle*. Previous literature found that risk aversion played a modest role in IAMs like DICE, even when using Epstein-Zin preferences (Ackerman et al., 2013), and in the case of non-linear threshold (Belaia et al., 2014). Our results suggest that too low levels of damages considered in these studies could be responsible for the the low influence of risk aversion.

We begin by laying out the model and methodology we use to model catastrophes and build a deterministic equivalent 2. Results using different welfare specifications are discussed in section 3. Section 4 concludes.

2. Methodology

We use a simple Integrated Assessment Model, which we present in section 2.1, to calculate the Social Cost of Carbon. We then explain in section 2.2 our methodology to assess the contribution of expected damages and risk, and the values we explore for the parameters of the model.

2.1. The model

An Integrated Assessment Model is meant to capture the main crossed interactions between the economy and the climate system. On the one hand, growth and technological choices drive the level of greenhouse gas emissions causing changes in the climate system, which affect back the economy. This allows to derive optimal emissions path from the point of view of a utilitarian social planer balancing costs of mitigation and damages of climate change, and to calculate the marginal damages caused by emissions – the SCC.

We use a classical DICE-like model, building on the Ramsey-Caas-Koopmans framework (Guivarch and Pottier, 2018). The economy produces a single good in quantity Q_t using two factors, capital K_t and labour L_t through a Cobb-Douglas function. The productivity is affected by climate change via a damage function Ω_t depending on temperature T_t, so that final production Q_t writes:

$$Q_t = \Omega(T_t) A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

The production induces emissions, which can be mitigated at a certain cost. The social planer trades off between consumption, mitigation costs (which represents a share Λ_t of production), and investment in capital (share s_t of production)

$$C_t = Q_t(1 - \Lambda_t - s_t) \tag{2}$$

$$\Lambda_t = \theta_1(t)\mu_t^{\theta_2} \tag{3}$$

$$K_{t+1} - K_t = -\delta.K_t + Q_t.s_t \tag{4}$$

where δ is capital depreciation, and μ_t the abatement rate. $\theta_1(t)$ measures total mitigation costs and decreases exogenously due to technical progress.

The difference with DICE equations concerns the climate system. There is growing evidence that temperature change depends linearly on cumulated emissions (Allen et al., 2009; Matthews et al., 2009; Goodwin et al., 2015), owing to the fact that effects from oceanic absorption of heat and carbon compensate. Modeling the temperature response to cumulated emissions with a linear function is common specification in the literature (Dietz, 2011; Lemoine and Traeger, 2014a), and it reduces computational burden of dynamic programming.

$$T_t = \beta.(CE_0 + \sum_{s=0}^{t} E_s) \tag{5}$$

where T_t is the global temperature increase at time t, CE_0 are cumulated emissions up to the first period of the model and E_s the emissions at time s.

$$E_t = \sigma_t (1 - \mu_t) Q_t \tag{6}$$

where σ_t is the carbon content of production that decreases exogenously over time, and μ_t the abatement rate.

The tipping point is described as a stochastic phenomenon leading to a productivity shock, in line with van der Ploeg and de Zeeuw (2013); Lontzek et al. (2015), because our purpose is to remain as general as possible. Such a change in the damage function can potentially apply to a large range of tipping points inducing larger damages than expected. It can be direct impact on the economy either caused by melting of ice caps, leading to severe sea-level rise; a slowing down of thermohaline circulation; or a social tipping point past which adaptation is no longer possible. Other studies consider that the tipping point can also affect climate variables such as climate sensitivity or depreciation rate of atmospheric carbon dioxide to reflect saturation of sinks (Lemoine and Traeger, 2014a).

$$\Omega(T_t) = \frac{1}{1 + \pi T_t^2} \tag{7}$$

Once the tipping point has been crossed, damages write:

$$\Omega(T_t) = \frac{1 - J}{1 + \pi T_t^2} \tag{8}$$

J is the strength of the productivity shock.

We model the trigger of the tipping point using an endogenous hazard rate. At each timestep there is a probability $h_t(T_t, T_{t-1})$ – conditional to non-crossing previously – to cross the tipping point. The probability is simply assumed to be equally distributed between two temperatures T_{min} and T_{max}. Besides, we suppose that the decision maker knows that visited temperature are safe, as in Lemoine and Traeger (2014a), so that she updates priors if the tipping point has not been crossed. This means that the marginal hazard rate tends to increase, as temperatures get warmer.

$$h_t = \frac{T_t - max(T_{min}, T_{t-1})}{T_{max} - max(T_{min}, T_{t-1})} \tag{9}$$

This learning is essential to prevent the crossing from being unavoidable, unlike in other works (van der Ploeg and de Zeeuw, 2013; Lontzek et al., 2015). Without any learning, there is no hedging strategy : mitigation actions only delay the expected time of crossing the tipping point, but there is no possibility to avoid it, even if temperatures stabilize. With learning, mitigation actions can avoid the tipping point.

The model seeks the welfare-maximizing path for two state variables, capital and cumulated emissions, choosing the path for the two control variables, the saving rate s_t and the abatement rate μ_t. We study two functional form for welfare: the classical Constant Relative Risk Aversion (CRRA), and Epstein-Zin preferences. In the CRRA representation, time and risk preferences are embedded in a single parameter, which gives both resistance to intertemporal substitution and risk aversion. However, both can induce opposing-directions effects in the presence of risks: while resistance to substitution favors the consumption of present generations, risk aversion encourages more abatement in the present to lower the risk of triggering the tipping point. For this reason, we also apply Epstein-Zin preferences, which allow to disentangle intertemporal substitution and risk aversion.

Welfare after time t, U_t, is defined recursively:

- For classical expected utility preferences

$$U_t = [(1 - \frac{1}{1+\rho})u_t + \frac{1}{1+\rho}\mathbb{E}(U_{t+1})] \tag{10}$$

where ρ is the pure time preference, and utility at each time step is given by:

$$u_t(C_t, L_t) = L_t \frac{(C_t/L_t)^{1-\eta}}{1-\eta} \tag{11}$$

η is the elasticity of marginal utility.

So that we can define Bellman functions as follows:

$$V_t = \max_{y_t}[u(x_t, y_t) + \frac{1}{1+\rho}\mathbb{E}(V_{t+1}(G(x_t, y_t)))] \tag{12}$$

- For Epstein Zin preferences. :[1]

$$U_t = [(1 - \frac{1}{1+\rho})u_t + \frac{1}{1+\rho}\mathbb{E}(U_{t+1}^{1-\gamma})^{\frac{1-\theta}{1-\gamma}}]^{\frac{1}{1-\theta}} \tag{13}$$

$$u_t(C_t, L_t) = L_t \frac{(C_t/L_t)^{1-\theta}}{1-\theta} \tag{14}$$

For the sake of clarity we use different notations in the Epstein-Zin case. We denote θ the inverse of the elasticity of intertemporal substitution, and γ the risk aversion parameter.

We can define Bellman functions in order to solve this dynamic program. $V_t = \frac{U_t^{1-\theta}}{1-\frac{1}{1+\rho}}$

$$V_t = \max_{y_t}[u(x_t, y_t) + \frac{1}{1+\rho}f(V_{t+1}(G(x_t, y_t)))] \tag{15}$$

f accounts for the decision maker's attitude toward the risk of tipping.[2]. $f(V_{t+1}) = [\mathbb{E}(V_{t+1}^{\frac{1-\gamma}{1-\theta}})]^{\frac{1-\theta}{1-\gamma}}$. It is the same formula as for CRRA preferences, in which $f = \mathbb{E}$.

[1]The formula holds for $\theta<1$. Otherwise when $\theta > 1$ utility function is negative, so that $U_t = -(-(1 - \frac{1}{1+\rho})u + \frac{1}{1+\rho}[\mathbb{E}_t(-U_{t+1})^{1-\gamma}]^{\frac{1-\theta}{1-\gamma}})^{\frac{1}{1-\theta}}$

[2]when $0 < \psi < 1$, the recursive formula involves $u_t - \frac{1}{1+\rho}f(-V_{t+1})$

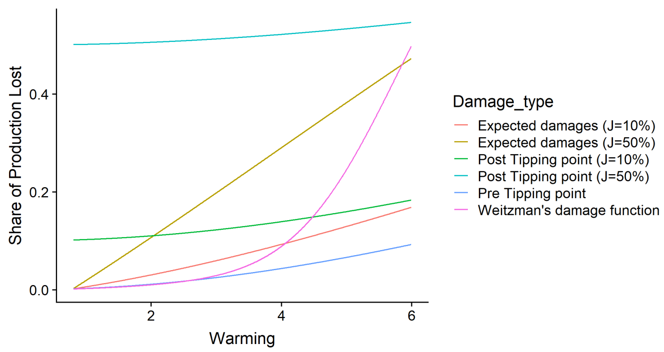

Figure 1: Comparison between different damage functions.

Using dynamic programming, we first approximate Bellman functions in the post-threshold world, and then in the pre-threshold world using expectations over the impact of the tipping event.

The Social Cost of Carbon can be expressed, thanks to envelop theorem, using Bellman function. If S_t is the stock of emissions, SCC writes:

$$SCC_t = -\frac{1}{1+\rho} \frac{\partial_S \mathbb{E}(V_{t+1})|_{x_{t+1}}}{\partial_C V_t|_{(x_t, y_t^*)}} \tag{16}$$

$y_t = (\Lambda, s_t)$ are control variables while $x_t = (S_t, K_t)$ are state variables. * denotes values that are taken along the optimal path.

2.2. Building a deterministic equivalent

We calculate the Social Cost of Carbon under two settings:

- SCC under the risk of a stochastic tipping point, as modelled above
- SCC with no risk, where damages at a given temperature are set at the expected level of damages accounting for a potential tipping point.

The second setting can be understood as a deterministic equivalent of a tipping point, using an 'expected damage function' instead of a damage function subject to a stochastic productivity shock. Thus, for each stochastic run with the risk of a tipping point, we run the optimization problem in a deterministic fashion, using an "expected damage function" instead of the damage function subject to a stochastic productivity shock (see fig 1). The expected damage function writes:

$$\Omega_d(T_t) = (1 - p(T_t))\Omega_1(T_t) + p(T_t)\Omega_2(T_t) \tag{17}$$

where $p(T_t)$ is the prior probability of having crossed the tipping point at temperature T_t. Comparing SCC between the stochastic and deterministic runs tells us how much SCC is explained by expected damages versus by risk aversion.

2.3. Calibration of the parameters

We use typical range of possible values for parameters related to attitude toward risk and time. Pure rate of time preference (ρ) can take three values: 0.1%, 0.5% and 1.5%. In the CRRA case, elasticity of marginal utility (η) ranges from 0.5 to 3. For the Epstein-Zin case, concerning the intertemporal substitution ($1/\theta$), θ is between 0.5 and 3, while γ ranges from 0.5 to as high as 20 .

For the parameters describing the tipping point, we acknowledge that the impacts of such phenomenon are very difficult to quantify and could be very large. We thus explore a large window for the productivity shock J, from 1 to 50%. The location of the trigger is also uncertain, and could be anywhere between current temperature and $T_{max} = 7°C$. Starting from an initial temperature increase of 0.8°C compared to pre-industrial times, this means for instance that a 2°C increase is associated with a 19% probability of triggering the tipping point. For robustess checks on this assumption, see Supplementary Information.

3. Results and discussion

In this section, we present the results, first when using CRRA preferences, then with Epstein-Zin preferences where risk aversion and the inverse of the elasticity of intertemporal substitution differ.

3.1. With CRRA preferences

We calculate the SCC under the two settings, in the deterministic case and for stochastic tipping point. We analyze the ratio between the two, for different values for the parameters representing preferences and damages due to the tipping point. The closer to one the ratio, the more expected damages explain the SCC. We plot contour lines for the ratio in the space of elasticity of marginal utility and damages due to tipping point (η, J) in figure 2 (top panel). We also plot contour lines for the absolute value of the SCC in the stochastic tipping point case (figure 2, bottom panel).

With CRRA preferences, elasticity of marginal utility (η) plays a role both in intermporal trade-offs and risk aversion. On the one hand, a higher elasticity of marginal utility tends to favor present consumption relative to future consumption of wealthier generations (and thus less abatement). On the other hand, it encourages mitigation of emissions to reduce the likelihood of triggering the tipping point. As far as the absolute level of the SCC in the stochastic case is concerned, of these opposing effects of elasticity of marginal utility, we find that the intertemporal substitution effect outweigh the risk aversion effect. For a given J, the SCC decreases when η increases.

As the decision maker only faces expected damages in the deterministic equivalent, there is no risk, and only the intertemporal substitution plays a role. This explains why a deterministic equivalent underestimates SCC compared to a stochastic run, and does so more as η increases. However, we find that it takes both high productivity shocks and high elasticity of marginal utility for the deterministic approach to significantly underestimate SCC. In fact, expected damages explain more than 90% of the SCC, as long as the productivity shock is inferior to 10%, whatever the value for risk aversion in the range explored. As impacts become greater than 10%, using a deterministic approach for underestimates SCC. Only with productivity shocks higher than 40% jointly with elasticity of marginal utility higher than 2 does risk amount to half of the contribution of SCC.

Though lower pure rate of time preference (ρ) significantly raises the level of the SCC, it does so with similar magnitudes in both methods, so that the ratio of SCC found with the two methods is similar in the cases of $\rho = 0.5$ and $\rho = 1.5$ (see graphs in Annex). The value of the time preference considered does not change the conclusion that it takes both high productivity shocks and high elasticity of marginal utility for the deterministic approach to significantly underestimate SCC.

3.2. When disentangling risk aversion and intertemporal substitution

We perform the same exercise when disentangling risk aversion and elasticity of intertemporal substitution, using Epstein-Zin specification for preferences. We present an illustration of the results in the space of risk aversion parameter and damages due to the tipping point (γ, J) for $\theta = 1.5$ and $\rho = 1.5\%$ in figure 3 (additional graphs for different values can be found in Appendix). For a given level of productivity shock, the value of the SCC increases with the risk aversion parameter γ, as this parameter has an intuitive influence in a single direction (figure 3, bottom panel). The contour plot of the ratio of SCC deterministic on SCC stochastic presents a hyperbolic shape (figure 3, top panel), reflecting that it takes both low risk aversion and low damages to have deterministic run be a good proxy for the SCC. For instance, for a productivity shock equal to 10%, the risk aversion parameter has to be lower than 4 to have the ratio of SCC deterministic on SCC stochastic higher than 0.9. Productivity shocks higher than 25%, combined with risk aversion parameters higher than 5, lead to the deterministic approach underestimating the SCC by a factor 2 at least. For a productivity shock equal to 40% and a risk aversion parameter equal to 5, the deterministic SCC represents only 20% of the stochastic SCC.

Graphs in Annex show the same results for alternative values for the elasticity of intertemporal substitution ($\theta = 1.5$) and for the pure time preference ($\rho = 0.5$). An decrease in the elasticity of substitution (a higher θ) tends to decrease the absolute value of SCC, but it does not affect the comparison between stochastic and deterministic runs. Indeed, θ plays a similar role in both types of runs by governing the trade-off between future consumption and present one. For the same reason, changes in utility discount rate (ρ) do not affect much the shape or position of the contours of the SCC ratios.

4. Discussion and Conclusion

Climate change is an issue in terms of inter-temporal distribution of welfare because of the damages it will impose on future generations. What makes these damages remarkable is that they are uncertain: is it their expected level or the uncertainty surrounding them that warrants undertaking mitigation actions? This question has been studied for many types of uncertainty, for instance regarding climate sensitivity or other critical aspects of the climate-economy system, but has not been applied to stochastic tipping points. Authors considering tipping points in Integrated Assessment Models have not

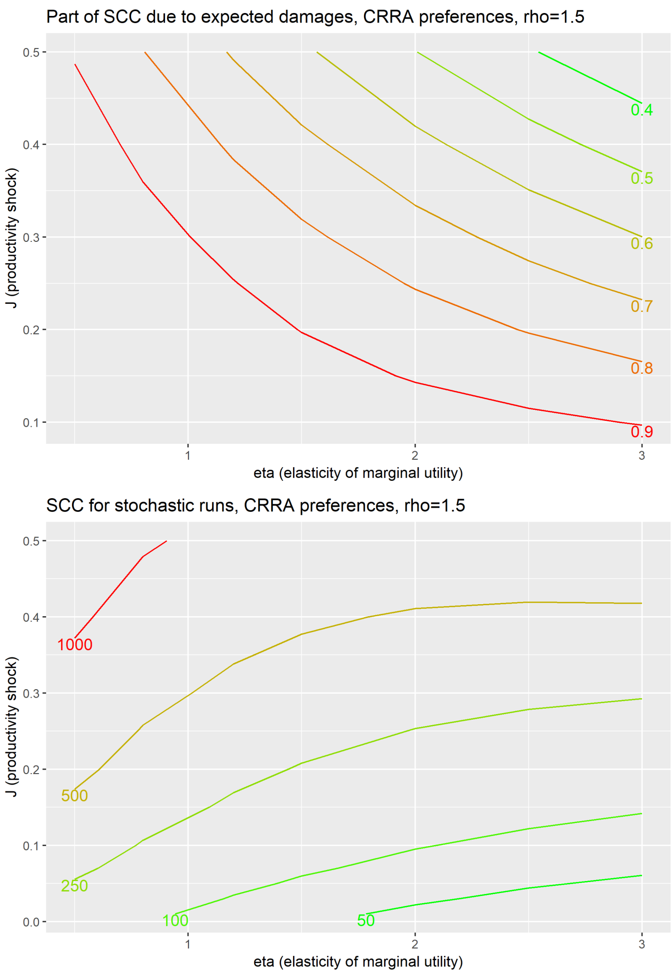

Figure 2: Contour of share of SCC explained by expected damages (ratio of SCC deterministic on SCC stochastic) and SCC (in US $2005) for stochastic runs, CRRA preferences

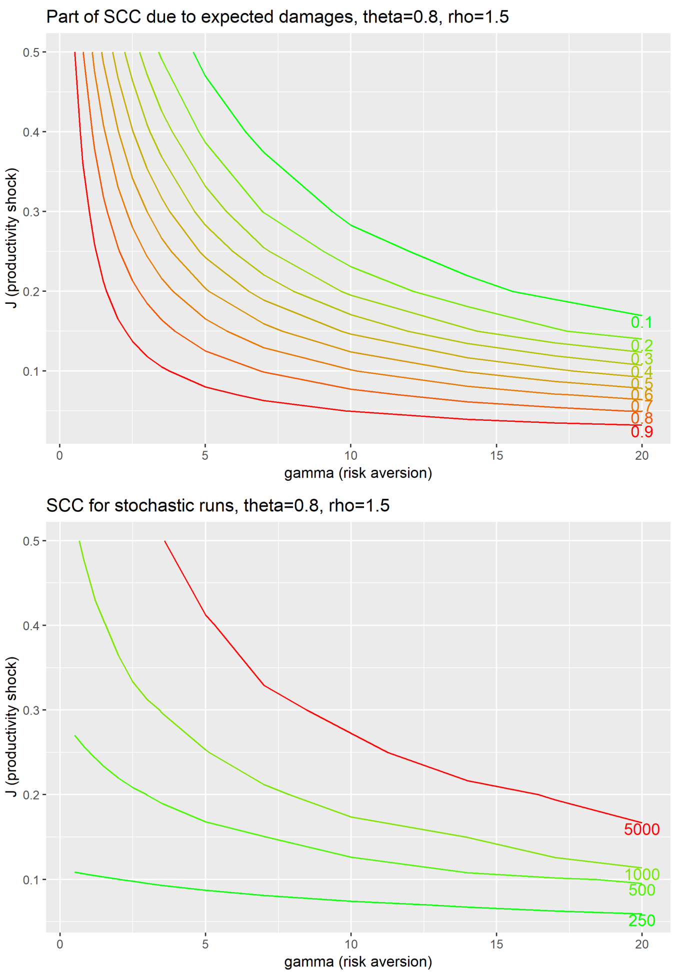

Figure 3: Contour plot of the share of SCC explained by expected damages (ratio of SCC deterministic on SCC stochastic) and SCC (in US $2005) for stochastic runs, Epstein-Zin preference

studied how explicit modeling of these phenomena differed from standard treatment of uncertainty via expected damage.

In this article, we have developed a methodology to evaluate how expected damages versus aversion to the realization of tipping points contribute to the Social Cost of Carbon. We compare a setting with explicit modeling of the tipping point to a deterministic setting using an equivalent damage function. Difference of SCC between the two methods are attributable to the effect of risk aversion.

Using conventional CRRA preferences, it takes high productivity shocks and risk aversion for a deterministic approach to underestimate SCC. Even when using Epstein-Zin preferences, the share of SCC attributable to risk aversion remains limited (less than 10%) under shocks affecting 10% of production and risk aversion of 10.

Productivity shock of 10% are in the range typically considered in the literature. For instance, in Lontzek et al. (2015), with a similar framework, authors consider the case of $J = 10\%$. Other modeling choices, in Lemoine and Traeger (2014a), make a tipping point induce a change to a sextic damage function, i.e. Weitzman's damage function. Weitzman's deterministic damage function relied on an expert panel that explicitly considered physical tipping points, but still leads to a loss of less than 10% of Gross World Product for 4°C. Our results suggest that the increase of SCC found in these studies are mostly due to a raise in expected damages, and that tipping points are rather a 'level' than a 'risk' problem.

Finally, our work sheds some light on the *risk aversion puzzle*, found in previous work, that is that risk aversion had a surprisingly little effect in Integrated Assessment Models (Ackerman et al., 2013), even in the case of tipping points (Belaia et al., 2014). We show that risk aversion only plays a role when considering very high possible damage levels, with the risk of losing a few tenths of production. Below these levels, an IAM is sensitive to expected damages, so that risk aversion plays a moderate role. Thus, we think that too low levels of possible damages considered in the literature explain the *risk aversion puzzle*.

Deterministic approaches using best-guess expected damages (together with sensitivity analyzes) are currently used to set a value for the SCC for regulations evaluations, and they have lower computational burden. Thus, knowing when deterministic approaches can be used as a good proxy for computing SCC under risk can guide policy making. Our results show that the Social Cost of Carbon comes primarily from the expected level of damages, when the shock induced by a potential tipping point remains lower than 10% or so. In that case, deterministic damage functions are appropriate. However, it is not possible to rule out higher shocks induced by tipping points. A very small number of studies explore the possibility of such large shocks as large as 90 % of consumption (Dietz, 2011), or possible extinction (Méjean et al., 2017). In the latter case, deterministic approaches are not suitable anymore and risk aversion plays a major role.

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