Tax policy, bubbles and unemployment

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Abstract

Following bubble asset crash, a great recession of economic activity took place and waves of job destruction were increasing. In this paper, a model with tax policy is presented to demonstrate that, asset bubbles decrease the unemployment level and increase the economic activity. We consider an OLG model with transfer, financed by tax burden on capital and labor income. Our results indicate that, the bubble promotes capital stock and reduces unemployment level if: The income and wealth redistribution is more in favor of young households and/or the tax rate on labor income is low, and the capital tax rate is high. Indeed, in the presence of bubble, a high level of tax burden on capital income, modifies significantly the relative price of capital, and this incites firms to favor the use of labor, due to the substitution effect. The income and wealth distribution effect and/or low labor income tax, allows the economy to sustain a higher capital stock.

Key words: Bubble, tax burden on capital and labor income, unemployment, capital stock, overlapping generations.

JEL classification: G12; H23

1 Introduction

The economic world has experienced multiple financial crises, such as Japan's crisis of the early 1990s. It is characterized by the bursting of financial asset bubbles and housing bubbles, after knowing a period of euphoria, i.e, an intense financial speculation between 1986-1990. Or again, the subprime crisis (the housing crisis) of 2007-2008 in the USA. It is a crisis and a more subtle bubble. It could be characterized by a significant collapse of housings' prices. It has been spread in other countries, generating an international financial crisis in 2008. Thus, the study of speculative bubbles has become a new field of investigation for economic theory.

The first works on speculative bubbles and economic growth, were carried out by Tirole (1985) and Weil (1987), they showed that, the deterministic and stochastic bubbles could have a recessionary effect on economic activity, they argued that, investing in a purely speculative asset allows to capture a part of the over-saving of the agents by absorbing it, which leads to a reduction of capital stock (crowding out effect), therefore, a decrease of economic activity. This theory is hard to reconcile with the observations, which show that the speculative bubbles have an economic expansion effect. Thus, recent works exhibited by different mechanisms, that the bubbly episodes are characterized by the boom of productive capital and its bursting causes depression. Martin and Ventura (2012) have relied on the existence of heterogeneous investment projects. Fahri and Tirole (2012) are used financial constraint. And more recently, Shi and Suen (2014) have shown that, considering labor as endogenous, the asset bubbles can promote per-worker capital and economic activity. They argued that, under such conditions, per-worker capital raises with the increase of labor supply, therefore, the economic activity raises.

All of these papers mentioned previously, were interested just on the impact that the asset bubbles may have on capital accumulation, capital per-worker, or on investment, but what about the relationship between speculative bubbles and unemployment?

During the period of the asset bubbles' crash, in particular the bursting of the Japanese asset price bubble and housings bubble in USA, the observations have shown that, this period was marked by a dramatic increase of the unemployment rate, where waves of job destruction were increasing more and more. According to World Bank figures (2016), the unemployment rate in Japan increased from 2.1% in 1991 to 4,8% in 2000, and in the US increased from 4,7% in 2007 to 9,7% in 2010. These figures lead us to say that, the fluctuations of employment are caused by the appearance and the burst of bubbles. Despite these historical observations, the impact of a bubble bursting on unemployment has not been fully theoretically investigated in macroeconomics.

To our knowledge, this issue has been known a very little interest for the moment. For this reason, the investigation of the relationship between asset bubbles and unemployment is a main theme of our paper. We will try to show a mechanism that will allow us to address the following main problematic: What are the effects of speculative bubbles on unemployment and economic activity?

The few literature that investigated this issue, has provided answers and shown by different mechanism that asset bubbles diminish the unemployment rate. Miao, Wang and Lifang (2015), have incorporated endogenous credit constraints. Hashimoto, Im and Kunieda (2017), have recourse to the existence of heterogeneous investment projects. And Kocherlakota (2011) has combined the OLG model with the DMP model. Our paper is different from the three previous works in three aspects. Unlike Miao, Wang and Lifang (2015) model, we focus in an intrinsically useless assets, but not on bubbles in the stock market value of the firm. According to Miao and al (2015), Hashimoto, Im and Kuneida (2017), Kocherlakota

(2011), the unemployment occurs in equilibrium because workers and firms face matching frictions. As far as we are concerned, we suppose that we have unemployment equilibrium. The third aspect concerns the introduction of tax policy. Indeed, we can demonstrate that, the existence of asset bubbles in the economy favors the economic activity, and reduces the unemployment level.

The tax burden on capital income reduces the capital supply at individual level, following that, the interest rate increases. Facing an increase in the relative price of capital stock, the firms substitute capital by labor. However, the supply of capital stock at the aggregate level increases, due to the low level of labor income and/or the high level of wealth transfers to the young investors. At equilibrium, on the one hand, the unemployment decreases due to the substitution effect, and on the other hand, the aggregate capital stock increases too (the increase of capital supply dominates the decrease of capital demand). Notice that this mechanism is powerful in bubbly than in bubbleless economy, the reason comes back to the existence of the bubble in the economy.

Our main innovation focuses on the introduction of tax policy. We consider that, the tax burden on capital and labor income finance the public spending and the transfers. To our knowledge, no economist has investigated the relationship between asset bubbles and unemployment by using this kind of tax. Capital income tax affects the level of the interest rate, and the labor income tax and/or the transfer modifies the young investors revenue. Both the interest rate and the revenue, are important for the existence of bubbles. The first determines the growth rate of the bubble, while the second is an essential determinant of the savings (it used in part to buy the asset bubbles).

In order to provide answers to the principal issue, we consider an overlapping generations model, each agent lives two periods, he works, consumes and saves in the first period, and in the second period, he consumes only. Assuming that, savers make a portfolio choice between the investment in productive capital, and the possession of a purely speculative asset. We present the preferences of representative households in the form of Cobb-Douglas technology. The choice of this function, allows us to determine the reservation wage, from which each household decides to enter into the labor market and offers 1 unit of work. Therefore, the supply of labor is perfectly elastic and endogenous. We consider also that, the tax burden on capital and labor income finance the government spending and the transfer to the households. Any variation of one or both taxes, it will modify the individual decision at the level of consumption and investment. Supposing in the first part of the paper that, the redistribution system is nil, at that case only the tax burden on labor income affects the income level of each households. In the second part, we suppose that the government spending is nil, at that case, the young investors revenues are affected by the labor income tax and the transfer.

Our paper is organized as follows: In the next section, we present our framework, which consists of three agents: households, firms and government. In each section 3 and 4, we determine the conditions under which the bubble can exist. We exhibit a new mechanism, which shows that, the asset bubbles are productive. In section 5, we calibrate the model. In section 6, we conclude the paper by providing a summary of all the results obtained during our research. Some technical details are relegated to an Appendix A, B.

2 Model

The economy is populated by three types of agents: Households, firms and Government. All markets are perfectly competitive.

2.1 Households

We consider an economy, inhabited by an overlapping generation of young and old. For each period of time t =0,1,2,3 ....∞, a new generation of identical consumers N is born, with a growth rate that is nil. The representative household of generation t, lives two periods. In the first period of his youth, he earns a wage income (w_t) taxed at a constant rate τ_{L_t}. This last is shared between consumption and saving, under two types of assets. Indeed, each saver makes a portfolio choice between the investment in a productive capital s_t, and the holding of m_t units of bubble asset. This asset is an intrinsically useless paper asset, specifically "money", which has a positive value $P_t > 0$, in spite of the fact that it is intrinsically useless. Its market fundamental is zero. So we can define money bubble as the difference between the market price and the market fundamental (Tirole 1985).
In the second period, the household consumes all its resources (the agent is selfish), which consists on the return from productive investment after-tax ($R_{t+1}(1-\tau_K)s_t$), and the return from speculative asset ($P_{t+1}m_t$), where R_{t+1}, τ_K and P_{t+1} denote, gross return from productive capital, tax rate on capital income and the price of the intrinsically worthless asset at time t + 1, respectively.

The preferences of the representative household are given by a Cobb Douglass utility function

$$U = c_{1t}^{\alpha} c_{2t+1}^{1-\alpha} - V d_t \quad \alpha \in [0.1]; \quad \forall t \geq 0 \tag{1}$$

where c_{1t} and c_{2t+1}, are the consumption levels while young and old respectively, V>0 denotes the disutility of work, and d_t =(0,1) is a binary choice variable, denoting the amount of hours supply. Each individual has a choice of offering either 0 or 1 unit of work, depending on the wage proposed by the firm. If this last is higher or equal to the after-tax reservation wage, the individual decides to enter into the labor market, and elastically offers one unit of work. The agent faces two budget constraints

$$c_{1t} + s_t + P_t m_t = (1-\tau_L) w_t dt + T_{1t}$$

$$c_{2t+1} = (1-\tau_K) R_{t+1} s_t + P_{t+1} m_t + T_{2t+1}$$

Solving the worker program, i.e. $d_t = 1$, we find the sharing between consumption and savings

$$c_{1t} = \alpha((1-\tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)}) \tag{2}$$

$$s_t + P_t m_t = (1-\alpha)((1-\tau_L)w_t + T_{1t}) - \frac{\alpha T_{2t+1}}{R_{t+1}(1-\tau_K)} \tag{3}$$

and we find also the no arbitrage condition

$$R_{t+1}(1-\tau_K) = \frac{P_{t+1}}{Pt} \tag{4}$$

It's a no-arbitrage condition between the investment in productive capital and the purchase of speculative bubble. The returns from productive capital after-tax must equal to the returns from speculative bubble. In other words, bubble must grow at the rate of interest after-tax for all $t \geq 0$.

Comparing the utility when $d_t = 1$ and $d_t = 0$, we deduce the reservation wage $\underline{w_t}$, which is the take-home pay required to make a worker indifferent between working and remaining unemployed.

$$\underline{w_t} = \frac{V}{\alpha^{\alpha}((1-\alpha)((1-\tau_K)R_{t+1})^{1-\alpha}(1-\tau_L)} \tag{5}$$

This equation shows that, we have an endogenous reservation wage, which depends positively on the disutility of work V and capital taxation τ_K, and negatively on forward looking expectation of interest rate R_{t+1}.

2.2 Firms

We assume that, there is z identical firms in the economy. In each period of $t \geq 0$, each firm hires labor L_t and purchases K_t, in order to produce out put Y_t according to the Cobb-Douglas production technology :

$$Y_t = A\overline{K_t}^{\nu} K_t^{\beta} L_t^{1-\beta}$$

where A>0 denotes scaling factor, and $\overline{K_t}$ is the average capital stock. The production function exhibits constant returns to scale, but there are an increasing returns to scale at the social level , due to the capital externalities $\nu > 0$. Note that ν is assumed to be arbitrarily small.

We assume that, physical capital (K_t) is fully depreciated after one period, and the purchasing of this capital from the perfect capital market costs R_t. We assume also that the hiring of labor L_t costs w_t. Therefore, the profits of each representative firm is presented as follows:

$$\pi = A\overline{K_t}^{\nu} K_t^{\beta} L_t^{1-\beta} - w_t L_t - R_t K_t \tag{6}$$

Profits maximization yields

$$w_t = (1-\beta)A\overline{K_t}^{\nu} K_t^{\beta} L_t^{-\beta} \quad and \quad R_t = \beta A\overline{K_t}^{\nu} K_t^{\beta-1} L_t^{1-\beta} \tag{7}$$

2.3 Government

The government revenues are financed by tax burden on capital and labor income. It balances the budget in each period so that:

$$\tau_K R_t K_t + \tau_L w_t L_t = G_t + NT_{1t} + NT_{2t} \tag{8}$$

where G_t represents the government spending, T_{1t} and T_{2t} are the transfers intended for each young and old household, respectively. From the government budget constraint, we

can distinguish two cases: The government uses all its revenues in public spending ($T_{1t} = 0$, $T_{2t} = 0$), or it uses all them in redistribution system ($G_t = 0$).
In each case, we investigate the effects of speculative bubbles on unemployment and production taking into account the tax policy.

3 Economy without redistribution system

Suppose that, the tax rate on capital and labor income finance only the public expenditures, i.e. $T_1 t = T_2 = 0$ for all $t \geq 0$

Aggregate supply of intrinsically worthless asset is normlized to 1, at price P_t. The demand side is determined by young workers, buying m_t units of asset bubbles, at price P_t. At equilibrium, we get:

$$L_t m_t = 1 \tag{9}$$

Equilibrium in capital market requires that, aggregate savings of the young worker, must be allocated between asset bubble and capital stock in the next period. Since all firms are identical, in equilibrium $K_t = \overline{K_t}$.

$$K_{t+1} + P_t = (1-\alpha)(1-\tau_L)(1-\beta)AK_t^{\nu}k_t^{\beta} \tag{10}$$

where $k_t = \frac{K_t}{L_t}$, denotes capital per worker

We focus on equilibrium with unemployment, i.e. $w_t = \underline{w_t}$ and $L_t < \frac{N}{z}$. From (5) and (7), we get

$$[\beta A {K_{t+1}}^{\nu} {k_{t+1}}^{\beta-1}]^{1-\alpha} = \frac{V}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\tau_K)^{1-\alpha}(1-\beta)(1-\tau_L)A{K_t}^{\nu}k_t^{\beta}} \tag{11}$$

Definition 1: *An intertemporal equilibrium is a sequence* $(K_t, k_t, P_t) \in R^3_{++}$ *satisfying (4), (9), (10) and (11)*

The stationary equilibrium, is an intertemporal equilibrium in which, $K_t = K, k_t = k$ and $P_t = P$, satisfying:

$$V = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(\beta(1-\tau_K))^{1-\alpha}(1-\beta)(1-\tau_L)(AK^{\nu})^{2-\alpha}k^{\beta-(1-\alpha)(1-\beta)} \tag{12}$$

$$1 = (1-\alpha)(1-\tau_L)(1-\beta)AK^{\nu}k^{\beta-1} - \frac{P}{K} \tag{13}$$

$$1 = (1-\tau_K)\beta AK^{\nu}k^{\beta-1} \tag{14}$$

We distinguish two types of stationary equilibrium. The first one without bubbles, i.e. the value of intrinsically worthless asset is $P = 0$. The second one with bubbles, in which case, the value of the asset is positive, i.e. P>0.

3.1 Fiscal policy and the existence of bubbles

In this subsection, we derive the main condition under which, the asset bubble can exist. We determine also the uniqueness of bubbly and bubbleless steady state.

Proposition 1 *A unique bubbly steady state exists, if* $\tilde{R} < 1$*, which is equivalent to* $\frac{\beta}{1-\beta}\frac{1-\tau_K}{1-\tau_L} < 1-\alpha$

Proof. See Appendix A2

where $\tilde{R}$, denotes the steady state interest rate in an economy without bubble.
According to proposition 1, $(1-\tau_K)\tilde{R} < 1$, is a necessary condition for the existence of bubbly steady state. When $\frac{1-\tau_K}{1-\tau_L} = 1$, i.e. the two taxes are identical. The existence of the bubble does not depend on the level of the two taxes. Our investigation focuses on the case when the two taxes are different.
From proposition 1, the bubble is more likely to appear, when τ_K is high and/or τ_L is low. The intuition is straightforward: When the tax burden on capital income increases, the return from productive investment becomes less attractive, which promotes the appearance of bubble. The after-tax income is reduced with a high level of labor income tax, which prevents the existence of bubble.
If τ_K is low in comparison to τ_L , i. e. $(1-\tau_K)\tilde{R} \geq (1-\alpha)$. Then, there is no bubble in the economy (i.e. $P = 0$), and the steady state $(\tilde{K}, \tilde{k})$ is unique.

$$\tilde{k} = \frac{V(1-\alpha)}{\alpha^\alpha}[\frac{(1-\beta)(1-\tau_L)}{\beta(1-\tau_K)}]^{1-\alpha} \tag{15}$$

$$\tilde{K} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(\beta(1-\tau_K)^{1-\alpha}}]^{\frac{1-\beta}{\nu}}[\frac{1}{1-\alpha}]^{\frac{\beta}{\nu}}[\frac{1}{(1-\beta)(1-\tau_L)}]^{\frac{1-(1-\alpha)(1-\beta)}{\nu}} \tag{16}$$

The value of $\tilde{k}$ and $\tilde{K}$, allows us to determine the labor, $\tilde{L}$.

$$\tilde{L} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(\beta(1-\tau_K))^{1-\alpha}}]^{\frac{(1-\beta)}{\nu}-1}[\frac{1}{1-\alpha}]^{\frac{\beta}{\nu}+1}[\frac{1}{(1-\beta)(1-\tau_L)}]^{\frac{1-(1-\alpha)(1-\beta)}{\nu}+(1-\alpha)} \tag{17}$$

Suppose that, the condition of the existence of bubbles is satisfied. Then, the bubbly steady state is given by:

$$k^* = \frac{V\beta(1-\tau_K)}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)} \tag{18}$$

$$K^* = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}}[\frac{1}{\beta(1-\tau_K)}]^{\frac{\beta}{\nu}} \tag{19}$$

$$P^* = [(1-\alpha)(1-\beta)(1-\tau_L)-\beta(1-\tau_K)][\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}}[\frac{1}{\beta(1-\tau_K)}]^{\frac{\beta}{\nu}+1} \tag{20}$$

we can get also,

$$L^* = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}-1}[\frac{1}{\beta(1-\tau_K)}]^{\frac{\beta}{\nu}+1} \tag{21}$$

The determination of bubbly and bubbleless steady state, allows us to see, whether the bubble is enhancing or productive.

3.2 Fiscal policy and productive bubbles

In this section, we show that, under such conditions, the asset bubbles are productive. They heighten capital stock (crowding out effect), and lessen the unemployment. To do this, we make a comparison between the two economies (with and without bubble) at the steady state, by using the following elements: capital stock and employment. If the capital stock in bubbly economy exceeds the capital stock in bubbleless economy, and the labor in bubbly economy dominates the labor in bubbleless economy. Thus, the bubble is productive. The results are summarized in the following proposition

Proposition 2: *1. If* $\frac{\beta}{1-\beta}\frac{1-\tau_K}{1-\tau_L} < \frac{\beta}{1-\beta} < 1-\alpha$*, there is a stationary non productive bubble, with* $K^* < \tilde{K}$ *and* $L^* < \tilde{L}$*.*

2. If $\frac{\beta}{1-\beta}\frac{1-\tau_K}{1-\tau_L} < 1-\alpha < \frac{\beta}{1-\beta}$*, there is a stationary productive bubble, with* $K^* > \tilde{K}$ *and* $L^* > \tilde{L}$ *.*

Proof. See Appendix A3

According to the proposition 2, the nature of bubble (productive or not) depends on the parameters α and β. If the savings rate $(1-\alpha)$ exceeds the ratio $\frac{\beta}{1-\beta}$. Then, the bubble has a crowding in effect (decreases the capital stock), and lessens the employment level. However, this relationship is hard to reconcile with the historical observations. Indeed, they tell us that, the level of savings must be high, but, not all households have a large savings capacity. So, we consider that, $(1-\alpha) < \frac{\beta}{1-\beta}$.

When τ_K is significantly high with respect to τ_L, the bubble appears, and its nature is productive (the condition $\frac{\beta}{1-\beta}\frac{1-\tau_K}{1-\tau_L} < 1-\alpha < \frac{\beta}{1-\beta}$ is satisfied). The mechanisms that allows us to explain these results are simple. In bubbleless steady state, when the return from the productive investment decreases following an increase in τ_K, we obtain an increase in $\underline{w_t}$ (see (5)). The households are facing a drop in τ_L, that in turn induces an decrease of the interest rate $\tilde{R}$ (see (35)). Facing the variation of $\underline{w_t}$ and $\tilde{R}$, firms tend to favor the use of capital to labor. At equilibrium, the employment level falls. Since capital accumulation comes from aggregate savings (Ls), the capital stock falls also.

At the bubbly economy, the level of capital per worker, k^*, comes from an arbitrage condition between the speculative asset and capital. With a high τ_K, the capital per worker decreases, leading to a raise of interest rate R^*. The reservation wage drops following the reduction in τ_L (see (5)). The firms substitute capital by labor. at equilibrium, the employment level increases. As mentioned previously that, capital stock is determined by aggregate savings. As this last increases with labor, thus, the capital stock increases too. In summary, with a high τ_K and low τ_K, the capital stock and labor decrease in bubbleless economy, unlike the bubbly economy, where the capital accumulation and labor raise. So, we end up with $K^*>\tilde{K}$ and $L^*>\tilde{L}$

4 Economy without government spending

Suppose now that, the tax rate on capital and labor income finance only the transfer. This last is shared between young and old households, at the fraction θ and $(1-\theta)$, respectively. Any variation of one or both taxes, it will modify the decision of the young households at the level of consumption and investment.

Definition 2: *An intertemporal equilibrium is a sequence* $(K_t^r, L_t^r, P_t^r) \in R^3_{++}$ *satisfying: the market clearing conditions:*

$$\begin{aligned} &Asset: \ Nm_t^r = 1 \\ &Capital\ stock: \ K_{t+1}^r = Ns_t^r \\ &Labor: \ w_t^r = \underline{w_t^r} \qquad ; L_t^r < \tfrac{N}{z} \end{aligned}$$

and the no-arbitrage condition:

$$R_{t+1}^r = \frac{P_{t+1}^r}{P_t^r}$$

Since all firms are identical, in equilibrium $K_t^r = \overline{K_t^r}$. Therefore the equilibrium system writes:

$$[\beta A {K_{t+1}^r}^{\nu+\beta-1} {L_{t+1}^r}^{1-\beta}]^{1-\alpha} = \frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\tau_K)^{1-\alpha}(1-\beta)(1-\tau_L)A K_t^{r\nu+\beta} L_t^{r-\beta}} \tag{22}$$

$$K_{t+1}^r = \frac{\beta(1-\tau_K)(1-\alpha)[(1-\tau_L)(1-\beta)+\theta(\beta\tau_K+(1-\beta)\tau_L)]}{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)} A K_t^{r\nu+\beta} L_t^{r1-\beta} - P_t^r \tag{23}$$

$$P_{t+1}^r = (1-\tau_K)\beta A {K_{t+1}^r}^{\nu+\beta-1} {L_{t+1}^r}^{1-\beta} P_t^r \tag{24}$$

We can determine the steady state equilibrium, which is defined as, the intertemporal equilibrium in which $K_t^r = K^r, L_t^r = L^r$ and $P_t^r = P^r$, satisfying:

$$V = \alpha^\alpha(1-\alpha)^{1-\alpha}(1-\tau_K)^{1-\alpha}(1-\beta)(1-\tau_L)\beta^{1-\alpha}(A)^{2-\alpha} K^{*r(\nu+\beta-1)(1-\alpha)+\nu+\beta} L^{*r(1-\alpha)(1-\beta)-\beta} \tag{25}$$

$$1 = \frac{\beta(1-\tau_K)(1-\alpha)[(1-\tau_L)(1-\beta)+\theta(\beta\tau_K+(1-\beta)\tau_L)]}{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)} A K^{*r(\nu+\beta-1)} L^{*r(1-\beta)} - \frac{P^{*r}}{K^{*r}} \tag{26}$$

$$1 = (1-\tau_K)\beta A K^{*r(\nu+\beta-1)} L^{*r(1-\beta)} \tag{27}$$

The bubbleless economy corresponds to our benchmark case, and it will be used to compare it with the bubbly economy, at the level of capital stock and employment.

4.1 Redistributive fiscal policy and the existence of bubbles

According to (3), the capital income tax, labor tax income and the transfer, determine the level of savings. This last is important for the existence of bubble. The following proposition presents the existence and uniqueness condition of bubbly steady state.

Proposition 3 *A unique bubbly steady state exists, if* $(1-\tau_K)\tilde{R}^r < 1$, *which is equivalent to* $\frac{\beta(1-\tau_K)+(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)}{1-\beta(1-\tau_K)} < (1-\alpha)$.

Proof. See Appendix B1

where $\tilde{R}^r$, denotes the steady state interest rate in a redistributive economy without bubble. The effect of τ_K, τ_L and θ on the existence of speculative bubble, is obtained from a simple comparative statics of $\lambda = \frac{\beta(1-\tau_K)+(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)}{1-\beta(1-\tau_K)} < 1$. As we can see, λ is decreasing in τ_K and θ, but increasing in τ_L. The bubble is more likely to appear, if one of the three conditions is satisfied: i) τ_K is high and τ_L is low, ii) τ_K and θ are high, iii) τ_K and θ are high and τ_L is low. Regarding the effect of the two taxes on the existence of bubble, the intuition is discussed previously. High transfer to the young households, allows them to have a sufficiently high income to sustain the existence of the bubble.
If the redistibutive fiscal policy, which is summarized by λ, does not satisfy the condition of the existence of bubble, i.e. $1-\tau_K)\tilde{R}^r \geq 1$, then, the bubble does not appear. There exists a unique bubbleless steady state $(\tilde{K^r}, \tilde{L^r})$, which is given by:

$$\tilde{K^r} = A^{-\nu}[\frac{V}{\alpha^\alpha(1-\beta)(1-\tau_L)}]^{\frac{(1-\beta)}{\nu}}[\frac{1}{(1-\alpha)(1-\tau_K)\beta}]^{\frac{\beta}{\nu}}[\frac{\omega}{\gamma}]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}} \tag{28}$$

$$\tilde{L^r} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\beta)(1-\tau_L)}]^{\frac{(1-\beta)}{\nu}-1}[\frac{1}{(1-\alpha)\beta(1-\tau_K)}]^{\frac{\beta}{\nu}+1}[\frac{\omega}{\gamma}]^{\frac{(1-\beta)(1-\alpha)-\beta}{\nu}-(2-\alpha)}$$

Therefore, capital per capita, $\tilde{k^r} = \frac{\tilde{K^r}}{N}$, can be determined systematically,

$$\tilde{k^r} = \frac{1}{N}A^{-\nu}[\frac{V}{\alpha^\alpha(1-\beta)(1-\tau_L)}]^{\frac{(1-\beta)}{\nu}}[\frac{1}{(1-\alpha)(1-\tau_K)\beta}]^{\frac{\beta}{\nu}}[\frac{\omega}{\gamma}]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}} \tag{29}$$

where $\gamma = \beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)$, and $\omega = (1-\beta)(1-\tau_L)+\theta(\beta\tau_K+(1-\beta)\tau_L)$

Suppose that, the condition of the existence of bubble is satisfied. Then, the uniqueness of the steady state is given by $(K^{*r}, L^{*r}, P^{*r}) \in R^3_{++}$:

$$K^{*r} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}}[\frac{1}{\beta(1-\tau_K)}]^{\frac{\beta}{\nu}} \tag{30}$$

$$L^{*r} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}-1}[\frac{1}{\beta(1-\tau_K)}]^{\frac{\beta}{\nu}+1} \tag{31}$$

$$P^{*r} = [\frac{1}{A}]^{\frac{1}{\nu}}\frac{(1-\alpha)(1-\tau_L)(1-\beta)-\beta(1-\tau_K)+(\theta-\alpha)(\beta\tau_K+(1-\beta)\tau_L)}{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)} \times [\frac{V}{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_{L^*})}]^{\frac{1-\beta}{\nu}}[\beta(1-\tau_{K^*})]^{\frac{-\beta}{\nu}} \tag{32}$$

we can determine uniquely capital per capita, k^{*r}.

$$k^{*r} = \frac{1}{N}[\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\beta)(1-\tau_L)}]^{\frac{1-\beta}{\nu}}[\beta(1-\tau_K)]^{\frac{-\beta}{\nu}} \tag{33}$$

4.2 Redistributive fiscal policy and productive bubbles

In this section, we show that, under a condition, the bubble is productive. To show so, we compare the the bubbly economy with the benchmark case, under two criteria: Capital accumulation and employment. The results are summarized in the following proposition

Proposition 4: *Moving from bubbleless to bubbly steady state leads to: 1) An increase of capital stock and employment if* $\lambda < (1-\alpha) < \frac{\beta}{(1-\beta)}$.
2) A decrease of capital stock and employment if $\lambda < \frac{\beta}{(1-\beta)} < (1-\alpha)$.

Proof. See Appendix B2

Depending on the parameter value of α and β, the nature of bubble can be determined. In fact, if $\frac{\beta}{(1-\beta)} < (1-\alpha)$, there is no stationary productive bubble, i.e. the bubble lessens capital accumulation (it has a crowding in effect), and heightens unemployment. And if $(1-\alpha) < \frac{\beta}{(1-\beta)}$, there is a stationary productive bubble.
Suppose that, τ_K and θ are high, and $(1-\alpha) < \frac{\beta}{(1-\beta)}$. Then, the bubble is productive (the condition $\lambda < (1-\alpha) < \frac{\beta}{(1-\beta)}$ is satisfied). The mechanism which can explain these results is simple: For a high level of τ_K, the interest rates R^{*r} and $\tilde{R^r}$ increase (see (27) and (39)). In the two economies, the firms favor the use of labor, due to substitution effect. At equilibrium, we have a raise of employment. Since $R^{*r} > \tilde{R^r}$, thus, the substitution effect is more powerful in bubbly economy than in bubbleless. We end up by $L^{*r}>\tilde{L^r}$. Capital accumulation comes from aggregate savings, which depends positively on the transfer, T and the interest rate, R. As $R^{*r} > \tilde{R^r}$, and $T^{*r} > \tilde{T^r}$ (see (8)). For a high value of θ, we conclude that $K^{*r}>\tilde{K^r}$,

5 Numerical analysis

The literature shows that, the tax burden on capital income reduces the capital stock. However, through our model with capital externality and endogenous labor supply, we demonstrate that, thanks to this kind of tax, the speculative bubble promotes capital stock and labor. In fact, a high level of capital income tax, sustained by a low level of labor income tax and/or high level of the transfer to the young investors, increases capital accumulation stock and labor. This has been proven in different countries as: Chile, Mexico, USA and Israel.
We fix the value of the parameters as follows, $\alpha = 0,54$, $\beta = 0,333$. A, V and ν are normalized to 1. We consider two sets of country, the first one is presented is the table 1 where, the distribution system is supposed to be nil (the income and wealth transfer is below the median).

Table 1: ***Tax burden on capital and labor income***

Countries	τ_K	τ_L
Chile	0.20	0.07
Korea	0.24	0.22
Mexico	0.3	0.19
Norway	0.28	0.37
Slovenia	0.17	0.42
Sweden	0.22	0.43
USA	0.39	0.31

Source: OECD Data base.

The second one is presented in the table 2, where there is no government spending (the redistribution is above the median).

Table 2: ***Results from the calibration***

Countries	$\tilde{K}$	K^*	$\tilde{L}$	L^*	F
Chile	193 777.43	197 280,77	204 872,39	230 074,73	0.03
Korea	767 791.58	$--$	859 636.95	$--$	-0.02
Mexico	760 555,33	773 352.39	805 812.14	897 750.46	0.02
Norway	3 984 252.05	$--$	4 800 473.76	$--$	-0.1
Slovenia	4 570 748.90	$--$	6 107 274.08	$--$	-0.25
Sweden	6 238 858.78	$--$	8 166 335.90	$--$	-0.22
USA	3 525 567.87	3 563 233.65	3 774 602.5	4 043 475.22	0.01

Source: OECD Data base.

The transfers towards the young, corresponds to the public expenditure on family (family allowances, maternity and parental leave), income maintenance and other cash benefits. Regarding the redistribution towards the old households, it consists the pension.
The calibration results are displayed in table 3 and 4

Table 3: ***Redistribution system and tax burden on capital and labor income***

Countries	τ_K	τ_L	θ	$1-\theta$
France	0.38	0.49	0.15	0.85
Iceland	0.2	0.34	0.49	0.51
Israel	0.25	0.20	0.57	0.43
Italy	0.31	0.47	0.13	0.87
Japan	0.37	0.31	0.1	0.9
United Kingdom	0.23	0.31	0.36	0.64

Source: OECD Data base.

Table 4: ***Results from the calibration***

Countries	$\tilde{K}^r$	K^{*r}	$\tilde{L}^r$	L^{*r}	F'
France	31 284 426.2	$--$	82 161 938.40	$--$	-0.28
Iceland	2 079 012.26	$--$	2 506 724.63	$--$	-0.1
Israel	666 441.51.51	667 524.02	706 842.88	714 310.2	0.003
Italy	17 071 566.74	$--$	43 575 706.49	$--$	-0.3
Japan	3 647 147.5	$--$	8 201 542.71	$--$	-0.18
United Kingdom	1 783 345.69	$--$	2 542 953.27	$--$	-0.12

Source: OECD Data base.

The two tables show the values of capital stock and labor in both economies (bubbly and bubbleless). The sign of F and F' which are in table 2 and 4, respectively, indicates the existence or not of the bubble (negative sign implying that the economy does not exhibit a bubble).
As we can see in table 3, the countries as Chile, Mexico and USA may exhibit a bubble, which is productive. It is obvious that, in these countries, the level of capital income tax is sufficiently high with respect to labor income tax . This encourages the investment in productive capital and speculative asset, and promotes also the employee recruitment.
The table 4 shows that, only Israel may exhibit a bubble. This bubble is productive. It follows from the fact that, the value of the capital income tax is high, sustained by an income and wealth redistribution to the young investors.
As a conclusion, firstly, we can say that, thanks to capital income tax, the asset bubble promotes capital stock. And secondly, the results obtained empirically match exactly our theoretical analysis.

6 Conclusion

The novelty presented in our research paper, is highlighted through the incorporation of tax policy. We thus consider that, the tax burden on capital and labor income finance the government spending and transfer. We have derived the condition under which the bubble can exist. High level of capital income tax, makes the investment into capital stock less attractive, which facilitates the existence of speculative bubble. Also low labor income tax and /or high transfer to the young investors, modifies the income upward, which promotes the appearance of the bubble. We have exhibited a new mechanism which shows that, the bubbles are productive. When the bubble exists, the interest rate increases following the raise of the capital income tax. Facing an increase of the relative price of capital, the firms tend to favor the use of labor. At equilibrium, the unemployment level decreases (due to the substitution effect), and the capital stock increases (it is sustained by low capital income tax and/or high transfer to the young households).
A way to extend our model, is to investigate the relationship between speculative bubble, unemployment and capital stock in an unionized economy. The union plays a role of capital income taxation. Thanks to its bargaining power, the union redistributes a share of rental cost of capital to wages. However, both the return on asset and wage, are important for the existence of bubble. In such a context, what is the effect of the union bargaining power on the existence and the size of the asset bubbles? Under what conditions, the bubbles can be productive? Can we explain the observed movements of unemployment following the appearance and the burst of asset bubbles?.

7 Appendix

7.1 Appendix A1

Determination of the reservation wage

Maximizing the utility under the budget constraint, we get from (51) and(2), we have,

$$c_{1t} = \alpha((1-\tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)}) \tag{34}$$

$$c_{2t+1} = (1-\alpha)(1-\tau_K)R_{t+1}((1-\tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)}) \tag{35}$$

Each individual decides to work if only if: $U(\underline{w_t}) = U(d_t = 1) - U(d_t = 0) \geq 0$

Using (34), (35) and:

$$U(d_t = 1) = [\alpha((1-\tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)})]^{\alpha}[(1-\alpha)(1-\tau_K)R_{t+1}((1-\tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)})]^{(1-\alpha)}$$

$$U(d_t = 0) = [\alpha(T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)})]^{\alpha}[(1-\alpha)(1-\tau_K)R_{t+1}(T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1-\tau_K)})]^{(1-\alpha)}$$

we obtain, $U(d_t = 1) - U(d_t \geq 0$ if and only if $w_t \geq \underline{w_t}$, with

$$\underline{w_t} = \frac{V}{\alpha^{\alpha}((1-\alpha)(1-\tau_K)R_{t+1})^{1-\alpha}(1-\tau_L)}$$

7.2 Appendix A2

- ***Condition for the existence and uniqueness of bubbly steady state***

Using (13) and (14), we have

$$1 = \frac{(1-\alpha)(1-\beta)(1-\tau_L)}{\beta(1-\tau_K)} - \frac{P}{K^*} \tag{36}$$

Substituting the steady state interest rate, $\tilde{R} = A\beta\tilde{K}^v\tilde{k}^{\beta-1}$ into (13) when P=0, we get:

$$1 = \frac{(1-\alpha)(1-\beta)(1-\tau_L)}{\beta}\tilde{R}$$

$$\tilde{R} = \frac{\beta}{(1-\alpha)(1-\beta)(1-\tau_L)} \tag{37}$$

Substituting (37) into (36), we get:

$$1 = \frac{1}{(1-\tau_K)\tilde{R}} - \frac{P^*}{K^*}$$

$P^* > 0 \Rightarrow$

$$\tilde{R}(1-\tau_K) < 1$$

7.3 Appendix A3

- ***Comparison between $\tilde{K}$ and K^****

We can rewrite $\tilde{K}$ as:

$$\tilde{K} = [\frac{(1-\alpha)(1-\beta)(1-\tau_L)}{\beta(1-\tau_K)}]^{(1-\alpha)(1-\beta)-\beta}K^*$$

$$\tilde{K} = [\frac{1}{(1-\tau_K)\tilde{R}}]^{(1-\alpha)(1-\beta)-\beta}K^*$$

$K^* > \tilde{K}$ requires that:

$$[\frac{1}{(1-\tau_K)\tilde{R}}]^{(1-\alpha)(1-\beta)-\beta} < 1$$

$$(1-\alpha)(1-\beta)-\beta < 0$$

$$1-\alpha < \frac{\beta}{1-\beta}$$

- ***Comparison between $\tilde{L}$ and L^****

We can rewrite $\tilde{L}$ as:

$$\tilde{L} = [\frac{(1-\alpha)(1-\beta)(1-\tau_L)}{\beta(1-\tau_K)}]^{(1-\alpha)[\frac{1-\beta}{\nu}-1]-\frac{\beta}{\nu}-1}L^*$$

$$\tilde{L} = [\frac{1}{(1-\tau_K)\tilde{R}}]^{(1-\alpha)[\frac{1-\beta}{\nu}-1]-\frac{\beta}{\nu}-1}L^*$$

$L^* > \tilde{L}$ requires:

$$[\frac{1}{(1-\tau_K)\tilde{R}}]^{(1-\alpha)[\frac{1-\beta}{\nu}-1]-\frac{\beta}{\nu}-1} < 1$$

$$(1-\alpha)[\frac{1-\beta}{\nu}-1]-\frac{\beta}{\nu}-1 < 0$$

$$(1-\alpha) < \frac{\beta+v}{1-\beta-v}$$

7.4 Appendix B1

- ***Condition for the existence and uniqueness of bubbly steady state***

Using (26) and (27), we have

$$1 = \frac{(1-\alpha)[(1-\tau_L)(1-\beta)+\theta(\beta\tau_K+(1-\beta)\tau_L)]}{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)} - \frac{P^r}{K^{*r}} \tag{38}$$

In the economy without bubble $\tilde{R}^r = A\beta\tilde{K}^{r(v+\beta-1)}\tilde{L}^{r(1-\beta)}$. Substitute this equation into (26) when P=0, we get:

$$1 = \frac{(1-\tau_K)(1-\alpha)[(1-\tau_L)(1-\beta)+\theta(\beta\tau_K+(1-\beta)\tau_L)]}{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)}\tilde{R}$$

$$\boxed{\tilde{R} = \frac{\beta(1-\tau_K)+\alpha(1-\theta)(\beta\tau_K+(1-\beta)\tau_L)}{(1-\tau_K)(1-\alpha)[(1-\tau_L)(1-\beta)+\theta(\beta\tau_K+(1-\beta)\tau_L)]}} \tag{39}$$

Substituting (39) into (38) , we obtain:

$$1 = \frac{1}{(1-\tau_K)\tilde{R}} - \frac{P^*}{K^*}$$

$P^* > 0 \Rightarrow$

$$\tilde{R}(1-\tau_K) < 1$$

7.5 Appendix B2

- ***Comparison between $\tilde{K}$ and K^****

We can rewrite $\tilde{K}$ as:

$$\tilde{K} = A^{-\nu}[\frac{V}{\alpha^\alpha(1-\beta)(1-\tau_L)}]^{\frac{(1-\beta)}{\nu}}[\frac{1}{(1-\alpha)(1-\tau_K)\tilde{R}}]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}}$$

$$K^* = \tilde{K}[(1-\tau_K)(1-\alpha)\tilde{R}]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}}(1-\alpha)^{\frac{\beta}{\nu}}(1-\alpha)^{-\frac{(1-\alpha)(1-\beta)}{\nu}}$$

$$K^* = \tilde{K}((1-\tau_K)\tilde{R})^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}}$$

$K^{*v} > \tilde{K}^v$ requires that:

$$((1-\tau_K)\tilde{R})^{(2-\alpha)(1-\beta)-1} > 1$$

$$(2-\alpha)(1-\beta)-1 < 0$$

$$\boxed{1-\alpha < \frac{\beta}{1-\beta}}$$

- ***Comparison between $\tilde{L}$ and L^****

We can rewrite $\tilde{L}$ as:

$$\tilde{L} = [\frac{1}{A}]^{\frac{1}{\nu}}[\frac{V}{\alpha^\alpha(1-\beta)(1-\tau_L)}]^{\frac{(1-\beta)}{\nu}-1}[\frac{1}{(1-\alpha)\beta(1-\tau_K)}]^{\frac{\beta}{\nu}+1}[\frac{1}{(1-\alpha)(1-\tau_K)\tilde{R}}]^{\frac{(1-\beta)(1-\alpha)-\beta}{\nu}-(2-\alpha)} \tag{40}$$

$$Ł^* = (1-\alpha)^{-(1-\alpha)[\frac{1-\beta}{\nu}-1]}(1-\alpha)^{\frac{\beta}{\nu}+1}[\tilde{R}(1-\tau_K)(1-\alpha)]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}-(2-\alpha)}\tilde{L}$$

$$Ł^* = [(1-\tau_K)\tilde{R}]^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}-(2-\alpha)}\tilde{L}$$

So, $L^* > \tilde{L}$ requires:

$$\tilde{R}^{\frac{(1-\alpha)(1-\beta)-\beta}{\nu}-(2-\alpha)} > 1$$

$$\frac{(1-\alpha)(1-\beta)-\beta}{\nu} - (2-\alpha) < 0$$

$$(1-\alpha) < \frac{\beta+v}{1-\beta-v}$$

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