Self-Inflicted Unemployment Scarring and Stigma*

Julien Hugonnier[1,4,5], Florian Pelgrin[2,6], and Pascal St-Amour[3,4,6]

[1]École Polytechnique Fédérale de Lausanne

[2]EDHEC Business School

[3]HEC Lausanne, University of Lausanne

[4]Swiss Finance Institute

[5]CEPR

[6]CIRANO

January 25, 2019

Preliminary

Do not quote without authors' permission

*We have benefited from very useful discussions with and comments from Rob Alessie, Marnix Amand, Tony Berrada, David Card, Rafael Lalive, Fabien Postel-Vinay, Jean-Marc Robin, Bruce Shearer and Nicolas Werquin, as well as from participants at the IZA World Labor 2018 Conference and the Society of Labor Economists 2017 Annual Meeting. Financial support from the Swiss Finance Institute is gratefully acknowledged. The usual disclaimer applies.

Abstract

Long-term scars of unemployment include higher ex-post displacement and income losses, as well as lower re-employment that increase in occurrence and duration of previous unemployment spells. Human capital explanations assume that its accumulation is valued by the market, but is impaired by non-employment. We retain the former assumption, yet relax the latter by considering continuous investment decisions made by workers across employment statuses, where wages, as well as likelihood and duration of unemployment spells are capital-dependent. We calculate analytically the joint optimal investment by the employed and the unemployed. We calibrate the model using NLSY79 data and identify two dynamically stable steady-state values with a lower one for the unemployed. Circular dynamics follow whereby human capital optimally falls during unemployment spells and increases again upon re-employment. Scarring and stigma are thus *self-inflicted*, i.e. endogenously induced through decisions made by agents only. A counter-factual exercise allows to gauge and confirm the importance of employment risks hedging in total demand for human capital and that of moral hazard issues in the design of UIB programs. We also show that status-dependent accumulation technology and capital specificity complement, but are not required for scarring and stigma.

Keywords— Human capital; Unemployment; Duration dependence; unemployment stigma and scarring; Displacement; Re-employment probability.

JEL classification— J24, J64, J65

1 Introduction

1.1 Motivation and overview

In addition to contemporaneous income losses associated with incomplete and temporary replacement,[1] unemployment (u) imposes long-term costs to workers. On the one hand, scarring refers to persistent detrimental labor market outcomes, such as earnings decline,[2] as well as lower employment (e), re-employment ($u \to e$) and higher displacement ($e \to u$) of workers with previous unemployment spells.[3] On the other hand, negative duration dependence (stigma) implies more unfavorable ex-post outcomes the longer agents are not working.[4]

Human capital is often invoked as an explanation for unemployment scarring and stigma. This conjecture relies on two postulates: (i) human capital is valued by employers and (ii) its accumulation is impaired by non-employment. Evidence for capital valuation include higher wages, lower displacement risk and faster re-employment transitions for

[1]The U.S. weighted average UI replacement rate in 2010-2011 was 0.41 and varied between 0.30 (AK, LA) and more than 0.49 (AZ, HI, RI) with median maximal duration of 26 weeks. Source: U.S. Department of Labor.

[2]Jacobson et al. (2005, Fig. 1) report that pre- vs post-displacement earnings losses are 10% for short-tenured, 23% for medium-tenured and 30% for long-tenured workers. See Kletzer (1998); Arulampalam et al. (2001); Abbott (2008); Quintini and Venn (2013); Carrington and Fallick (2014) for reviews of US and international evidence on post-unemployment income losses. Additional discussion of income scars is presented in Jacobson et al. (1993); Neal (1995); von Wachter et al. (2009); Farber (2011); Davis and von Wachter (2011); Fang and Silos (2012); Huckfeldt (2016). Corresponding welfare costs are found to be substantial by Rogerson and Schindler (2002); Krebs (2007).

[3]Ruhm (1991a) finds that displacement entails a three times higher risk of future unemployment. Stevens (1997) shows that displacement induces multiple additional displacement, resulting in long-term earnings losses. Krueger et al. (2014, Fig. 3) show that the long-term unemployed ($>$ 26 weeks) have an exit rate to employment less than half that of the very short-term ($<$ 5 weeks). Guvenen et al. (2017) emphasize the persistence of (voluntary and involuntary) non-employment statuses in explaining earnings losses. Fujita and Moscarini (2017) distinguish between recalled and new hires in analyzing $e \to u \to e$ transitions, showing that recalled workers had more tenure, received offers faster and stayed longer with their employer, while experiencing more duration dependence than new hires. See also Nilsen and Reiso (2011); Eliason and Storrie (2006) for Scandinavian and Arulampalam (2001) for British evidence on employment scarring. Seniority rules determining Last-in-First-Out termination policies are discussed in Kletzer (1998); Medoff and Abraham (1981); Carmichael (1983).

[4]Kroft et al. (2013) rely on fictitious CV's sent to prospective employers advertising openings and find that call-backs were 45% lower for 8-month unemployment spells, compared to 1-month. Similar effects through low call-backs are identified in Eriksson and Rooth (2014) for Swedish data. See also Eubanks and Wiczer (2016); Alvarez et al. (2016); Nekoei and Weber (2015); Huttunen et al. (2011); van den Berg and van Ours (1996); Ruhm (1991b) for discussions of the role of sample composition effects and unobserved heterogeneity in explaining duration dependence.

skilled workers.[5] Reasons for slower capital accumulation for the unemployed include learning-by-doing, faster skills depreciation and access to different learning technologies in non-employment, as well as human capital specificity, technological obsolescence, and unemployment insurance (UI) incentives distortions. The relative depreciation of the unemployed workers' capital is sanctioned by employers who rely on observable spell occurrence and duration as a screening mechanism to identify existence and magnitude of imperfectly observed human capital losses. Firms are consequently less willing to hire and pay high wages to, as well as are more inclined to lay off previously unemployed workers, especially the long-duration ones.

Our main research question is whether these long-term unemployment costs are still relevant when the first postulate of valuable capital is retained, but not the second assumption of exogenous accumulation wedges across employment statuses. In particular, we ask whether unemployment scarring and stigma can persist an environment where measurable human capital (i) is associated with both a lower likelihood and expected duration of unemployment spells, in addition to higher wages and (ii) can be continuously adjusted by agents in both employment and unemployment states. Whereas the first assumption is well justified empirically and in the literature, the second postulate can be rationalized through workers' decisions at the extensive (i.e. participation) and intensive (i.e. effort) margins with respect to on-the-job training, continuing education and active UI programs. Indeed, both evidence and theoretical rationalization for human capital decision- and cost-sharing in employment is provided by Becker (1962, 1993); Acemoglu and Pischke (1999); Fu (2011); Marotzke (2014); Kräkel (2016) whereas unemployed agents' participation in active UI policies is reviewed by Heckman et al. (1999); Jacobson et al. (2005). To the extent that capital positively affects wages, as well as reduces unfavorable employment risks and that its accumulation is decided by the agent, exposure to unemployment scarring and stigma should be minimized by investing more when

[5] See Mincer (1974) for education, tenure and experience gradients of wages. See Neal (1995); Kletzer (1998); Farber (2005, 2011); Riddell and Song (2011); Gomes (2012); Fang and Silos (2012); Quintini and Venn (2013) for evidence on role of human capital in mitigating exposure to labor market risks,

employed (to prevent displacement), as well as when unemployed (to accelerate re-employment and counter duration dependence). If the optimal strategy nonetheless admits long-term unemployment costs, then any residual scarring and stigma must be optimally *self-inflicted* by the agent.

To answer this question, we address unemployment scarring and stigma through the lens of classical Human Capital (HK) investment theory, to which we append endogenous exposure to employment risks. We rely on four modeling choices. First, we take as primitive the assumption that human capital induces better wages, as well as lower displacement risk and faster re-employment transitions for the better-skilled agents. Second, we internalize both the income and employment risks motives in a HK setup with Ben-Porath (1967) accumulation featuring stochastic employment states and endogenous transition densities. Third, a realistic specification of unemployment insurance benefits provides both the resources and the incentives for investing during unemployment spells. Finally, we allow for (but do not impose) differences in human capital technology across employment statuses, as well as for firm- or sector-specific capital losses incurred upon occurrence of displacement. Abstracting from both in our baseline setup lets us emphasize scarring dynamics resulting from *optimal investment* policies, instead of from arbitrary *parametric restrictions*. We later reinstate status-dependent technology and capital specificity to gauge their respective contributions.

We compute interior investment rules for this problem and characterize the wages and employment dynamics resulting from the optimal choices. Solving this dynamic model is particularly challenging for two reasons. First, as is the case for Diamond (1982); Mortensen and Pissarides (1994) (DMP) Search and Matching models – and unlike standard HK models –, the employment and unemployment value functions are non-separably intertwined with one another, as the returns to investing when employed depend on what is selected when unemployed and vice versa. Second and more importantly, both the displacement and re-employment arrival rates are endogenous functions of the human capital decided by the agent, which enriches the motives for investing, but significantly complicates the model's solution. We circumvent this problem through two-step expansion methods developed in Hugonnier, Pelgrin and St-Amour (2013). We start by solving analytically a restricted version (referred to as order-0) where the arrival rates governing displacement and re-employment are exogenously set. We then do an expansion on this solution (order-1) where the perturbation concerns the key parameter governing the endogeneity of the arrival rates.

We first show that the order-0 solution captures only a subset of the stylized facts on scarring and stigma. The exogenous employment risks case yields two separate and constant human capital growth; consequently no steady-state exists. To illustrate its shortcomings, we abstract from ad-hoc depreciation and productivity differences across employment statuses, as well as from capital specificity in our baseline scenario. Importantly, a sufficiently high capital gradient of income entails that both investment and growth are lower for the unemployed than for the employed. Since capital positively affects employment revenues, the gap in constant growth rates generates positive income wedges that are increasing in unemployment duration, consistent with *income* scarring and stigma. However, because displacement and re-employment intensities cannot be adjusted, slower capital growth during unemployment spells is inconsequential for future employment risks exposure. The restricted model is thus unable to reproduce *employment* scarring and stigma observed in the data. Moreover constant growth rates levels and differentials entail permanent effects of unemployment, at odds with the persistent, but temporary nature of observed scarring and stigma.

We next reinstate endogenous displacement and re-employment intensities in calculating, calibrating and simulating the order-1 solutions to assess whether these shortcomings can be addressed. The calibration is selected to match employment and income dynamics from NLSY79 data. Again abstracting from technological differences and capital specificity, our baseline results confirm that the optimal human capital dynamics are now fully consistent with *both* income and employment scarring and stigma, as well as their non-permanent features. This finding rests on two main results. First, investment by the unemployed is positive, but lower than for the employed. Second, distinct employed and unemployed steady-state levels of human capital exist, are dynamically stable and lower for the unemployed. Combining the two entails circular optimal wages and risks dynamics. Upon unemployment, human capital optimally falls towards the lower unemployed steady state and increases towards the higher employed steady state upon re-employment. Since re-employment (resp. displacement) and wages are increasing (resp. decreasing) functions, unemployment spells thus internally induce lower recall rates and lower wages and higher displacement upon re-employment (scarring). Moreover, since human capital falls continuously until either re-employment occurs or the steady state is reached, duration dependence (stigma) obtains internally. Because scarring and stigma depend on displacement and re-employment events whose joint likelihood is human capital-dependent and since the investment in the capital is decided by workers exclusively, scarring and stigma are self-inflicted in the sense that both arise through an optimal dynamic strategy of workers, with minimal and realistic assumption on market valuation of skills.

Since our model innovates from standard human capital theory in that dimension, we gauge the importance of displacement and re-employment risks control in total demand for human capital. By removing endogenous exposure and adjusting the parameters to maintain the mean displacement/reemployment rates constant, we show that the marginal effects of risk exposure adjustment strongly complements any wage considerations in investment decisions. Moreover, we also measure the policy effects of UI generosity and of base income on total investment. Standard search models associate more generous programs with reduced search efforts and longer unemployment spells (e.g. Chetty, 2008; Daly et al., 2012). We offer an alternative moral hazard explanation whereby less generous UIB increases the motives for investing, decreasing unemployment through lower displacement and higher re-employment. Finally, our baseline results assume employment status independent technologies and no capital specificity. We assess the importance the these restrictions by re-introducing both in turn. Our results show that unemployment disadvantages are complementary, but not necessary for self-imposed scarring and stigma.

This paper contributes to discussions of human capital in labor market dynamics. We highlight the importance of endogenous employment risks exposure as additional motivation for investing in one's own human capital as a complement to the traditional higher wages argument. These employment risks are widely assumed to be the result of systemic macro shocks and cannot be insured against through market instruments, thereby justifying both active macro stabilization and UIB policies. We show instead that displacement and re-employment risks *can* be adjusted through agents' decisions and that long-term scars can obtain optimally through investment choices made by workers only. Finally, we highlight the strong moral hazard risks in making the UIB programs more generous. This results in lowering the incentives for investing, with ensuing higher displacement and lower wages and re-employment.

1.2 Related literature

HK models Our paper is most directly related to the HK literature where agents make continuous decisions on their human capital accumulation subject to Ben-Porath (1967) technology. A first strand emphasizes the role of specificity, of capital complementarities and of market frictions in optimal cost- and decision-sharing by workers and firms (Becker, 1962, 1993; Acemoglu and Pischke, 1999; Fu, 2011; Marotzke, 2014; Kräkel, 2016). A second strand focuses on heterogeneity in human capital production, both in terms of abilities and in types of acquired capital (Ingram and Neumann, 2006; Cunha and Heckman, 2007; Heckman, 1976, 2008; Hu and Taber, 2011; Yamaguchi, 2012; Polachek et al., 2015; Jones, 2014; Stantcheva, 2017; Guvenen et al., 2018). A third subset of HK contributions is primarily concerned with the life cycle of wages and earnings, notably how pre-employment education, finite employment and life horizons reduces human capital investment late in life and yields hump-shaped earnings profiles (Heckman, 1976, 2008; Keane and Wolpin, 1997; Huggett et al., 2006, 2011; Cervellati and Sunde, 2013; Hendricks, 2013; Kredler, 2014). A fourth strand of the HK literature measures the impact of non-diversifiable depreciation and income shocks to the accumulation process (Rogerson and Schindler, 2002; Krebs, 2003; Pavoni, 2009; Huggett et al., 2011).

We follow the classical HK approach in letting capital investment decisions be made and costs be incurred by agents exclusively. In addition, the model is flexible enough to allow for differences in abilities or technology, as well as between general and specific capital. However, we do not emphasize heterogeneity in the primitives as the main driving force. Rather heterogeneous income and employment outcomes stem exclusively from optimal investment and idiosyncratic shocks whose distributions are endogenously determined through the agents' choices. Moreover, although the HK framework we resort to is by definition a life cycle model, we do not emphasize its life cycle properties. In particular, we neither focus on education decisions made prior to labor market entry, nor do we rely on the earnings profile by age to identify the properties of the law of motion. Finally, the distribution of human capital shocks found in the literature is exogenously set and cannot be altered. One exception is Keane and Wolpin (1997) where agents select between finite alternative distributions on human capital returns. However, our choices are continuous, rather than among a fixed set of alternatives (e.g. working, not working) and the shocks we consider are exclusively driven by employment status, with any variability in capital resulting from corresponding optimal choices.

DMP models Our paper is indirectly related to the strand of the DMP Search and Matching models either explictly or implicitly emphasizing human capital (DMP-HK). Explicit DMP-HK literature[6] primarily adopts a learning-by-doing perspective whereby skills reflect work experience that improve match quality and wages and that accumulate if employed and stagnate or decline during non-employment spells (either voluntary or not).

[6]Examples of DMP settings with explicit human capital considerations include Ljungqvist and Sargent (1998); Shimer and Werning (2006); Pavoni (2009); Yamaguchi (2010); Burdett et al. (2011); Esteban-Pretel and Fujimoto (2014); Bagger et al. (2014); Ortego-Marti (2017); Fujita (2018); Guvenen et al. (2018). Capital depreciation can further be accelerated in "micro-turbulent" periods where workers suffer from specific skills obsolescence (Ljungqvist and Sargent, 1998; Kitao et al., 2017).

Human capital accumulation in DMP-HK models is best characterized as a by-product of workers' job acceptance decisions and on- and off-the-job search efforts, rather than as a consequence of explicit investment choices by agents.[7] Exposure to employment risk is also indirectly affected by workers decisions, such as in the case of endogenous separation, where matches are not consumed in light of insufficient *ex-post* quality (Esteban-Pretel and Fujimoto, 2014; Fujita and Moscarini, 2017) or in unemployment search efforts that are combined with market tightness conditions (Mukoyama et al., 2018), as well as human capital specificity (Fujita and Moscarini, 2017; Fujita, 2018) to determine the job arrival rate.

We also draw from the DMP literature with implicit references to human capital. For example, the match quality in Pissarides (1992) depends on past employment status and is higher for previously employed workers, thereby mimicking additional skills depreciation during unemployment. Recall models such as Fujita and Moscarini (2017) emphasize dynamics for match productivity that persist as long as a worker does not find employment outside a given firm, thereby capturing firm-specific human capital that can be drawn upon when recalled. Kroft et al. (2016) implicitly mimic unemployment depreciation by directly appending negative duration dependence to model UE transitions in a search framework. Finally, Job Ladders models (Lise, 2013; Pinheiro and Visschers, 2015; Moscarini and Postel-Vinay, 2016; Krolikowski, 2017) emphasize slow resolution of mismatches between demanded and offered skills to explain wages and employment risks dynamics. These papers have implicit references to human capital where displaced workers suffer from jumps to less favorable employment ladders and slowly climb back up when their capital is replenished following re-employment.

[7]Exceptions in DMP-HK setups with explicit investment decisions include Flinn and Mullins (2015) who consider binary schooling choices made prior to market entry and Kitao et al. (2017) who allow for direct investment at the mid-life (Experienced) phase. Flinn et al. (2017); Fu (2011) analyse joint training decisions by workers/employers, whereas agents decide on job offers that include training opportunities, as well as wages, whereas Lentz and Roys (2015) consider training decisions made by firms exclusively. Guvenen et al. (2018) let workers select accumulation through directional search for firms with different skills requirements that augment human capital. This literature considers income motives only for accumulation, with no effects on the distribution of employment risks internalized in workers decisions.

We indirectly borrow from the DMP paradigm in letting agents' decisions affect their employment outcomes and from the DMP-HK segment by channeling this influence through their human capital. We also implicitly assume that match quality is improved by the latter, resulting in better employment opportunities (wages/risks) for high-capital agents. Moreover, the circular optimal wage and employment dynamics we uncover share strong similarities with those obtained under the Job Ladders approaches.

However, several differences with DMP are worth mentioning. First, we abandon the learning-by-doing perspective by making capital accumulation a product of deliberate and continuous decisions by agents across the employment statuses. Equivalently, whereas DMP models focus on extensive margin adjustments associated with changes in statuses, we emphasize intensive adjustments where agents can continuously fine-tune their human capital throughout the employment or unemployment spells. Second, we depart from DMP in taking a partial-equilibrium and agents-focused perspective. Indeed, firms, rather than agents, act mechanically in our setup, supplying the wage, displacement and re-employment functions that are taken as primitives and are not stemming from general equilibrium. Finally, we put forward an idiosyncratic, rather than systemic stochastic environment where the capital-induced distributions are agent-specific and do not encompass equilibrium variables such as the market tightness rate.

2 Some NLSY79 evidence on employment risks and human capital

We resort to National Longitudinal Survey of Youth (NLSY79) data to provide *prima facie* evidence of scarring and stigma, as well as to compute empirical moments that will be used in the calibration exercise below. NLSY79 is a widely-used[8] panel of 9,964 respondents aged between 14-22 in 1979, and followed up to 2014, providing longitudinal information on employment statuses and income, as well on socio-economic variables (see

[8]See Guvenen et al. (2018); Lise (2013) and references therein for recent applications.

Appendix A for details). Summary statistics in Table 1 shows that our sample is evenly balanced on gender, composed mainly of white, US citizens, of average age 32, and living in urban areas. Human capital measures include close to 13 years of highest completed grade, with 14% of respondents indicating vocational or professional training. Overall, 91% were employed with mean income less than $17 thousands in real terms.

Table 2 identifies employment scarring and stigma by reporting current employment probabilities by past statuses (panel a) and by human capital (panels b and c). First, $t-1$ unemployment lowers current employment by 26.4% (95.13%-68.73%), whereas $t-2$ unemployment reduces it by 17.2% (78.25%-95.44%). Duration dependence (stigma) is apparent as being continuously unemployed in the last two periods reduces current employment by 38.7% (96.31%-57.58%).

Second, panels b and c show the mitigating effects of human capital on unemployment level and persistence. Agents with less than high school had 14.3% (97.70%-83.39%) lower employment rates in general compared with those having college degree. They also faced a 30.9% (89.36%-58.43%) lower employment if previously unemployed, compared with only a 8.9% (97.96%-89.07%) gap for those with college degrees. Vocational and professional training also provides some attenuating effects, although of lower magnitude compared to education. Trained agents had higher employment by 2.7% (95.15%-92.46%), and faced a past unemployment gap of 24.0% (96.75%-72.77%), compared with 28.1% (94.82%-66.72%) for untrained respondents.

Table 3 reveals similar scarring and stigma when measured in terms of income. Declining persistence is apparent with $t-1$ unemployment resulting in 64.7% lower income, while $t-2$ spells lower income by 52.2%. Income stigma is also apparent with continuous unemployment in the last two periods leading to a 74.2% drop in current revenues. The mitigating human capital effect on income is less striking compared to that on employment. Although college graduates earn 61.8% more than those without high school, the effects of education on the income gap associated with $t-1$ unemployment is relatively constant, ranging between 57.3% and 61.6%. Again, the effect of training appears more limited.

These statistical findings are confirmed by longitudinal regression analysis. Table 4 shows the marginal effects from panel Probit regressions of current employment statuses, controlling for socio-demographic characteristics and year fixed effects. The dependent variables are (1) the unconditional, i.e. $\Pr(i_t = e)$, (2) re-employment, i.e. $\Pr(i_t = e \mid i_{t-1} = u)$ and (3) continuing employment, i.e. non-displacement $\Pr(i_t = e \mid i_{t-1} = e)$. First, in panel a, past employment statuses improve current employment, re-employment and continuing employment. The temporary nature of scarring is apparent with weaker effects associated with time $t-2$ statuses, compared to $t-1$. Second, in panel b, human capital measured either through lagged work experience (i.e. cumulated past statuses up to $t-1$), education or training significantly augment current employment, re-employment, and continuing employment probabilities. Table 5 makes similar findings for current income via panel GLS regressions. Positive gradients are also found for being employed in the last two periods, with stronger effects for more recent statuses. Again, human capital proxied by work experience, education, or training improve current, re-employment, and continuing employment incomes.

Overall, we conclude that the employment and income scarring costs associated with previous unemployment are significant, more important for recent events and that duration of spells compounds these costs (stigma). Human capital augments both employment and income and is a significant hedge against these scarring and stigma costs. The next section describes a theoretical model incorporating these elements. Consistent with Tables 4, and 5, we assume that labor demand values human capital with higher re-employment, lower displacement probabilities, as well as higher wages. Taking these labor market characteristics as given, we let agents select their investment in human capital and verify whether the resulting dynamics are consistent with scarring and stigma costs identified with NLSY79 data.

3 Model

Overview Consider an economy where agents are characterized by two sources of heterogeneity: Human capital $H_t \in \mathbb{R}_+$ and labor market status $i_t \in \{e, u\}$ (i.e. employed, unemployed). The former is defined as the publicly measurable set of skills accumulated by workers over their lifetime. We assume that investment in human capital is decided by agents and takes place both within (e.g. through experience or voluntary training) and outside (e.g. through formal and informal education) employment. The pecuniary (e.g. tuition fees, books, software, ...) and indirect (e.g. opportunity cost of time and effort spent acquiring skills) investment costs are borne by individuals. Human capital provides no direct utility flows to the agent, but is valued by employers, as reflected in more favorable conditions with respect to wages, firing and hiring for highly-skilled agents. Although our perspective is on general human capital, we allow for part of that capital to be immediately depreciated upon a displacement event in order to reflect firm- or industry-specific components that have limited value to outside employers.

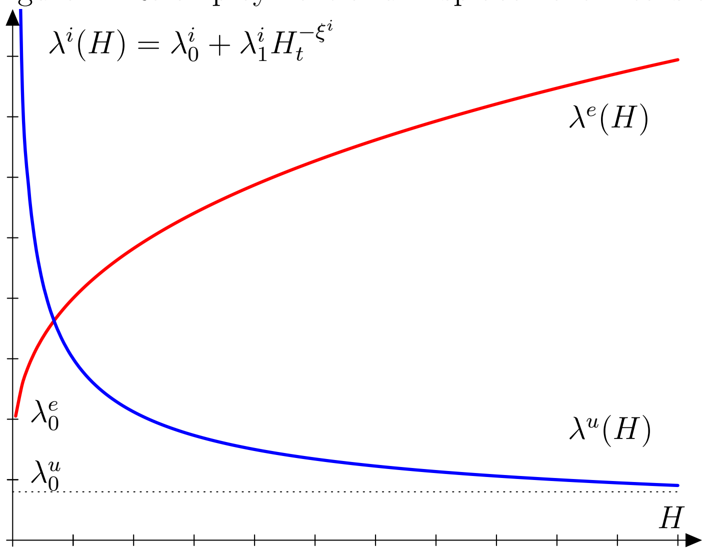
Labor market statuses are stochastic and the transition matrix between employment and unemployment spells is agent-specific, in that it depends on the accumulated level of human capital. Employed agents receive an income that is continuously adjusted to reflect changes in human capital. Conversely, unemployed agents receive unemployment benefits that are set at a fraction of the last employment revenue; the benefits are constant for the duration of the unemployment spell. Risk-neutral agents thus select optimal investment paths taking into account its joint benefits in terms of income premia and employment risk adjustments.

Employment statuses A person's time-t labor market status i_t follows a Poisson stochastic process. Importantly, the arrival intensity is assumed to be dependent of the observable human capital level H_t. More specifically, let T^i, be the random time of job displacement ($i_t = u$) from current employment, or re-employment ($i_t = e$) from current unemployment, with Poisson arrival intensities $\lambda^i : \mathbb{R}_+ \to \mathbb{R}_{++}$ defined as:

$$\begin{aligned}\lambda^i(H_t) &= \lim_{\tau\to 0}\frac{1}{\tau}\Pr\left[t < T^i < t+\tau \mid H_t\right], \quad i \in \{e,u\} \\ &= \lambda_0^i + \lambda_1^i H_t^{-\xi^i}, \quad \lambda_0^i, \lambda_1^i \geq 0; \quad \xi^i > -1. \end{aligned} \tag{1}$$

Hence, imposing $\xi^u > 0$ in (1) entails decreasing and convex work displacement intensities, whereas $\xi^e \in (-1, 0)$ yields increasing concave re-employment intensities.

Figure 1: Re-employment and Displacement Intensities



Notes: $\lambda^e(H)$: re-employment intensity. $\lambda^u(H)$: displacement intensity.

As shown in Figure 1, an agent can thus reduce his exposure to conditional employment risks by investing in his human capital which decreases his displacement intensity $\lambda^u(H)$, as well as increases his re-employment intensity $\lambda^e(H)$. On the one hand, the parameters λ_0^i represent unadjustable exposure to displacement and re-employment hazard. On the other hand, the parameters λ_1^i capture the endogeneity of the employment risks exposure and play a key role in the solution method discussed below, with ξ^i governing the extent of diminishing returns to investment against employment shocks.

Income process The income process $Y_t = Y(H_t, \overline{H}, i_t) \in \mathbb{R}^+$ is status- and human-capital-dependent:

$$Y(H_t, \overline{H}, e) = Y^e(H_t) = y_0 + y_1 H_t, \tag{2a}$$

$$Y(H_t, \overline{H}, u) = Y^u(\overline{H}) = \eta Y^e(\overline{H}), \tag{2b}$$

where $\eta \in (0,1)$ is the UI replacement rate and where $\overline{H}$ is the last measurable human capital level at the beginning of the unemployment spell (i.e. *lock-in* capital).

Figure 2: Employed and Unemployed Income

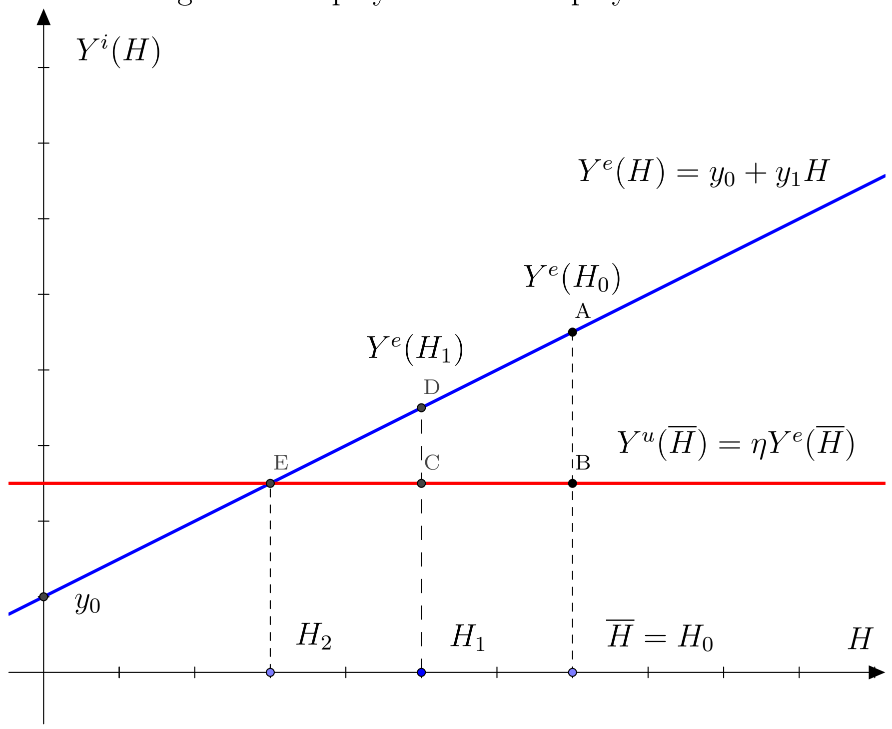


Figure 2 shows that employment income $Y^e(H)$ increases in human capital which can be continuously altered through the agent's investment decisions. Upon job loss at human capital level H_0, unemployment income at point B is a fraction η of the last employment income $Y^u(\overline{H}) = \eta Y^e(H_0)$ and remains fixed throughout the duration of the unemployment spell. For example, if human capital declines to H_1 during unemployment, UI income remains constant, whereas the income upon re-employment income at point D is lower than previously, $Y^e(H_1) < Y^e(H_0)$. Consistent with passive UI policies, investment decisions during the unemployment spell thus affect the displacement and re-employment probabilities, as well as the re-employment wage, but not the UI benefits.[9]

Note further that the income loss (resp. gain) associated with displacement (resp. re-employment):

$$\begin{aligned}\Delta Y(H,\overline{H}) &= Y^e(H) - Y^u(\overline{H}) \\ &= (1-\eta)y_0 + y_1(H - \eta\overline{H}) \end{aligned} \tag{3}$$

is an increasing function of H and can become negative if human capital depreciates sufficiently during the unemployment spell, i.e. for $H < H_2$ in Figure 2. Indeed, beyond point E, UIB benefits are more generous than what would be earned upon re-employment, thereby lowering incentives to invest in order to augment re-employment probability.

Human capital dynamics The law of motion for the agent's human capitals, $\mathrm{d}H_t = \mathrm{d}H_t(I_t, H_t, i_t)$, is status-dependent and is given by:

$$\mathrm{d}H_t = -\delta^i H_t \mathrm{d}t + P^i I_t^{\alpha} H_t^{1-\alpha}\mathrm{d}t, \quad \alpha, \delta^i \in (0,1) \tag{4}$$

The accumulation process (4) is in the spirit of the HK literature, (e.g. Ben-Porath, 1967; Heckman, 1976; Huggett et al., 2006; Kredler, 2014) and captures continuous, as opposed to period-specific (e.g. pre-employment education) investment I_t decided by the agent. The Cobb-Douglas gross investment function $P^i I_t^{\alpha} H_t^{1-\alpha}\mathrm{d}t$ is monotone increasing and concave in its arguments. The productivity term P^i can be equivalently interpreted as an ability or as the inverse of an investment price, whereas depreciation δ^i can be interpreted as technological obsolescence of acquired skills.

Unlike most models who assume on-the-job training only (i.e. $I_t(i_t = u) \equiv 0$), or active unemployment training decided by UI planners (e.g. Spinnewijn, 2013), the agent's

[9] See St-Amour (2015) for an alternative UIB policy with continuous adjustments in $Y^u(H) = \eta Y^e(H)$, instead of lock-in capital. The results for this specification are qualitatively similar.

investment decisions extend across employment statuses. Differences in productivity and depreciation capture status-dependent returns to investment. For example, faster depreciation, and/or lower productivity when unemployed[10] can be attained by imposing $\delta^u > \delta^e$ and $P^u < P^e$. We remain agnostic by not imposing such restrictions and instead solving the model for any δ^i, P^i combinations.

The literature also puts forward distinctions between general and firm- or industry-specific human capital, where the latter has a lower outside value (Hamermesh, 1987; Becker, 1993; Neal, 1995; Ljungqvist and Sargent, 1998; Wasmer, 2006; Decreuse and Granier, 2013). We can incorporate this feature by defining a transferability share $\phi \in (0, 1]$ representing the general capita. In the spirit of Ljungqvist and Sargent (1998), a newly displaced agent's capital is thus only valued $\phi H_t < H_t$ to prospective employers for income and reemployment intensity purposes. This non-stochastic jump can capture firm- or industry-specific capital that is foregone when employment is terminated. Alternatively, the loss $(1 - \phi)H_t$ can also be interpreted as discrimination or branding against unemployed workers whereby the actual capital is under-estimated by prospective employers following an unemployment spell. Both the effects on displacement/re-employment and on firm-specific capital loss are fully internalized in the agent's investment decisions, as will be seen below.

Preferences All agents are infinitely-lived and select dynamic investment in human capital I_t to maximize the expected discounted (at rate ρ)[11] value of net income flow, taking as given the dynamics for human capital, the distributional assumptions and income function. More specifically, the value function can be written as:

$$V(H_0, \overline{H}, i_0) = \sup_I \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[Y(H_t, \overline{H}, i_t) - I_t \right] \mathrm{d}t \geq 0, \tag{5}$$

[10]See Pissarides (1992); Acemoglu (1995); Ljungqvist and Sargent (1998); Pavoni and Violante (2007); Pavoni (2009); Spinnewijn (2013) for discussions of unemployment disadvantages in capital accumulation.

[11]Restricting to finite lives is easily adaptable by assuming Poisson death intensity λ^m and augmenting discounting at rate $\rho + \lambda^m$ over an infinite horizon.

subject to the intensities (1), the income rate (2) and the human capital law of motion (4).

We remain in the HK tradition in assuming risk-neutral preferences in (5), with two important implications. First, observe that negative net income $Y_t - I_t < 0$ always remains feasible and can be achieved by implicit borrowing (at rate $r = \rho$), as long as the expected net present value $V(H_0, \overline{H}, i_0)$ remains non-negative.[12] Second, risk neutrality implies that any incremental demand for human capital (above that related to higher income) induced by endogenous displacement and re-employment risks cannot strictly be justified by self-insurance motives. Rather, this demand stems from a duration service procured by additional human capital which augments the expected time spent in the employed state (with associated high income $Y^e(H)$), and reduces that spent in unemployment (with associated low income $Y^u(\overline{H})$). Observe that this duration service comes at no extra cost (aside from the increase in marginal price due to convex adjustment costs) and can thus be interpreted as positive side benefit of investment over and above income considerations.

Letting $V^e(H), V^u(H, \overline{H})$ denote the pair of value functions and invoking the Law of Iterated Expectations with Poisson distributions allows the agent's problem (5) to be written as a joint optimization system:

$$V^e(H_0) = \sup_I \int_0^\infty e^{-\int_0^t (\rho + \lambda^u(H_s)) \mathrm{d}s} \left[Y^e(H_t) - I_t + \lambda^u(H_t) V^u(\phi H_t, H_t) \right] \mathrm{d}t, \quad \text{(6a)}$$

$$V^u(H_0, \overline{H}) = \sup_I \int_0^\infty e^{-\int_0^t (\rho + \lambda^e(H_s)) \mathrm{d}s} \left[Y^u(\overline{H}) - I_t + \lambda^e(H_t) V^e(H_t) \right] \mathrm{d}t. \quad \text{(6b)}$$

The presence of $V^u(\phi H, H)$ in the employed agent's problem (6a) highlights the additional depreciation that is associated with employment-specific capital $(1 - \phi)H$ that is foregone upon the displacement event occurring with intensity $\lambda^u(H_t)$. The UI income in (6b) is calculated at locked-in capital $\overline{H}$ until re-employment occurs with intensity $\lambda^e(H_t)$, after

[12] As will be seen shortly, the optimal strategy never involves borrowing at the parameter set used below, such that non-negative value function is never binding. St-Amour (2015) considers the case where risk-averse agents have no access to borrowing for human capital investment. The main findings obtained through numerical solutions remain qualitatively similar to the ones of this paper.

which the agent returns to $V^e(H)$. The program (6) features endogenous discounting at augmented rates $\rho + \lambda^i(H)$ induced by the Poisson distributional assumption.

The corresponding Hamilton-Jacobi-Bellman (HJB) representation of (6) is:

$$\begin{aligned}
0 = \sup_I & - \rho V^e(H) - \lambda^u(H)\left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) - I \\
& + V^e_H(H)\left[-\delta^e H + P^e I^\alpha H^{1-\alpha}\right], \\
0 = \sup_I & - \rho V^u(H,\overline{H}) - \lambda^e(H)\left[V^u(H,\overline{H}) - V^e(H)\right] + Y^u(\overline{H}) - I \\
& + V^u_H(H,\overline{H})\left[-\delta^u H + P^u I^\alpha H^{1-\alpha}\right].
\end{aligned}$$

Calculating the first-order conditions and substituting back into the objective function reveals that the joint HJB system simplifies to:

$$\begin{aligned}
0 = & - \rho V^e(H) - \lambda^u(H)\left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) \\
& - \delta^e H V^e_H(H) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^e V^e_H(H)\right]^{\frac{1}{1-\alpha}}, \qquad (7a)\\
0 = & - \rho V^u(H,\overline{H}) - \lambda^e(H)\left[V^u(H,\overline{H}) - V^e(H)\right] + Y^u(\overline{H}) \\
& - \delta^u H V^u_H(H,\overline{H}) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^u V^u_H(H,\overline{H})\right]^{\frac{1}{1-\alpha}}. \qquad (7b)
\end{aligned}$$

The bi-variate system of first-order differential equations (7) has no analytical solution due to the endogeneity and nonlinear functional forms used for the intensity functions (1). St-Amour (2015) relies on Chebyshev polynomials to calculate numerical solutions to a similar program. We resort instead to a two-step approximate closed-form solution method developed in Hugonnier, Pelgrin and St-Amour (2013). First we remove the endogeneity in the employment intensities by imposing $\lambda^i_1 = 0$ in (1). This exogenous employment risks case yields a closed-form solution (referred to as order-0 solution) for $V^i_0(H,\overline{H}), I^i_0(H,\overline{H})$. Second, we rewrite the endogenous intensity component as $\lambda^i_1 = \epsilon\overline{\lambda}^i_1, i = e, u$ for some constants $\overline{\lambda}^i_1$ and perturbation ϵ and perform a first-order expansion of the value functions around the $\epsilon = 0$ solution:

$$V^e(H,\epsilon) \approx V^e(H,0) + \epsilon V^e_\epsilon(H,0),$$
$$V^u(H,\overline{H},\epsilon) \approx V^u(H,\overline{H},0) + \epsilon V^u_\epsilon(H,\overline{H},0).$$

Once the approximate solution (referred to as order-1) for the value functions is obtained, any relevant associated variable such as investment and human capital growth is thus recovered through a similar expansion. In particular, any function F involving the value functions can be approximated as:

$$F^e(H,\epsilon) \approx F^e(H,0) + \epsilon F^e_\epsilon(H,0),$$
$$F^u(H,\overline{H},\epsilon) \approx F^u(H,\overline{H},0) + \epsilon F^u_\epsilon(H,\overline{H},0).$$

4 Optimal human capital investment and growth

We now calculate the optimal investment, starting first with the exogenous displacement and re-employment (order-0), followed by the more general case where both are endogenous (order-1).

4.1 Exogenous displacement and re-employment (order-0)

Theorem 1 (exogenous employment risks) *Let $\lambda^e_1 = \lambda^u_1 = 0$ and assume that the order-0 transversality and regularity conditions conditions* (15) *in Appendix C hold. Then:*

1. *The indirect utility functions of employed and unemployed agents are given as:*

$$V^e_0(H) = A^e_0 + A^e_h H \tag{8a}$$
$$V^u_0(H,\overline{H}) = A^u_0 + A^u_h H + A^u_b \overline{H} \tag{8b}$$

2. The optimal investment functions are given as:

$$I_0^e(H) = H\left(P^e \alpha A_h^e\right)^{\frac{1}{1-\alpha}} \tag{9a}$$

$$I_0^u(H) = H\left(P^u \alpha A_h^u\right)^{\frac{1}{1-\alpha}} \tag{9b}$$

3. The optimal human capital growth functions are given as:

$$g_0^e = -\delta^e + P^{e\frac{1}{1-\alpha}}\left(\alpha A_h^e\right)^{\frac{\alpha}{1-\alpha}} \tag{10a}$$

$$g_0^u = -\delta^u + P^{u\frac{1}{1-\alpha}}\left(\alpha A_h^u\right)^{\frac{\alpha}{1-\alpha}} \tag{10b}$$

where the parameters (A^e, A^u) *are given in closed form in Appendix D.*

The expression A_h^i in the indirect utility functions (8) capture the marginal value (i.e. shadow price), corresponding to the status-dependent Tobin's-q of human capital that jointly solve (19). The last measurable human capital level before the unemployment spell begins $\overline{H}$ is valued under unemployment, but not for employed agents. Indeed, the UIB program sets $\overline{H} = H$ when unemployment begins, such that the value function simplifies to a function of H only from the employed agent's perspective. The optimal investment in (9) shows that the investment-to-capital ratio is constant and increasing in the shadow price. Consequently, the growth rates (10) are constant, so that no steady-state exists at the order zero.

The restricted case with exogenous exposure to employment risks solved in Theorem 1 captures only a subset of the unemployment scarring and stigma stylized facts. To see why, consider the baseline scenario (14) of status-independent human capital technology and purely general capital. The optimal dynamics in (9), (10) then show that, conditional on H, human capital investment and growth are both higher when employed than unemployed if the Tobin's-q satisfy $A_h^e > A_h^u$, a condition that is verified when the human capital gradient of income y_1 is sufficiently large.[13] Since income (2) is increasing in H, the slower growth when unemployed is penalized by lower wages upon re-employment, and because growth is constant under both statuses, the magnitude of the income wedge is increasing in the duration of the unemployment spell.

However, the exogenous displacement and re-employment case has two important shortcomings with respect to observed patterns. First that constant growth rates differentials imply permanent scars as the unemployed expect no catching-up of their wages following re-employment, contrary to observed patterns of persistent, but non-permanent income wedges (e.g. Jacobson et al., 2005; Davis and von Wachter, 2011; Carrington and Fallick, 2014). Second, because the order-zero case has exogenous exposure to employment risks (i.e. $\lambda^i(H) = \lambda^i_0$), the slower growth is inconsequential for post-unemployment displacement and re-employment risks exposure. Equivalently, the exogenous employment risks case replicates *income* scarring and duration dependence when the shadow price is higher for the employed and does so without requiring ad-hoc assumptions such as lower productivity or higher depreciation when unemployed. However, the restricted case fails to capture the *employment* scars and stigma associated with unemployment, as well as the non-permanent nature of both income and employment scars.

4.2 Endogenous displacement and re-employment (order-1)

We now consider the more general case of endogenous exposure to gauge whether the shortcomings of the exogenous employment risks exposure model can be addressed.

Theorem 2 (endogenous employment risks) *Assume that the order-0 transversality and regularity conditions conditions* (15) *in Appendix C hold. Then, up to a first-order approximation,*

[13] See Corollary 1 for discussion. This condition is verified in our calibration discussed below with $A^e_h = 9.0262$ and $A^u_h = 2.5789$.

1. The indirect utility functions of employed and unemployed agents are given as:

$$\begin{aligned} V^e(H) =& V_0^e(H) + B_u^e\lambda_1^u H^{-\xi^u} + B_{1u}^e\lambda_1^u H^{1-\xi^u} + B_e^e\lambda_1^e H^{-\xi^e} \\ &+ B_{1e}^e\lambda_1^e H^{1-\xi^e}, \end{aligned} \tag{11a}$$

$$\begin{aligned} V^u(H,\overline{H}) =& V_0^u(H,\overline{H}) + B_u^u\lambda_1^u H^{-\xi^u} + B_{1u}^u\lambda_1^u H^{1-\xi^u} + B_e^u\lambda_1^e H^{-\xi^e} \\ &+ B_{1e}^u\lambda_1^e H^{1-\xi^e} + B_b^u\overline{H}\lambda_1^e H^{-\xi^e}, \end{aligned} \tag{11b}$$

2. The optimal investment functions are given as:

$$\begin{aligned} I^e(H) =& I_0^e(H) + C_u^e\lambda_1^u H^{-\xi^u} + C_{1u}^e\lambda_1^u H^{1-\xi^u} + C_e^e\lambda_1^e H^{-\xi^e} \\ &+ C_{1e}^e\lambda_1^e H^{1-\xi^e}, \end{aligned} \tag{12a}$$

$$\begin{aligned} I^u(H,\overline{H}) =& I_0^u(H) + C_u^u\lambda_1^u H^{-\xi^u} + C_{1u}^u\lambda_1^u H^{1-\xi^u} + C_e^u\lambda_1^e H^{-\xi^e} \\ &+ C_{1e}^u\lambda_1^e H^{1-\xi^e} + C_b^u\overline{H}\lambda_1^e H^{-\xi^e}. \end{aligned} \tag{12b}$$

3. The optimal human capital growth functions are given as:

$$\begin{aligned} g^e(H) =& g_0^e + D_u^e\lambda_1^u H^{-1-\xi^u} + D_{1u}^e\lambda_1^u H^{-\xi^u} + D_e^e\lambda_1^e H^{-1-\xi^e} \\ &+ D_{1e}^e\lambda_1^e H^{-\xi^e}, \end{aligned} \tag{13a}$$

$$\begin{aligned} g^u(H,\overline{H}) =& g_0^u + D_u^u\lambda_1^u H^{-1-\xi^u} + D_{1u}^u\lambda_1^u H^{-\xi^u} + D_e^u\lambda_1^e H^{-1-\xi^e} \\ &+ D_{1e}^u\lambda_1^e H^{-\xi^e} + D_b^u\overline{H}\lambda_1^e H^{-1-\xi^e}. \end{aligned} \tag{13b}$$

where the order-0 values $V_0^e(H), V_0^u(H,\overline{H}), I_0^e(H), I_0^u(H,\overline{H})$ and $g_0^e(H), g_0^u(H,\overline{H})$ are given in Theorem 1 and where the parameters (B^e, B^u), (C^e, C^u) and (D^e, D^u) are given in closed form in Appendix E.

When contrasted with Theorem 1, the order-1 results of Theorem 2 show that the investment shares of human capital $I^i(H,\overline{H})/H$ are no longer constant. It follows that neither are the optimal growth functions $g^i(H,\overline{H})$, such that steady state values $H^i_{SS}(\overline{H})$

may exist, contrary to the exogenous employment risks case. Moreover, a role for the lock-in capital $\overline{H}$ is reinstated for optimal investment and growth for the unemployed; employed investment and growth remain unaffected for reasons that have been discussed before. Importantly, generalizing $\lambda_1^i \neq 0$ permits feedback effects of changes in H for employment risks exposure. In addition to income wedges identified for the order-0 case, any gaps in the optimal dynamics $g^e(H) - g^u(H, \overline{H})$ will be penalized in both displacement and re-employment intensities, thereby reinstating potential employment scarring and stigma.

5 Simulated human capital dynamics

We rely on the order-1 optimal rules in Theorem 2 to simulate the model and identify the dynamics of employment statuses and income induced by those of the human capital. In order to emphasize capital dynamics resulting from optimal investment, rather than from technological differences, it will be useful to define a baseline scenario of status-independent technologies:

$$\delta^i = \delta, \ P^i = P, \quad i \in \{e, u\} \tag{14a}$$

and of no capital specificity:

$$\phi = 1. \tag{14b}$$

We first simulate the model for the baseline scenario (14). We will reinstate both status-dependent technology and firm-specific capital loss in the comparative statics exercise in Section 7.3.

Our simulation follows the Monte Carlo procedure outlined in Appendix F. In short, conditional on status i_t, current H_t and locked-in capital $\overline{H}_t$, the optimal investment $I^i(H_t, \overline{H}_t)$ from Theorem 2 are selected, and the Poisson intensities $\lambda^i(H_t)$ are set.

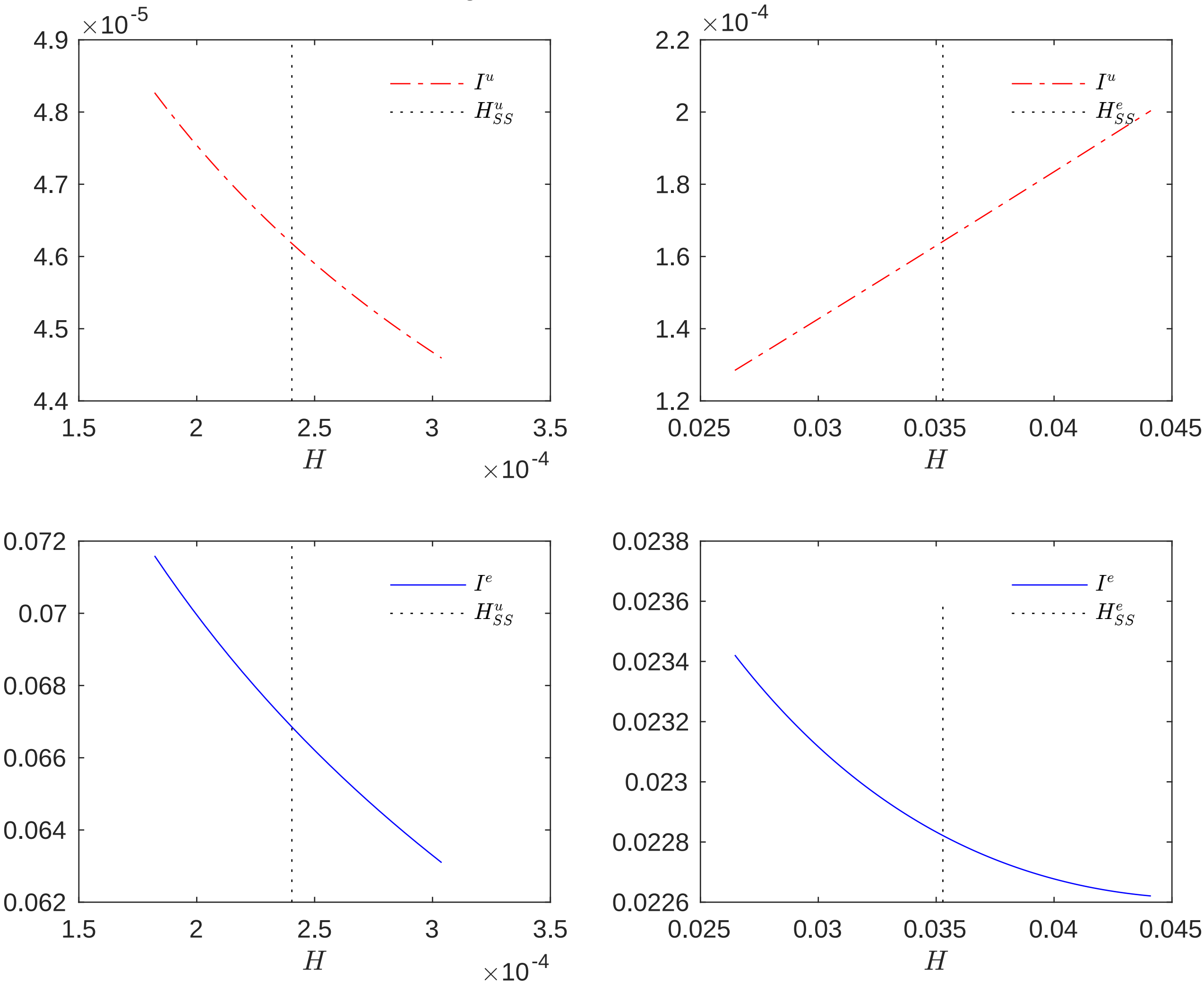
The agent's status and capital are then updated to i_{t+1}, H_{t+1}, with special provision – when applicable – for a share $(1-\phi)$ of employment-specific capital being lost upon new displacement events. The procedure is iterated upon for $T = 200$ periods over $n = 10'000$ individuals. The calibration is selected so as to match the theoretical moments calculated from the simulated histories of employment statuses $i_j = \{i_{j,t}\}_{t=1}^{T}$ to their observed counterparts. The moments to be matched in Table 7 are (a) the conditional employment probabilities by (i_{t-1}, i_{t-2}) statuses; (b) the continuation employment and unemployment probabilities from $t-\tau$ to t; (c) the income by current status. Finally, the calibration is undertaken subject to the three order-0 transversality and regularity conditions conditions (15) in Appendix C. The calibrated parameters for the displacement and re-employment intensity functions $\lambda^i(H_t)$ in (1), the income functions $Y^i(H_t)$ in (2), as well as the human capital production function (4) match the moments reasonably well in Table 7. Moreover, the law of motion (4) calibration is in the habitual range of estimates for Ben-Porath (1967) technology.[14]

Optimal investment Figure 3 plots the optimal investment in human capital for employed (blue, solid line) and unemployed (red, dashed-dotted line) agents, in function of H and for mid-level $\overline{H}$ lock-in capital level, in the vicinity of the employed and unemployed steady-states H^i_{SS} (discussed below).

First, we find that investment for unemployed agents is lower for all H and $\overline{H}$ than for employed workers. Second, investment is falling in human capital for the employed, but is U-shaped for the unemployed due to conflicting income and employment risks effects. Indeed, on the one hand, increasing H reduces the likelihood of displacement, while increasing the re-employment probability, thereby reducing the incentives for investment. Diminishing returns in adjusting the arrival intensities $\lambda^i(H)$ entail that the marginal effect on employment risk is stronger at low H. One the other hand, an increase in H raises the employed agent's revenues $Y^e(H)$ – and thus available resources for investing

[14] Estimates for α vary between 0.35 and 0.80, whereas δ estimates range between 0.027 and 0.07 (see the references cited in Polachek et al., 2015, p. 1425).

Figure 3: Investment



Notes: Optimal investment for the unemployed ($I^u(H, \overline{H})$ in red, dashed-dotted) and for the employed ($I^e(H)$ in blue, solid line) in the vicinity of the unemployed (H^u_{SS}) and employed (H^e_{SS}) steady-states.

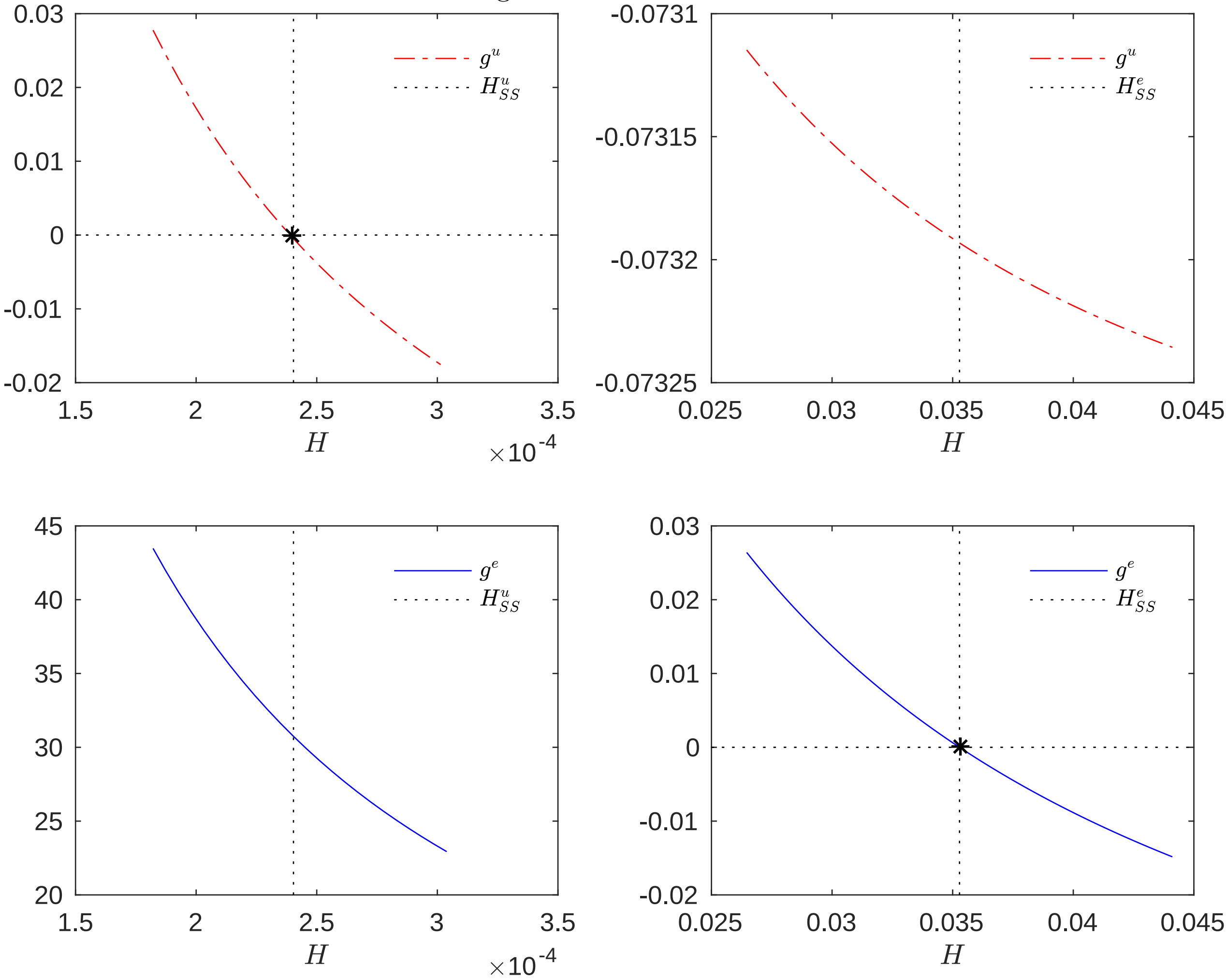
for the employed – without affecting UI income fixed at lock-in level $Y^u(\overline{H})$. Moreover, equation (3) shows that it also raises the income wedge $\Delta Y(H, \overline{H})$, i.e. the value at risk in case of unemployment, and potential income gain if re-employed. The income level and gain both concur to increase investment. Our calibration reveals that the employment risk effect dominates the income effect for the employed, as well as for the unemployed with low human capital. At high H however, diminishing returns entail that the income effect is stronger for the unemployed and investment increases in human capital.

Third, our calibration entails that $C_b^u, D_b^u < 0$, indicating that the investment and growth are both lower for unemployed agents with high lock-in capital, although the net effect is weak due to two opposing forces. On the one hand, a high lock-in capital raises UI revenues available for investing. On the other hand, the discussion of (3) revealed that the attractiveness of investing in order to raise the likelihood of re-employment is reduced due to more generous UIB income for high $\overline{H}$. Our results indicate that the two effects more or less offset one another.

Optimal growth Figure 4 shows the optimal human capital dynamics for employed (blue, solid line) and unemployed (red, dashed-dotted line) agents, where the latter are evaluated at mid-level lock-in capital levels. These results show that two distinct steady-state levels exist, are unique given status and $\overline{H}$ and are dynamically stable. In particular, the higher levels of investment for the employed workers translate into higher steady-states $H_{SS}^e = 0.0353 > H_{SS}^u(\overline{H}) = 2.4\text{e-}4$. Importantly, dynamic stability implies that a displaced worker will optimally choose a depletion of his human capital until either a new lower steady state H_{SS}^u obtains, or he is re-employed, after which human capital will grow again up to H_{SS}^e.

Simulated trajectories These dynamics are illustrated in Figure 5 which plots a sample of the simulated optimal trajectories for human capital $\{H_{j,t}\}$. Consistent with Figure 4.c and d, dynamic paths converge rapidly towards the dynamically stable steady-state level associated with employment $H_{SS}^e = 0.0353$ (dotted line). Each dip in $H_{j,t}$ is caused by a job displacement; once re-employed, the paths converge again towards H_{SS}^e. A prolonged unemployment spell is associated with a constant fall in capital towards the unemployment steady state $H_{SS}^u = 2.4\text{e-}04$. Since the predicted unemployment probability $\Pr(u) = 6.84\%$ is low, most of the dynamic paths hover around the employed steady-state value H_{SS}^e.

Figure 4: Growth

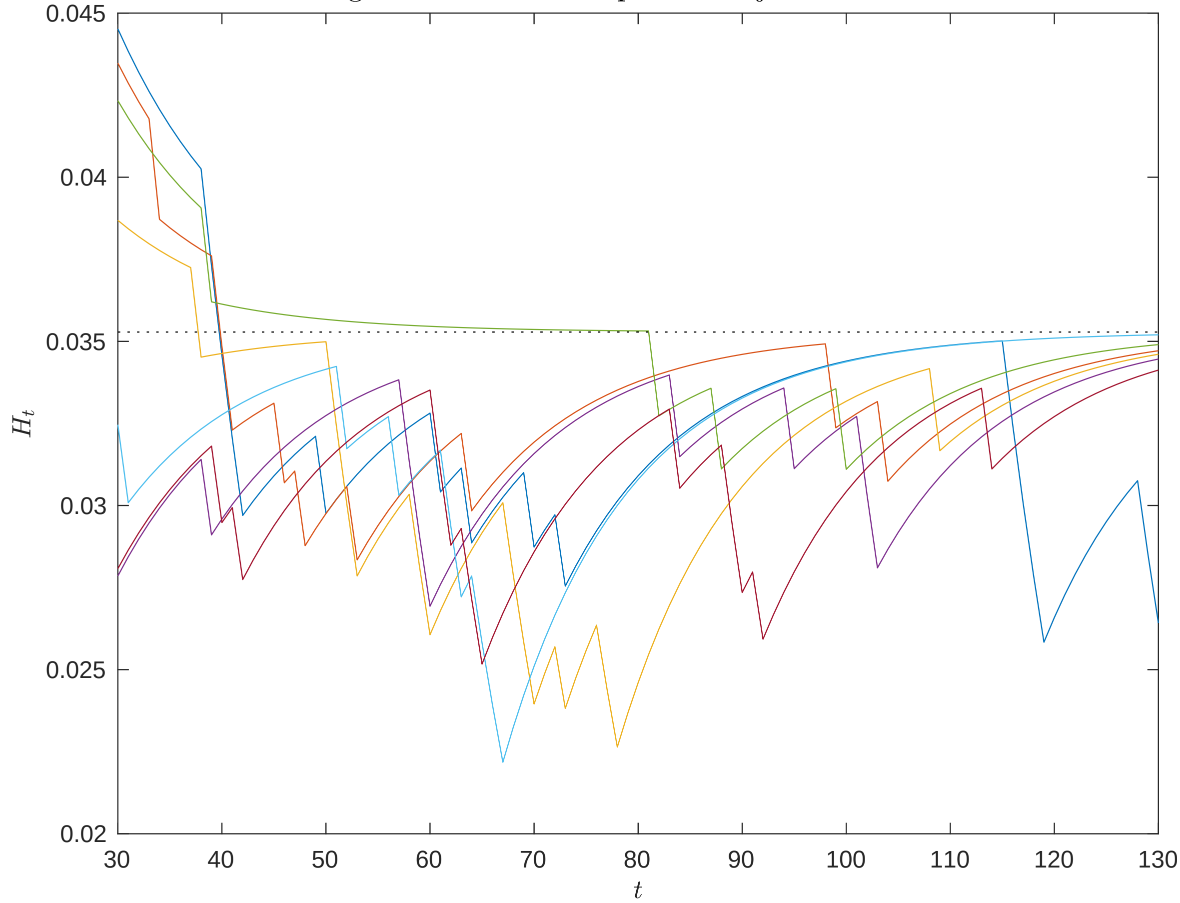


Notes: Optimal growth rates for the unemployed ($g^u(H, \overline{H})$ in red, dashed-dotted) and for the employed ($g^e(H)$ in blue, solid line) in the vicinity of the unemployed (H^u_{SS}) and employed (H^e_{SS}) steady-states.

6 Self-inflicted unemployment scars and stigma

Figure 6 plots the optimal phase diagrams of human capital stemming from Figure 4. These paths are consistent with circular dynamics. First, a long-tenured worker with steady-state capital H^e_{SS} and who is displaced moves from a to b on the optimal human capital growth path. From the previous analysis, human capital then optimally depletes for the entire duration of the unemployment spell and moves towards the new lower steady state in c. Once attained, the capital remains at steady-state H^u_{SS} for the duration of the

Figure 5: Simulated optimal trajectories

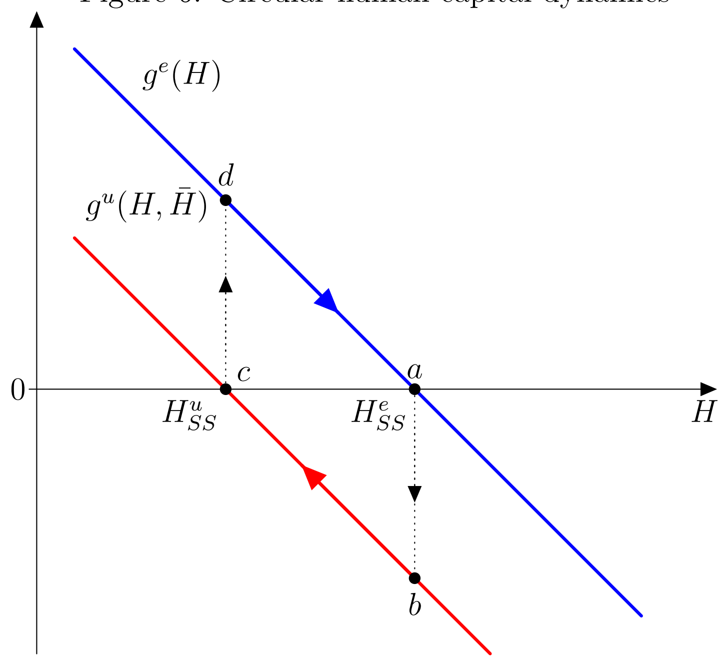


Notes: Sample of simulated $\{H_{jt}\}$ optimal paths for Monte-Carlo procedure in Appendix F.

unemployment event. Upon re-employment, the agent's capital moves to point d after which capital increases again back to the former steady-state H^e_{SS}.

Figure 7 next shows how these human capital dynamics translate into employment scarring and stigma. The long-tenured displaced worker moves from a to b on the re-employment intensity function. As human capital optimally falls, so does the recall probability with intensity moving towards c. Duration dependence endogenously obtains as the longer the duration spell, the more important is the associated unemployment stigma, i.e. the fall in $\lambda^e(H)$. Upon re-employment, the agent moves to point d on the $\lambda^u(H)$ intensity and is subject to a higher displacement probability due to the optimal

Figure 6: Circular human capital dynamics



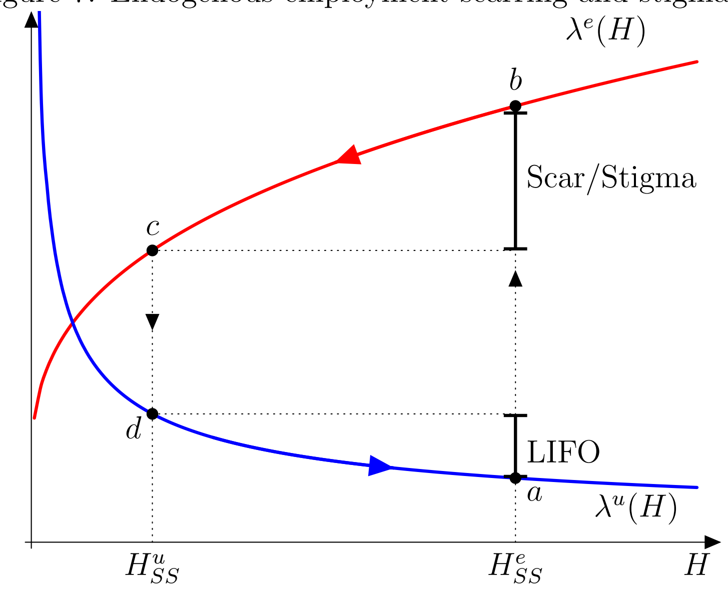
Notes: $g^e(H)$: Optimal human capital growth conditional on employment in (13a). $g^u(H, \overline{H})$: Optimal human capital growth conditional on unemployment in (13b), for capital H and UIB lock-in capital $\overline{H}$.

fall in human capital. This last-in-first-out (LIFO) effect persists up to the period where the former steady state H^e_{SS} is attained in point a.

The model also generates endogenous income scarring and stigma effects of unemployment, as evidenced in Figure 8. A displaced long-tenured worker suffers a drop in income from a to b. As human capital is optimally depleted towards c, the UIB revenues remain unaffected due to the lock-in feature. However, upon re-employment, the agent's labor income is now lower at d, with the longer the unemployment spell, the more important the drop in wages upon re-employment. The model thus endogenously generates wage dynamics that are consistent with income scarring and stigma effects of unemployment.

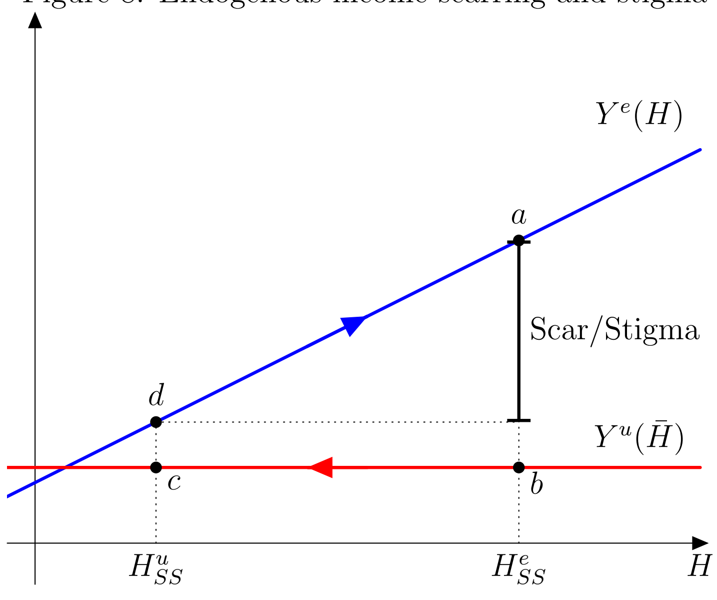
The predicted unemployment scars and stigma can thus be characterized as *self-inflicted*, to the extent that they stem from optimal human capital dynamics decided by agents exclusively. Indeed, we have relied on simple and empirically motivated characterization of labor demand whereby observable human capital is valued by employers,

Figure 7: Endogenous employment scarring and stigma



Notes: $\lambda^e(H)$: re-employment intensity; $\lambda^u(H)$: displacement intensity, under dynamics described in Figure 6.

Figure 8: Endogenous income scarring and stigma



Notes: $Y^e(H)$: employment income. $Y^u(\bar{H})$: unemployment income, under dynamics described in Figure 6.

resulting in higher wages, lower displacement and higher re-employment probabilities. Traditional explanations of scarring and stigma based on screening practices by employers are therefore not required to explain this phenomenon. Importantly, neither are ad-hoc parametric hypotheses, such as (i) more important depreciation rates, (ii) capital specificity, (iii) less efficient production technology of human capital, or (iv) learning-by-doing. Indeed, our baseline calibration assumes identical laws of motion for human capital under employed and unemployed statuses and depletion or growth is decided optimally by employed and unemployed workers. Finally, unlike the exogenous risk model in Theorem 1, the predicted unemployment scarring and stigma are persistent, but not permanent. Indeed, a sufficiently long employment history pushes human capital up to its former steady-state level H^e_{SS}, thereby reinstating former wages and exposure to displacement and re-employment risks.

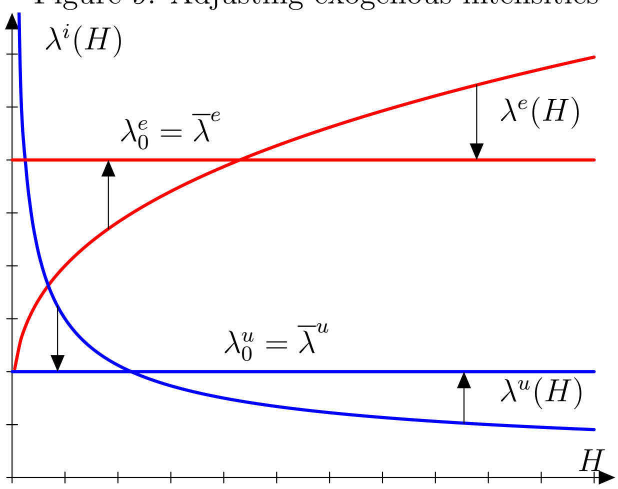
7 Counter-factual analysis

We now conduct a counter-factual analysis to gauge the effects of parametric changes on our results. In particular, starting with the optimal allocation $I = I(H, \overline{H}; \theta)$, we modify the deep parameters θ to $\tilde{\theta}$ and recompute the optimal rules $\tilde{I} = I(H, \overline{H}; \tilde{\theta})$. Three exercises are performed. We first assess the effects of the endogenous exposure to employment risks on the demand for human capital. We next measure the changes in optimal dynamics resulting from policy changes in the UIB, and base income regimes. Finally, we gauge the effects of additional unemployment costs in the form of a lower productivity in the HK technology, a higher depreciation rate and of firm-specific human capital that is depleted upon displacement. The effects on the baseline results are reported in Table 8.

7.1 Gauging the risks adjustment motives

Traditional HK models focus on higher wages as primary motives and incorporate at most undiversifiable employment risks. A main contribution of our model is thus to allow for possible adjustment of these risks by agents, in addition to the usual income motives for human capital accumulation. We assess its marginal contributions to investment, human capital, unemployment, displacement and re-employment. This exercise is performed by first removing only the re-employment ($\lambda_1^e = 0$) and second only the displacement ($\lambda_1^u = 0$) endogeneity in (1), with corresponding solutions given in Theorem 1. Since the intensities are mechanically lowered, we re-adjust the base intensity so as to maintain the mean theoretical displacement and re-employment rates in Table 7.a. This adjustment is however not neutral and tends to benefit low human capital agents by providing them with higher re-employment and lower displacement rates; high human capital agents are disadvantaged for the opposite reasons (see Figure 9).

Figure 9: Adjusting exogenous intensities



The first two columns of Table 8 reports how the variables of interest are affected by exogenous employment risks, relative to baseline levels. First, removing the capacity to accelerate re-employment in column (1) lowers the attractiveness of investing in human capital and results in a narrowing of the steady-state gaps, as well as a 84% drop in both investment and capital levels. By construction, the re-employment $\Pr(e|u)$ is unaffected, while displacement $\Pr(u|e)$ is increased by 2.4% due to the sharp drop in human capital, resulting in a 3.2% increase in unemployment $\Pr(u)$. Second, exogenous displacement in column (2) also lowers the incentives to invest with I, H falling by 80%. By construction the displacement risk $\Pr(u|e)$ is unaffected, but re-employment $\Pr(e|u)$ falls by 2.4%, leading to a modest increase in unemployment rate. For both cases, the fall in investment under exogenous employment risks is caused by lower returns. Indeed, Figure 9 shows that higher re-employment and lower displacement probabilities reduce the incentives for investing for those agents with low human capital. Moreover, agents with high human capital witness a strong drop in the returns to investment when hedging capacities are removed; they respond by decreasing investment.

7.2 UIB and base income policies

In Table 8, column (3), we investigate the effect of less generous unemployment insurance by decreasing the UI replacement rate η from 0.50 to 0.33 in (2b). The outcome is a 18% increase in investment and capital, inducing lower displacement and improvements in re-employment and unemployment. In column (4), we next analyze changes in the base income y_0 in (2a) by allowing an increase in the latter from 0.10 to 0.15. The increase in disposable income leads to 40% increases in investment and human capital leading to improvements in labor market outcomes.

The reason for these similar effects of less (more) generous UI (base income) policies on investment and capital can be deduced from (3) which shows that the income loss associated with unemployment $\Delta Y(H, \overline{H})$ is a decreasing function of η and is increasing in base income y_0. Less generous UIB and/or higher base income thus both increase the income gap of unemployment and gains from re-employment, thereby raising the incentives for investing. Our results are thus consistent with strong moral hazard responses to UIB generosity, whereby both employed and unemployed agents invest less in their human capital and face higher displacement and lower re-employment probabilities in more generous regimes. These effects are similar in spirit to Davidson and Woodbury (1993); Belzil (1995); Ljungqvist and Sargent (1998); Chetty (2008); Daly et al. (2012); Spinnewijn (2013) who argue that more generous UI benefits (e.g. in Europe) distort incentives away from job search and favor remaining long-term unemployed where skills are mechanically depreciated.

7.3 Additional costs of unemployment

Our baseline results obtained under restrictions (14) have thus far abstracted from additional disadvantages of being unemployed, such as lower returns to investment and loss of firm-specific human capital. However, Theorems 1 and 2 make it possible to calculate the effects of such costs.

First, in column (5) we augment the depreciation rate of human capital when unemployed to $\delta^u = 0.1313 > \delta^e = 0.0750$. Second, in column (6), we introduce depletion of firm-specific human capital by imposing a $1 - \phi = 50\%$ loss on the capital stock upon displacement. Both comparative statics convey the same message. Faster capital depreciation rate once unemployed or immediate cut in capital upon the displacement event lower both I, H, leading to deterioration in labor market outcomes, with increased displacement and reduced re-employment leading to higher unemployment.

8 Conclusion

In addition to the contemporaneous drop in income due to incomplete and temporary UI replacement, unemployment imposes significant long-term scarring and stigma costs on agents. In particular, displacement (re-employment) probabilities are higher (lower), whereas wages upon re-employment are lower following unemployment spells. Moreover, the duration of unemployment spells significantly compounds the magnitude of these costs.

Human capital loss has long been suspected as potential rationale for these costs. Accelerated depreciation during unemployment associated with screening by employers for imperfectly observed human capital levels have been invoked as the main drivers for scarring and stigma. This explanation has notably been advocated in DMP models with human capital appended, where a learning-by-doing perspective minimizes accumulation outside of employment. Traditional HK models allow for explicit investment by agents, but fail to account for effects on employment risks exposure.

This paper has taken the alternative approach or endogenizing human capital decisions by employed and unemployed workers alike and by internalizing their exposure to displacement and re-employment risks. Contrary to others, our model can integrate or abstract from status-dependent human capital accumulation technology and from firm- or sector-specific capital depletion upon displacement. For our baseline scenario, these additional tolls of unemployment are shut down. It follows that any acquisition and depletion of human capital and resulting unemployment scarring and stigma are entirely endogenous, rather than mechanic.

We first investigated whether and confirmed that this framework is capable of generating unemployment scarring and stigma at the optima. The two key theoretical elements behind this result are that investment is positive, but lower when unemployed than when employed and that the model generates two status-dependent and dynamically stable steady-states for human capital, with the one for the unemployed always being lower. Changes in employment statuses thus trigger circular dynamics characterized by endogenous depletion of acquired human capital when unemployed and accumulation upon re-employment. Since re-employment (displacement), as well as wages intensities are increasing (decreasing) functions of human capital, scarification and stigmatisation are internally generated. Because they depend entirely on optimal decisions made by workers instead of by employers, scarring and stigma are therefore self-inflicted.

To the extent that scarring and stigma both impose substantial costs to workers, that they depend on accumulated human capital and that the latter can be adjusted by agents, the optimal strategy could have been to minimize exposure to these risks by investing more to prevent displacement if employed and in favor of re-employment if unemployed. However, our results show that this is not the case. The cushioning against downward income risks offered by UI programs, as well as imperfect replacement rates entails that moral hazard and low income prevent the unemployed from investing more to avoid long-term costs. Incorporating incremental tolls of displacement, such as added depreciation and/or depletion of firm-specific capital for the unemployed is complementary, but not essential for self-inflicted costs.

References

Abbott, Michael (2008) 'Worker displacement: A literature review and potential policy rrsponses.' In 'A Festschrift in Honour of David Dodge' (Bank of Canada) pp. 237–263

Acemoglu, Daron (1995) 'Public policy in a model of long-term unemployment.' *Economica* 62(246), 161 – 178

Acemoglu, Daron, and Jorn-Steffen Pischke (1999) 'The structure of wages and investment in general training.' *Journal of Political Economy* 107(3), 539 – 572

Alvarez, Fernando E., Katarína Borovičková, and Robert Shimer (2016) 'Decomposing duration dependence in a stopping time model.' NBER Working Paper 22188, National Bureau of Economic Research, April

Arulampalam, Wiji (2001) 'Is unemployment really scarring? effects of unemployment experiences on wages.' *The Economic Journal* 111(475), F585–F606

Arulampalam, Wiji, Paul Gregg, and Mary Gregory (2001) 'Unemployment scarring.' *The Economic Journal* 111(475), 577 – 584

Bagger, Jesper, Francois Fontaine, Fabien Postel-Vinay, and Jean-Marc Robin (2014) 'Tenure, experience, human capital, and wages: A tractable equilibrium search model of wage dynamics.' *American Economic Review* 104(6), 1551 – 1596

Becker, Gary S. (1962) 'Investment in human capital: a theoretical analysis.' *Journal of Political Economy* 70(5), 9 – 49

_ (1993) *Human capital: A theoretical and empirical analysis, with special reference to education.,* 3 ed. (Chicago: NBER and the University of Chicago Press)

Belzil, Christian (1995) 'Unemployment insurance and unemployment over time: An analysis with event history data.' *The Review of Economics and Statistics* 77(1), 113–126

Ben-Porath, Y. (1967) 'The production of human capital and the life cycle of earnings.' *Journal of Political Economy* 75(4), 352 – 365

Burdett, Kenneth, Carlos Carrillo-Tudela, and Melvyn G. Coles (2011) 'Human capital accumulation and labor market equilibrium.' *International Economic Review* 52(3), 657 – 677

Carmichael, Lorne (1983) 'Does rising productivity explain seniority rules for layoffs?' *American Economic Review* 73(5), 1127 – 1131

Carrington, William J., and Bruce C. Fallick (2014) 'Why do earnings fall with job displacement?' Working Paper 14-05, Federal Reserve Bank of Cleveland, June

Cervellati, Matteo, and Uwe Sunde (2013) 'Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited.' *Econometrica* 81(5), 2055 – 2086

Chetty, Raj (2008) 'Moral hazard versus liquidity and optimal unemployment insurance.' *Journal of Political Economy* 116(2), 173 – 234

Cunha, Flavio, and James Heckman (2007) 'The technology of skill formation.' *American Economic Review* 97(2), 31 – 47

Daly, Mary C., Bart Hobijn, Aysegul Sahin, and Robert G. Valletta (2012) 'A search and matching approach to labor markets: Did the natural rate of unemployment rise?.' *Journal of Economic Perspectives* 26(3), 3 – 26

Davidson, Carl, and Stephen A. Woodbury (1993) 'The displacement effect of reemployment bonus programs.' *Journal of Labor Economics* 11(4), 575 – 605

Davis, Steven J., and Till M. von Wachter (2011) 'Recessions and the cost of job loss.' Working Paper 17638, National Bureau of Economic Research, December

Decreuse, Bruno, and Pierre Granier (2013) 'Unemployment benefits, job protection, and the nature of educational investment.' *Labour Economics* 23, 20 – 29

Diamond, Peter A. (1982) ‘Wage determination and efficiency in search equilibrium.’ *Review of Economic Studies* 49(2), 217 – 227

Eliason, Marcus, and Donald Storrie (2006) ‘Lasting or latent scars? Swedish evidence on the long-term effects of job displacement.’ *Journal of Labor Economics* 24(4), 831 – 856

Eriksson, Stefan, and Dan-Olof Rooth (2014) ‘Do employers use unemployment as a sorting criterion when hiring? evidence from a field experiment.’ *American Economic Review* 104(3), 1014 – 1039

Esteban-Pretel, Julen, and Junichi Fujimoto (2014) ‘Life-cycle labor search with stochastic match quality.’ *International Economic Review* 55(2), 575 – 599

Eubanks, James D., and David Wiczer (2016) ‘Duration dependence and composition in unemployment spells.’ Review 98(4), Federal Reserve Bank of St. Louis

Fang, Lei, and Pedro Silos (2012) ‘Wages and unemployment across business cycles: A high-frequency investigation.’ Working Paper 2012-16, Federal Reserve Bank of Atlanta, October

Farber, Henry S. (2005) ‘What do we know about job loss in the United States? Evidence from the displaced workers survey, 1984-2004.’ *Federal Reserve Bank of Chicago Economic Perspectives* 29(2), 13 – 28

_ (2011) ‘Job loss in the great recession: Historical perspective from the displaced workers survey, 1984-2010.’ Working paper 17040, National Bureau of Economic Research

Flinn, Christopher, Ahu Gemici, and Steven Laufer (2017) ‘Search, matching and training.’ *Review of Economic Dynamics* 25, 260 – 297

Flinn, Christopher, and Joseph Mullins (2015) ‘Labor market search and schooling investment.’ *International Economic Review* 56(2), 359 – 398

Fu, Chao (2011) 'Training, search and wage dispersion.' *Review of Economic Dynamics* 14(4), 650 – 666

Fujita, Shigeru (2018) 'Declining labor turnover and turbulence.' Working paper WP 18-06, Federal Reserve Bank of Philadelphia Research Department, January

Fujita, Shigeru, and Giuseppe Moscarini (2017) 'Recall and unemployment.' *American Economic Review* 107(12), 3875 – 3916

Gomes, Pedro (2012) 'Labour market flows: Facts from the United Kingdom.' *Labour Economics* 19(2), 165 – 175

Guvenen, Fatih, Burhan Kuruscu, Satoshi Tanaka, and David Wiczer (2018) 'Multidimensional skill mismatch.' manuscript, University of Minnesota, March

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song (2017) 'Heterogeneous scarring effects of full-year nonemployment.' *American Economic Review* 107(5), 369 – 373

Hamermesh, Daniel S. (1987) 'The costs of worker displacement.' *Quarterly Journal of Economics* 102(1), 51 – 75

Heckman, James J. (1976) 'A life-cycle model of earnings, learning, and consumption.' *Journal of Political Economy* 84(4), S11 – 44

Heckman, James J. (2008) 'School, skill, and synapses.' Working paper 14064, National Bureau of Economic Research, June

Heckman, James J., Robert J. Lalonde, and Jeffrey A. Smith (1999) 'The economics and econometrics of active labor market programs.' In *Handbook of Labor Economics,* ed. Orley C. Ashenfelter and David Card, vol. 3 (Elsevier) chapter 31, pp. 1865 – 2097

Hendricks, Lutz (2013) 'The Ben-Porath model and age-wage profiles.' Manuscript, University of North Carolina, Chapel Hill, January

Hu, Luojia, and Christopher Taber (2011) 'Displacement, asymmetric information and heterogeneous human capital.' *Journal of Labor Economics* 29(1), 113–152

Huckfeldt, Christopher (2016) 'Understanding the scarring effects of recessions.' Manuscript, Department of Economics, Cornell University, March

Huggett, Mark, Gustavo Ventura, and Amir Yaron (2006) 'Human capital and earnings distribution dynamics.' *Journal of Monetary Economics* 53(2), 265 – 290

_ (2011) 'Sources of lifetime inequality.' *American Economic Review* 101(7), 2923 – 2954

Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour (2013) 'Health and (other) asset holdings.' *The Review of Economic Studies* 80(2), 663–710

Huttunen, Kristiina, Jarle Moen, and Kjell G. Salvanes (2011) 'How destructive is creative destruction? effects of job loss on job mobility, withdrawal and income.' *Journal of the European Economic Association* 9(5), 840 – 870

Ingram, Beth F., and George R. Neumann (2006) 'The returns to skill.' *Labour Economics* 13(1), 35 – 59

Jacobson, Louis S., Robert J. LaLonde, and Daniel G. Sullivan (1993) 'Earnings losses of displaced workers.' *American Economic Review* 83(4), 685 – 709

Jacobson, Louis S., Robert LaLonde, and Daniel Sullivan (2005) 'Is retraining displaced workers a good investment?.' *Federal Reserve Bank of Chicago Economic Perspectives* 29(2), 47 – 66

Jones, Benjamin F. (2014) 'The human capital stock: A generalized approach.' *The American Economic Review* 104(11), 3752–3777

Keane, Michael P., and Kenneth I. Wolpin (1997) 'The career decisions of young men.' *Journal of Political Economy* 105(3), 473 – 522

Kitao, Sagiri, Lars Ljungqvist, and Thomas J. Sargent (2017) 'A life-cycle model of trans-atlantic employment experiences.' *Review of Economic Dynamics* 25, 320 – 349. Special Issue on Human Capital and Inequality

Kletzer, Lori G. (1998) 'Job displacement.' *Journal of Economic Perspectives* 12(1), 115–136

Kräkel, Mathias (2016) 'Human capital investment and work incentives.' *Journal of Economics and Management Strategy* 25(3), 627–651

Krebs, Tom (2003) 'Human capital risk and economic growth.' *Quarterly Journal of Economics* 118(2), 709 – 744

_ (2007) 'Job displacement risk and the cost of business cycles.' *American Economic Review* 97(3), 664 – 686

Kredler, Matthias (2014) 'Experience vs. obsolescence: A vintage-human-capital model.' *Journal of Economic Theory* 150, 709 – 739

Kroft, Kory, Fabian Lange, and Matthew J. Notowidigdo (2013) 'Duration dependence and labor market conditions: Evidence from a field experiment.' *Quarterly Journal of Economics* 128(3), 1123 – 1167

Kroft, Kory, Fabian Lange, Matthew J. Notowidigdo, and Lawrence F. Katz (2016) 'Long-term unemployment and the great recession: The role of composition, duration dependence, and nonparticipation.' *Journal of Labor Economics* 34(1), S7 – 54

Krolikowski, Pawel (2017) 'Job ladders and earnings of displaced workers.' *American Economic Journal: Macro* 9(2), 1–31

Krueger, Allan B., Judd Cramer, and David Cho (2014) 'Are the long-term unemployed on the margins of the labor market?' Brookings Papers on Economic Activity, Brookings Institute

Lentz, Rasmus, and Nicolas Roys (2015) 'Training and search on the job.' WP 21702, National Bureau of Economic Research, November

Lise, Jeremy (2013) 'On-the-job search and precautionary savings.' *Review of Economic Studies* 80(3), 1086 – 1113

Ljungqvist, Lars, and Thomas J. Sargent (1998) 'The european unemployment dilemma.' *Journal of Political Economy* 106(3), 514 – 550

Marotzke, Petra (2014) 'Human capital investments and worker mobility over the life cycle.' Manuscript, Deutsche Bundesbank, October

Medoff, James L., and Katharine G. Abraham (1981) 'The role of seniority at U.S. work places: A report on some new evidence.' Working Paper 618, National Bureau of Economic Research, 1981

Mincer, Jacob (1974) *Schooling, Experience and Earnings* (National Bureau of Economic Research)

Mortensen, Dale T., and Christopher A. Pissarides (1994) 'Job creation and job destruction in the theory of unemployment.' *Review of Economic Studies* 61(3), 397 – 415

Moscarini, Giuseppe, and Fabien Postel-Vinay (2016) 'Did the job ladder fail after the great recession?.' *Journal of Labor Economics* 34(1), S55 – 93

Mukoyama, Toshihiko, Christina Patterson, and Aysegul Sahin (2018) 'Job search behavior over the business cycle.' *American Economic Journal: Macroeconomics* 10(1), 190 – 215

Neal, Derek (1995) 'Industry-specific human capital: Evidence from displaced workers.' *Journal of Labor Economics* 13(4), 653–677

Nekoei, Arash, and Andrea Weber (2015) 'Recall expectations and duration dependence.' *American Economic Review* 105(5), 142 – 146

Nilsen, Oivind A., and Katrine Holm Reiso (2011) 'Scarring effects of unemployment.' Discussion Paper 6198, IZA, December

Ortego-Marti, Victor (2017) 'The cyclical behavior of unemployment and vacancies with loss of skills during unemployment.' *Macroeconomic Dynamics* 21(6), 1277 – 1304

Pavoni, Nicola (2009) 'Optimal unemployment insurance, with human capital depreciation, and duration dependence.' *International Economic Review* 50(2), 323 – 362

Pavoni, Nicola, and G. L. Violante (2007) 'Optimal welfare-to-work programs.' *Review of Economic Studies* 74(1), 283 – 318

Pinheiro, Roberto, and Ludo Visschers (2015) 'Unemployment risk and wage differentials.' *Journal of Economic Theory* 157, 397 – 424

Pissarides, Christopher A. (1992) 'Loss of skill during unemployment and the persistence of employment shocks.' *Quarterly Journal of Economics* 107(4), 1371 – 1391

Polachek, Solomon W., Tirthatanmoy Das, and Rewat Thamma-Apiroam (2015) 'Micro- and macroeconomic implications of heterogeneity in the production of human capital.' *Journal of Political Economy* 123(6), 1410 – 1455

Quintini, Glenda, and Danielle Venn (2013) 'Back to work: Re-employment, earnings and skill use after job displacement.' Final report, OECD, October

Riddell, W. Craig, and Xueda Song (2011) 'The impact of education on unemployment incidence and re-employment success: Evidence from the U.S. labour market.' *Labour Economics* 18(4), 453 – 463

Rogerson, Richard, and Martin Schindler (2002) 'The welfare costs of worker displacement.' *Journal of Monetary Economics* 49(6), 1213 – 1234

Ruhm, Christopher J. (1991a) 'Are workers permanently scarred by job displacements?' *American Economic Review* 81(1), 319 – 324

— (1991b) 'Displacement induced joblessness.' *Review of Economics and Statistics* 73(3), 517 – 522

Shimer, Robert, and Ivn Werning (2006) 'On the optimal timing of benefits with heterogeneous workers and human capital depreciation.' Working Paper 12230, National Bureau of Economic Research, May

Spinnewijn, Johannes (2013) 'Training and search during unemployment.' *Journal of Public Economics* 99, 49 – 65

St-Amour, Pascal (2015) 'Human capital and employment risks diversification.' SFI Research Paper 15-18, Swiss Finance Institute, June

Stantcheva, Stefanie (2017) 'Optimal taxation and human capital policies over the life cycle.' *Journal of Political Economy* 125(6), 1931–1990

Stevens, Ann Huff (1997) 'Persistent effects of job displacement: The importance of multiple job losses.' *Journal of Labor Economics* 15(1), 165 – 188

van den Berg, Gerard J., and Jan C. van Ours (1996) 'Unemployment dynamics and duration dependence.' *Journal of Labor Economics* 14(1), 100 – 125

von Wachter, Till, Jae Song, and Joyce Manchester (2009) 'Long-term earnings losses due to mass layoffs during the 1982 recession: An analysis using U.S. administrative data from 1974 to 2004.' manuscript UCLA

Wasmer, Etienne (2006) 'General versus specific skills in labor markets with search frictions and firing costs.' *American Economic Review* 96(3), 811 – 831

Yamaguchi, Shintaro (2010) 'Job search, bargaining, and wage dynamics.' *Journal of Labor Economics* 28(3), 595–631

— (2012) 'Tasks and heterogeneous human capital.' *Journal of Labor Economics* 30(1), 1–53

A Data

We rely on the National Longitudinal Survey of Youth (NLSY79), a panel of 9,964 respondents aged 15-22 in 1979 (Round 1), followed up to year 2014 (Round 26), annual from 1979-1994, biennial afterwards. The principal variables are constructed as follows:

s Employment status (binary), from `ESR_COL_` employment status recode (collapsed) equal to 1 if employed (`ESR_COL_==1`), 0 if unemployed (`ESR_COL_==2`). One- and two-period lagged statuses (`s_lag_1, s_lag_2`), as well as lagged cumulated statuses (`exper_lag_1`) are used in the panel regressions.

y Income, scaled by 1.0e-05, in real terms, from:

- `Q13_5_` total income wages and salary, years 1979-1981,
- `Q13_5_TRUNC_REVISED_` (truncated, revised), years 1982-2000,
- `Q13_5_TRUNC_` amount of respondent's salary wages and tips, years 2002-2014.

male Gender (binary), from `SAMPLE_SEX_1979==1`

white Race (binary), from `SAMPLE_RACE_78SCRN==3`

citizen US citizen (binary), from `CITIZEN_1990==1`, is respondent of US citizenship.

educ Education level, from `HGC_`, highest grade completed by 05.01 of survey year. Less than high school ($HGC < 12$); High school ($HGC = 12$); Some college/associate degree ($12 < HGC < 16$); College ($16 \leq HGC$).

weight Sampling weight, from `SAMPWEIGHT_`

training Training (binary) from `Q8_18_`, any vocational/technical training for more than one month.

urban Urban (binary), from `URBAN_RURAL_==1` current residence urban/rural.

age Age, from `AGEATINT_`, age of respondent at interview date.

B Tables

Table 1: Summary statistics

Variable	Year-pers. obs.	Mean	Std. Dev.	Min	Max
Employment	149 242	0.9103	0.2858	0	1
Income	184 317	16 970	18 596	0	156 449
Age	250 709	32.62	10.74	14	58
Male	329 836	0.5086	0.4999	0	1
White	329 836	0.7941	0.4044	0	1
US citizen	329 836	0.9119	0.2835	0	1
HGC	249 936	12.95	2.4596	0	20
Training	181 333	0.1367	0.3435	0	1
Urban	234 958	0.7588	0.4278	0	1

Notes: See Appendix A for NLSY79 data details. Balanced panel of 9 964 individuals over period 1979-2014 (biennial after 1994). Sample moments are weighted with frequency weights.

Table 2: Current employment probability by previous statuses and by human capital

	i_{t-1} status: Unemployed	i_{t-1} status: Employed	All
All	0.6873	0.9513	0.9298
a. By i_{t-2} status			
- Unemployed	0.5758	0.8669	0.7825
- Employed	0.7716	0.9631	0.9544
b. By education			
- Less than high school	0.5843	0.8936	0.8339
- High school	0.6925	0.9464	0.9251
- Some college	0.7836	0.9642	0.9539
- College	0.8907	0.9796	0.9770
c. By training			
- No	0.6672	0.9482	0.9249
- Yes	0.7277	0.9675	0.9515

Notes: Current employment probability by previous employment status $i_{t-1} \in \{u, e\}$. Panel a: by $i_{t-2} \in \{u, e\}$ status. Panel b: by education level from highest grade completed (HGC). Panel c: By professional/vocational and on-the-job training.

Table 3: Current real annual income by previous statuses and by human capital

	i_{t-1} status: Unemployed	i_{t-1} status: Employed	All
All	5 724	16 220	15 519
a. By i_{t-2} status			
- Unemployed	4 372	9 037	7 924
- Employed	7 086	16 969	16 560
b. By education			
- Less than high school	3 976	10 329	9 416
- High school	5 697	13 883	13 279
- Some college	5 808	15 118	14 614
- College	10 670	24 997	24 622
c. By training			
- No	6 590	14 690	14 114
- Yes	7 565	17 733	17 248

Notes: Average current real annual income by previous employment status $i_{t-1} \in \{u, e\}$. Panel a: by $i_{t-2} \in \{u, e\}$ status. Panel b: by education level from highest grade completed (HGC). Panel c: By professional/vocational and on-the-job training.

Table 4: Current employment: Marginal effects

	(1) All $\Pr(i_t = e)$	(2) Re-employment $\Pr(i_t = e \mid i_{t-1} = u)$	(3) Cont. employment $\Pr(i_t = e \mid i_{t-1} = e)$
	a. Past employment statuses		
s_lag_1	0.0928***		
	(34.79)		
s_lag_2	0.0420***	0.0582***	0.0362***
	(15.38)	(4.25)	(13.04)
	b. Human capital		
exper_lag_1	0.0103***	0.0310***	0.00824***
	(19.74)	(8.49)	(17.13)
educ_	0.00890***	0.0292***	0.00698***
	(17.72)	(8.97)	(15.11)
training_	0.0173***	0.0323	0.0156***
	(5.22)	(1.53)	(5.10)
	c. Other controls		
age_	0.00951***	0.0153	0.0107***
	(3.55)	(0.92)	(4.22)
age_sq_	−0.000195***	−0.000472	−0.000202***
	(−3.85)	(−1.44)	(−4.27)
white	0.0231***	0.0650***	0.0190***
	(10.84)	(4.93)	(9.60)
male	−0.00615**	−0.000551	−0.00624**
	(−2.95)	(−0.04)	(−3.25)
citizen	−0.0169***	−0.0725***	−0.0116***
	(−5.12)	(−3.58)	(−3.82)
urban_	0.00725**	0.0194	0.00611**
	(2.86)	(1.24)	(2.60)
Observations	67 629	5 930	61 699

Notes: Marginal effects calculated from panel Probit with year fixed effects and population-averaged effects. Variables s_lag_j are $t - j$ employment statuses, while exper_lag_1 are cumulated lagged statuses up to time $t-1$. t-statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 5: Current income

	(1) Employed $y(i_t = e)$	(2) Re-employed $y(i_t = e \mid i_{t-1} = u)$	(3) Cont. employed $y(i_t = e \mid i_{t-1} = e)$
	a. Past employment statuses		
s_lag_1	0.0323***		
	(27.76)		
s_lag_2	0.00814***	0.00844***	0.00985***
	(8.06)	(3.77)	(8.80)
	b. Human capital		
exper_lag_1	0.00903***	0.00429***	0.00881***
	(35.03)	(7.54)	(32.47)
educ_	0.0126***	0.00427***	0.0129***
	(47.17)	(8.89)	(46.63)
training_	0.00472***	0.00414	0.00482***
	(6.22)	(1.37)	(6.19)
	c. Other controls		
age_	0.0164***	0.00752**	0.0178***
	(19.18)	(2.82)	(19.74)
age_sq_	−0.000238***	−0.0000968	−0.000258***
	(−16.27)	(−1.85)	(−16.87)
white	0.0128***	0.0103***	0.0134***
	(8.87)	(5.14)	(8.97)
male	0.0468***	0.0206***	0.0481***
	(33.90)	(10.29)	(33.61)
citizen	−0.0141***	−0.00437	−0.0157***
	(−7.11)	(−1.49)	(−7.55)
urban_	0.0110***	0.00340	0.0114***
	(9.63)	(1.39)	(9.63)
Observations	59 086	3 285	55 801

Notes: Estimates from panel GLS with year fixed-effects and population-averaged effects. t-statistics in parentheses. Variables s_lag_j are $t-j$ employment statuses, while exper_lag_1 are cumulated lagged statuses up to time $t-1$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 6: Calibrated parameters

(a) Intensities (1)

λ_0^e	λ_1^e	ξ^e	λ_0^u	λ_1^u	ξ^u
0.05	1.15	-0.04	0.002	0.02	0.25

(b) Income (2), Dynamics (4) and HJB (7)

y_0	y_1	η	δ^e, δ^u	α	P^e, P^u	ρ	ϕ
0.1	1.1	0.5	0.075	0.85	0.1	0.05	1

Table 7: Moments matching

(a) Conditional probabilities

	i_{t-1} status:		
By i_{t-2} status	Unemployed	Employed	All
a. Data			
- Unemployed	0.5758	0.8669	0.7825
- Employed	0.7716	0.9631	0.9544
- All	0.7031	0.9574	0.9414
b. Model			
- Unemployed	0.6477	0.9499	0.8443
- Employed	0.6516	0.9524	0.9380
- All	0.6502	0.9523	0.9316

(b) Continuation probabilities from $t - \tau$ to t

	1. Unemployment		2. Employment	
τ	a. Data	b. Model	a. Data	b. Model
0	0.0897	0.0684	0.9103	0.9316
1	0.0542	0.0239	0.8657	0.8872
2	0.0430	0.0084	0.8268	0.8450
3	0.0379	0.0029	0.7924	0.8048

(c) Income (in 100'000$)

By i_t status	a. Data	b. Model
- Unemployed	0.0664	0.0683
- Employed	0.1526	0.1365
- All	0.1470	0.1318

Notes: (a) Transition probabilities by i_{t-1}, i_{t-2} statuses. (b) Probability of continued unemployment $\Pr(i_{t-\tau}, \ldots, i_t = u)$ and employment $\Pr(i_{t-\tau}, \ldots, i_t = e)$. (c) Income by current status i_t.

Table 8: Hedging motives and comparative statics

Variable	Base	Risks (1)	Risks (2)	Policy (3)	Policy (4)	Unempl. costs (5)	Unempl. costs (6)
		(a) Human capital					
I	0.0214	−83.84	−79.93	18.34	40.16	−5.04	−7.58
H	0.0331	−83.74	−79.43	18.68	40.20	−9.78	−29.88
		(b) Employment					
$\Pr(u)$	0.0686	3.20	0.19	−0.31	−0.56	0.18	0.68
$\Pr(e\|u)$	0.6498	−0.09	−2.39	0.43	0.56	−0.34	−1.53
$\Pr(u\|e)$	0.0479	2.47	−0.04	−0.20	−0.38	0.11	0.39

Notes: (a) Percentage and (b) basis points changes from base scenario. (1) Exogenous re-employment, $(\lambda_0^e, \lambda_1^e) = (1.0441, 0)$ instead of $(0.05, 1.15)$. (2) Exogenous displacement, $(\lambda_0^u, \lambda_1^u) = (0.0486, 0)$ instead of $(0.002, 0.02)$. (3) UIB low, $\eta = 0.33$ instead of 0.50. (4) Base income high, $y_0 = 0.15$ instead of 0.10. (5) High unemployment depreciation, $\delta^u = 0.1313$ instead of 0.075. (6) General human capital share $\phi = 0.50$ instead of 1.0.

C Order-0 transversality and regularity conditions

The required transversality and regularity conditions for the order-0 solutions are:

$$0 < \rho + \lambda_0^e + \delta^u - \left(\alpha P^u{}^{\frac{1}{\alpha}} A_h^u\right)^{\frac{\alpha}{1-\alpha}}, \tag{15a}$$

$$0 < \rho + \lambda_0^u + \delta^e - \left(\alpha P^e{}^{\frac{1}{\alpha}} A_h^e\right)^{\frac{\alpha}{1-\alpha}}, \tag{15b}$$

$$\phi\lambda_0^e\lambda_0^u < \left(\rho + \lambda_0^e + \delta^u - \left(\alpha P^u{}^{\frac{1}{\alpha}} A_h^u\right)^{\frac{\alpha}{1-\alpha}}\right)\left(\rho + \lambda_0^u + \delta^e - \left(\alpha P^e{}^{\frac{1}{\alpha}} A_h^e\right)^{\frac{\alpha}{1-\alpha}}\right), \tag{15c}$$

D Order-0 parameters

Proof. At the optimum, the order-0 HJB (7) corresponding to $\lambda_1^e, \lambda_1^u = 0$ can be written as:

$$\begin{aligned} 0 = &- \rho V^e(H) - \lambda_0^u \left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) \\ &- \delta^e H V_H^e(H) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^e V_H^e(H)\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{16a}$$

$$\begin{aligned} 0 = &- \rho V^u(H, \overline{H}) - \lambda_0^e \left[V^u(H, \overline{H}) - V^e(H)\right] + Y^u(H) \\ &- \delta^u H V_H^u(H, \overline{H}) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^u V_H^u(H, \overline{H})\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{16b}$$

Consider candidate solution:

$$V_0^e(H) = A_0^e + A_h^e H \tag{17a}$$

$$V_0^u(H, \overline{H}) = A_0^u + A_h^u H + A_b^u \overline{H} \tag{17b}$$

Substituting the candidate solutions (17) in (16) yields:

$$0 = \tilde{A}_0^e + \tilde{A}_h^e H \tag{18a}$$

$$0 = \tilde{A}_0^u + \tilde{A}_h^u H + \tilde{A}_b^u \overline{H} \tag{18b}$$

Assuming the transversality and regularity conditions conditions (15) hold, we can individually set the implicit parameters $\tilde{A}^e, \tilde{A}^u$ to zero in (18) and obtain that the parameters in Theorem 1 are:

$$A_0^u = \frac{y_0 \left(\lambda_0^e + \eta \left(\rho + \lambda_0^u\right)\right)}{\rho \left(\lambda_0^e + \rho + \lambda_0^u\right)}; \quad A_b^u = \frac{\eta y_1}{\lambda_0^e + \rho}; \quad A_0^e = \frac{y_0 \left(\lambda_0^e + \rho + \eta \lambda_0^u\right)}{\rho \left(\lambda_0^e + \rho + \lambda_0^u\right)}$$

and where A_h^e, A_h^u jointly solve:

$$0 = A_h^e \lambda_0^e - A_h^u \left(\delta^u + \lambda_0^e + \rho\right) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \left(P^u A_h^u\right)^{\frac{1}{1-\alpha}} \tag{19a}$$

$$0 = \lambda_0^u \left(\phi A_h^u + \frac{\eta y_1}{\lambda_0^e + \rho}\right) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \left(P^e A_h^e\right)^{\frac{1}{1-\alpha}} - A_h^e \left(\delta^e + \rho + \lambda_0^u\right) + y_1 \tag{19b}$$

The optimal investment and growth functions follow directly by substituting (A_h^e, A_h^u) in (9) and (10). The following result shows that a large income sensitivity y_1 is sufficient to generate a lower Tobin's-q when unemployed.

Corollary 1 *Define:*

$$b_0^u = \frac{(\lambda_0^e + \rho + \lambda_0^u \eta) y_1}{(\lambda_0^e + \rho)\phi \lambda_0^u}, \quad b_1^u = \frac{\delta^e + \lambda_0^u + \rho}{\phi \lambda_0^u}, \quad b_2^u = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} P^{e\frac{1}{1-\alpha}}}{\phi \lambda_0^u}.$$

If the following condition holds:

$$\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(b_1^u - 1\right)^{\frac{1}{\alpha}} b_2^{u\frac{\alpha-1}{\alpha}} < b_0^u \tag{20}$$

then the solutions A_h^e, A_h^u to (19) satisfy

$$A_h^u < A_h^e.$$

The proof is obtained by rewriting (19) as:

$$A_h^e = b_1^e A_h^u - b_2^e A_h^{u\,\frac{1}{1-\alpha}}$$

$$A_h^u = b_1^u A_h^e - b_2^u A_h^{e\,\frac{1}{1-\alpha}} - b_0^u$$

The two functions $A_h^e = A_h^e(A_h^u)$ and $A_h^u = A_h^u(A_h^e)$ are increasing at the solution, by transversality conditions (15) and are concave. They intersect at the solution to (19) in the (A_h^e, A_h^u) space. A sufficient condition for the intersection to lie below the 45 degree line is for the function $A_h^u(A_h^e) < A_h^e$ everywhere, or:

$$\max_{A_h^e} (b_1^u - 1)A_h^e - b_2^u A_h^{e\,\frac{1}{1-\alpha}} - b_0^u < 0.$$

Taking derivatives, setting to zero and substituting back yields sufficient condition (20). Observe from the closed-form solution to b_0^u that the latter can also be rewritten as

$$\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\frac{\delta^e + (1-\phi)\lambda_0^u + \rho}{\phi\lambda_0^u} \right)^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} P^{e\,\frac{1}{1-\alpha}}}{\phi\lambda_0^u} \right]^{\frac{\alpha-1}{\alpha}} \frac{(\lambda_0^e + \rho)\phi\lambda_0^u}{(\lambda_0^e + \rho + \lambda_0^u\eta)} < y_1$$

i.e. a large capital gradient of income y_1 is a sufficient condition for $A_h^u < A_h^e$ and therefore lower capital growth when unemployed than employed $g_0^u < g_0^e$. ■

E Order-1 parameters

Proof. Without loss of generality, rewrite the endogenous component in intensities (1) as $\lambda_1^i = \epsilon\overline{\lambda}_1^i, i = e, u$ for some constants $\overline{\lambda}_1^i$ and perturbation ϵ. The order-1 solution proceed as a first-order Taylor expansion around the order-0 solution corresponding to $\epsilon = 0$. First, the corresponding order-1 HJB can be written as:

$$0 = \sup_I -\rho V^e(H) - \left(\lambda_0^u + \epsilon\overline{\lambda}_1^u H^{-\xi^u}\right) \left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) - I \\ + V_H^e(H)\left[-\delta^e H + P^e I^\alpha H^{1-\alpha}\right], \quad (21a)$$

and

$$0 = \sup_I - \rho V^u(H,\overline{H}) - \left(\lambda_0^e + \epsilon\overline{\lambda}_1^e H^{-\xi^e}\right)\left[V^u(H,\overline{H}) - V^e(H)\right] + Y^u(\overline{H}) - I \\ + V_H^u(H,\overline{H})\left[-\delta^u H + P^u I^\alpha H^{1-\alpha}\right]. \tag{21b}$$

Second, consider candidate solutions given by:

$$V^e(H) = V_0^e(H) + \epsilon\Big(B^e H + B_u^e\overline{\lambda}_1^u H^{-\xi^u} + B_{1u}^e\overline{\lambda}_1^u H^{1-\xi^u} + B_e^e\overline{\lambda}_1^e H^{-\xi^e} \\ + B_{1e}^e\overline{\lambda}_1^e H^{1-\xi^e}\Big), \tag{22a}$$

and

$$V^u(H,\overline{H}) = V_0^u(H,\overline{H}) + \epsilon\Big(B^u H + B_u^u\overline{\lambda}_1^u H^{-\xi^u} + B_{1u}^u\overline{\lambda}_1^u H^{1-\xi^u} + B_e^u\overline{\lambda}_1^e H^{-\xi^e} \\ + B_{1e}^u\overline{\lambda}_1^e H^{1-\xi^e} + B_b^u\overline{H}\ \overline{\lambda}_1^e H^{-\xi^e}\Big). \tag{22b}$$

Third, we solve for I^e, I^u using guess (22) in HJB (21) and express optimal investment as a first-order expansion around $\epsilon = 0$. Fourth, we substitute this first-order solution back in the HJB, again do a first-order expansion around $\epsilon = 0$ and individually solve the implicit parameters B as follows:
where the (A^i, g_0^i) parameters are given in Appendix D and Theorem 1. Substituting back for $\lambda_1^i = \epsilon\overline{\lambda}_1^i$ yields the optimal solution in Theorem 2.

Investment and growth Given the parameters (B^e, B^u), the parameters (C^e, C^u) for the investment functions are obtained as:

$$C^e = \begin{pmatrix} C_u^e \\ C_{1u}^e \\ C_e^e \\ C_{1e}^e \end{pmatrix} = \kappa^e \begin{pmatrix} -\xi^u B_u^e \\ (1-\xi^u)B_{1u}^e \\ -\xi^e B_e^e \\ (1-\xi^e)B_{1e}^e \end{pmatrix}, \quad C^u = \begin{pmatrix} C_u^u \\ C_{1u}^u \\ C_e^u \\ C_{1e}^u \\ C_b^u \end{pmatrix} = \kappa^u \begin{pmatrix} -\xi^u B_u^u \\ (1-\xi^u)B_{1u}^u \\ -\xi^e B_e^u \\ (1-\xi^e)B_{1e}^u \\ -\xi^e B_b^e \end{pmatrix}$$

B^e	0
B^e_u	$\frac{(\eta-1)y_0\phi^{\xi^u}(\lambda_0^e+g_0^u\xi^u+\rho)}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^u}(g_0^e\xi^u+\rho+\lambda_0^u)(\lambda_0^e+g_0^u\xi^u+\rho)-\lambda_0^e\lambda_0^u)}$
B^e_{1u}	$-\frac{\phi^{\xi^u}(\lambda_0^e+g_0^u(\xi^u-1)+\rho)((\lambda_0^e+\rho)(A_h^e-\phi A_h^u)-\eta y_1)}{(\lambda_0^e+\rho)(\phi^{\xi^u}(g_0^e(\xi^u-1)+\rho+\lambda_0^u)(\lambda_0^e+g_0^u(\xi^u-1)+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B^e_e	$-\frac{(\eta-1)y_0\lambda_0^u}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_0^e+\rho)-\lambda_0^e\lambda_0^u)}$
B^e_{1e}	$\frac{\lambda_0^u(\phi A_h^e(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)-\phi A_h^u(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)-\eta y_1((\xi^e-1)g_0^u+\lambda_0^e+\rho))}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)(\phi^{\xi^e}((\xi^e-1)g_0^e+\rho+\lambda_0^u)((\xi^e-1)g_0^u+\lambda_0^e+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B^u	$0,$
B^u_u	$\frac{(\eta-1)y_0\lambda_0^e\phi^{\xi^u}}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^u}(g_0^e\xi^u+\rho+\lambda_0^u)(\lambda_0^e+g_0^u\xi^u+\rho)-\lambda_0^e\lambda_0^u)}$
B^u_{1u}	$-\frac{\lambda_0^e\phi^{\xi^u}((\lambda_0^e+\rho)(A_h^e-\phi A_h^u)-\eta y_1)}{(\lambda_0^e+\rho)(\phi^{\xi^u}(g_0^e(\xi^u-1)+\rho+\lambda_{\mathrm{u0}})(\lambda_0^e+g_0^u(\xi^u-1)+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B^u_b	$-\frac{\eta y_1}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)}$
B^u_e	$-\frac{(\eta-1)y_0\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_{\mathrm{e0}}+\rho)-\lambda_0^e\lambda_0^u)}$
B^u_{1e}	$\frac{\phi^{\xi^e}((A_h^e-A_h^u)(\lambda_0^e+\rho)((\xi^e-1)g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_0^e+\rho)-\eta y_1\lambda_{\mathrm{e0}}\lambda_0^u\phi^{-\xi^e})}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)(\phi^{\xi^e}((\xi^e-1)g_0^e+\rho+\lambda_0^u)((\xi^e-1)g_0^u+\lambda_0^e+\rho)-\phi\lambda_0^e\lambda_0^u)}$

where we have set:

$$\kappa^i \equiv \frac{\left[P^i\alpha\left(A_h^i\right)^{\alpha}\right]^{\frac{1}{1-\alpha}}}{1-\alpha}, \quad i=e,u$$

Given the parameters (C^e, C^u), the parameters (D^e, D^u) for the growth functions are obtained as:

$$D^i = \frac{C^i}{A_h^i}, \quad i=e,u.$$

■

F Simulation

We begin by calibrating the main parameters and by initializing the employment status and human capital for a population of agents $j=1,2,\ldots,n$:

- The employment status is drawn from the unconditional population rates: $i_{j,0} \sim \{e,u\}$.
- Both the initial capital $H_{j,0}$ and the initial lock-in capital $\overline{H}_{j,0}$ are independently drawn from a uniform distribution over interval $[a,b]$.

Next, the recursive phase is obtained for $\forall j$ and $\forall t = 0, 1, 2, \ldots, T$ as follows:

1. Set the employment status $i = i_{j,t}$, in order to compute the optimal investment (12) and welfare (11), as well as the displacement/re-employment exposures and income as:

$$I_{j,t} = I^i(H_{j,t}, \overline{H}_{j,t}), \qquad V_{j,t} = V^i(H_{j,t}, \overline{H}_{j,t}),$$
$$\lambda_{j,t} = \lambda^i(H_{j,t}), \qquad Y_{j,t} = Y^i(H_{j,t}, \overline{H}_{j,t}).$$

2. Use the law of motion (4) to update human capital and the Poisson distribution to update employment status as:

$$H_{j,t+1} = H_{t+1}(I_{j,t}, H_{j,t}), \qquad \overline{H}_{j,t+1} = \mathbb{1}_t^e H_{j,t+1} + \mathbb{1}_t^u \overline{H}_{j,t},$$
$$i_{j,t+1} \sim \text{Poisson}(\lambda_{j,t}).$$

   Note that each new displacement event $(e \to u)$ adds an additional loss of $(1-\phi)H$ associated with capital specificity. These losses are abstracted from when the agent remains unemployed $(u \to u)$.