Spatial Polarization*

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Abstract

In this paper we use a general equilibrium model to show that two well known phenomena, i) the spatial sorting of workers with heterogeneous skills across U.S. metropolitan areas and ii) aggregate employment polarization in the U.S. are tightly linked and are both largely driven by the emergence of skill-biased technological change (SBTC). In particular, we aim at rationalizing together the following facts: 1) the skill distribution of workers is similar across metropolitan areas with different size between 1960 and 1980, while between 1980 and 2008 large cities display a higher fraction of both high- and low-skilled workers relative to small cities, which in turn display a higher fraction of medium-skilled; 2) aggregate employment polarization starts emerging in the 1980s and is more pronounced in larger cities relative to smaller ones over the 1980-2008 period; and 3) skill-biased technological change grows faster in larger metropolitan areas. We connect these facts in a spatial general equilibrium model and we assess the impact of SBTC and other competing channels on the dynamics of spatial sorting and employment polarization.

*The usual disclaimers apply.

1 Introduction

The emergence of employment polarization in the U.S. after 1980 has been extensively documented. Several explanations have been provided in the literature for this phenomenon, which consists in an increase in employment shares both at the bottom and the top of the skill distribution, with a decline in employment shares in the middle of the distribution. Among these explanations, the spatial sorting of workers has received little attention in the literature.[1] In this paper we fill this gap by showing first that the allocation of workers by skill level across U.S. metropolitan areas in the last decades can be accounted for by the emergence of skill-biased technological change (SBTC) after 1980, and second that employment polarization emerges lo a large extent as a result of such sorting process.

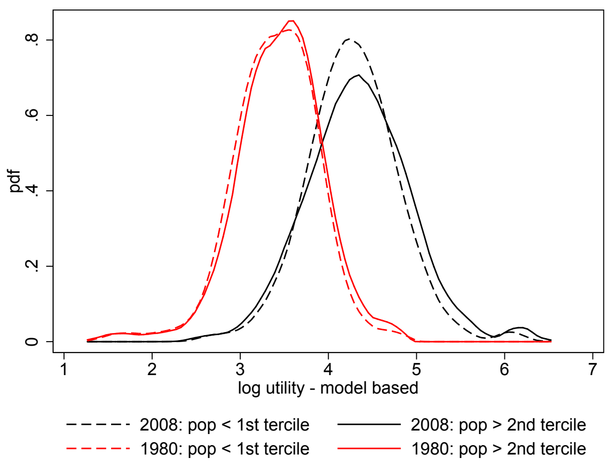

Figure 1: Skill distribution in 1980 (red) and 2008 (black) in small (dashed lines) and large (thick lines) cities. The figure compares metropolitan areas with population above the 2nd tercile and below the 1st tercile in the reference year.

We first document how spatial sorting of workers by skill level changes after 1980 in the U.S. To do this we follow a methodology similar to Eeckhout et al. (2014), who study how the skill distribution changes across U.S. metropolitan areas in the year 2000. They first construct a model-based measure of skills that takes into account wages corrected by housing prices. Once endowed with this price-theoretic measure of skills, they document that while the skill level of the median worker does not differ significantly between large and small cities, the skill distribution in larger cities displays fatter tails relative to that of smaller cities. We apply a similar methodology focusing on the years 1980 and 2008. Figure 1 shows that in 1980 the skill distribution is similar between small and large cities, while

[1] An exception is Blien and Dauth (2016)

in 2008 large cities display fatter tails with respect to smaller cities. We also compute the same measure for the 1960 and find that, as in 1980, the skill distribution is similar across city size. The results are robust to the definition of "large" and "small" city and suggest that after 1980 the spatial sorting of workers starts changing, with larger cities attracting proportionally more high- and low-skilled workers with respect to small cities.

Then, by using broad occupation categories we report that employment polarization is more pronounced in larger cities relative to smaller ones over the 1980-2008 period. More precisely, while the employment shares of broad groups of occupations are similar across cities in 1980, they diverge substantially in 2008 with larger cities displaying a larger increase in the share of both low- and high-skilled occupations and a larger decrease in the share of middle-skilled occupations. Also, we document that this fact is not only robust to different definitions of small and large cities but that this divergence increases with the relative size of large over small cities. Thus, our empirical evidence suggests that the emergence of fat tails in the skill distribution and higher degree of employment polarization from 1980 in large cities are intimately linked.

From the theoretical side, we provide a theory that is able to account for the emergence of the empirical patterns described above. Our theory accounts for the timing of the phenomenon, which is absent between 1960 and 1980 and the spatial heterogeneity of it, i.e. that the dispersion of skills over time occurs at a faster pace in larger cities. We build on elements in Eeckhout et al. (2014) and Cerina et al. (2017). From the first, we borrow a multi-location environment in which three types of workers, high-, medium- and low-skilled decide where to locate to maximize utility. In doing this agents consider both the wage they receive and the price of housing in the specific location. In addition, agents consume a tradable good that is produced in all locations, and by its nature follows the law of one price at the economy level. Utility equalization by skill type determines the allocation of workers across locations and so it allows to construct a model based measure of skills that is used to construct the skill distributions by location in the data. We extend this setting by following Cerina et al. (2017) and introducing a home/market labor time decision, and a multisector environment, in which each agent consumes, in addition to housing and the tradable good, services produced at home and services produced in the market. These extensions become crucial to link the rise in the share of high-skilled workers to that of the low-skilled within the same city. As SBTC occurs in a location, it attracts high-skilled workers by raising, *ceteris paribus*, their wages. This implies also that the opportunity cost of working at home rises for high-skilled workers, who react by reducing the amount of home produced services, and increase the demand for services produced in the market. As low-skilled services are typically produced by low-skilled workers[2], the model generates a correlation between employment shares of high- and low-skilled workers, and a decline in the employment shares of middle-skilled workers. Thus, once this mechanism is introduced in a general equilibrium model with different locations and mobility of workers, it has the potential to explain spatial sorting because: i) SBTC emerges in the middle/late 1970s in the U.S. (Acemoglu and Autor (2011) and Heathcote et al. (2010)); and ii) Baum-Snow and Pavan (2013) document that the education wage premium increased faster in larger relative to smaller metropolitan areas in the U.S from 1980 onward. Coupled with the observation that larger cities typically attract a larger fraction of high-skilled workers with respect to smaller cities, this evidence points to faster SBTC in larger cities. In a similar vein as in Eeckhout et al. (2014), we use the theory to construct the model based measure that allows us to construct the empirical skill distributions described above.

We then quantitatively assess the ability of the model to replicate our empirical findings. We consider a version of the model with two locations and focus on two equilibria, calibrated to the years 1980 and 2008. The data counterparts of the two locations correspond to the sets of cities with population above and below the median of the distribution of city size. The only difference between the two equilibria is the level of skill-biased technological change, which grows in both cities over time but at a faster pace in city 2. The calibration imposes that aggregate employment polarization is matched, while the spatial sorting of workers, and so employment polarization at the city level is an endogenous outcome of the equilibrium. The model can account for employment polarization in the large city, although the magnitude of the change in high- and medium-skilled is larger than in the data. In the small city, instead, the model accounts for a similar increase of the low-skilled as in the data, but produces a counterfactual increase of middle-skilled and decrease of high-skilled.

The reminder of the paper is as follows. In section 2 we present the empirical evidence on the emergence of fat tails over time, and on employment polarization by metropolitan areas; in section 3 we present the model and in section 4 we discuss the quantitative analysis. Finally, section 5 concludes.

2 Empirical Evidence

In this section we present evidence on spatial sorting and employment polarization across city size and time.

[2] Almost 50% of employment in low-skilled services is represented by low-skilled occupations.

2.1 Spatial Sorting

In this subsection we investigate how the spatial sorting of workers with heterogeneous skills changes across time (between 1980 and 2008) and space (large and small cities). The measures we present are model based, and we referthe reader to Section 3 for details of the theory. For the time being it is sufficient to note that, as in Eeckhout et al. (2014), workers spatial allocation in equilibrium is such that two workers of the same skill type in two different cities share the same level of utility. Thus, using equilibrium conditions it is possible to derive a price-theoretic measure of skills, which maps wage and price levels into a well defined skill level. In this way it is possible to reconstruct the entire skill distribution in each city (or groups of cities) by using prices and wages observed in the data. We first discuss the wage and skill distributions for different city size and year. Second, we run quantile regressions to provide a quantitative assessment on the change in the shapes of the wage and skill distributions across time and space. Details of the data used can be found in the Data Appendix.

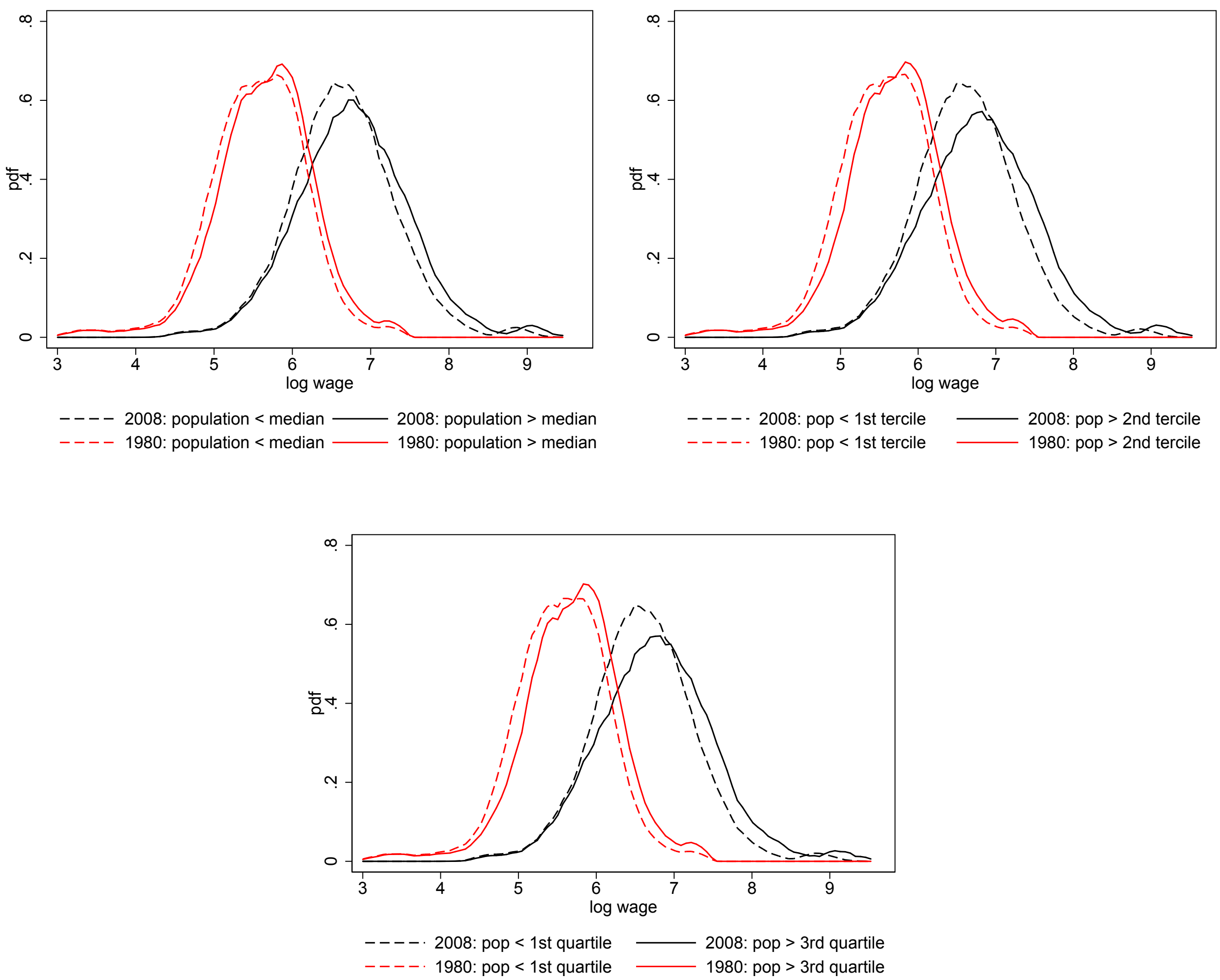

Figure 2: Wage distribution in 1980 (red) and 2008 (black) in small (dashed lines) and large (thick lines) cities. The Left panel compares metropolitan areas with population above and below the median in the reference year while right panel compares metropolitan areas with population above the 2nd tercile and below the 1st tercile in the reference year.

2.1.1 Wage and skill distributions across time and space

Figure 2 shows the wage distribution across time and space. The three panels split cities into two groups. The first one groups cities into those above the median city size and those below. The second panel considers the group of cities below the first tercile and that above the second tercile while the third panel compares the group below the first quartile and above third quartile. Consistent with previous literature, we find a city-size wage premium both in 1980 and in 2008. Average wages are higher and there is a first-order stochastic dominance of the wage distribution in large cities relative to that of small ones. That is, for each wage level x, the fraction of people earning a wage lower than x is larger in small cities than in large cities. In addition, we observe a divergence in the shape of skill distributions overtime. In 1980 the wage distribution of large cities appears to have the same shape as that of small cities. In 2008 instead, the tails of the distribution are fatter in large cities than in small ones. The result emerges in the three panels of Figure 2, but the difference is more pronounced when considering quartiles with respect to terciles, or terciles with respect to the median split, which suggests that the divergence between small and large cities is increasing with cities relative size.

Figure 3 reports the corresponding evidence for the skill distribution. Using a similar model-based measure of skills, Eeckhout et al. (2014) find that in 2000 the average and the median worker have the same level of skill in large and small cities but, crucially, the skill distribution in larger cities has fatter tails both at the top and at the bottom of the distribution. We find a similar result for 2008: the first panel of Figure 3 shows that in 2008 cities with population above the median (black thick line) display fatter tails with respect to cities with population below the median (black dashed line). The middle panel shows that the divergence in skill distribution between large and small cities is increasing in relative size: the difference in the tails' mass between cities with population above the 2nd tercile and cities with population below the 1st tercile is substantially larger than the same difference computed for the groups of cities with population above and below the median. By considering cities with population above the 3rd quartile and cities with population below the 1st quartile the divergence in tails is even more pronounced.

While the observations for 2008 appear consistent with the results for 2000 shown in Eeckhout et al. (2014), the evidence for 1980 is substantially different. In 1980 (red lines) the skill distributions of large and small cities are remarkably similar and almost overlap. Thus, there is no evidence of fat tails in larger cities, either by comparing cities with population above and below the median, above the second and below the first tercile, and above the third and below the first quartile. If anything, there is a slight first-order stochastic dominance of cities with population above the third quartile over those below the first quartile, and above the second tercile over those below the first tercile, while the skill distribution of cities with size above and below the median are virtually identical.

These results suggest that the emergence of fat tails in the skill distribution of large cities is a phenomenon which emerged in the last decades. This is confirmed by the analysis of the skill distribution in 1960. In Figure 4 we document for 1960 a similar picture as for 1980: the skill distribution is similar in small and large cities.[3] The larger dispersion in 1980

[3]Due to a space constraint we report here the results for the terciles grouping. However, results with the median and quartiles grouping are very similar and available upon request.

relative to 1960 is a phenomenon which is common to all cities regardless of their size. Thus, the emergence of fat tails which increase with city size should be related to changes in the economic structure that occurred after 1980.

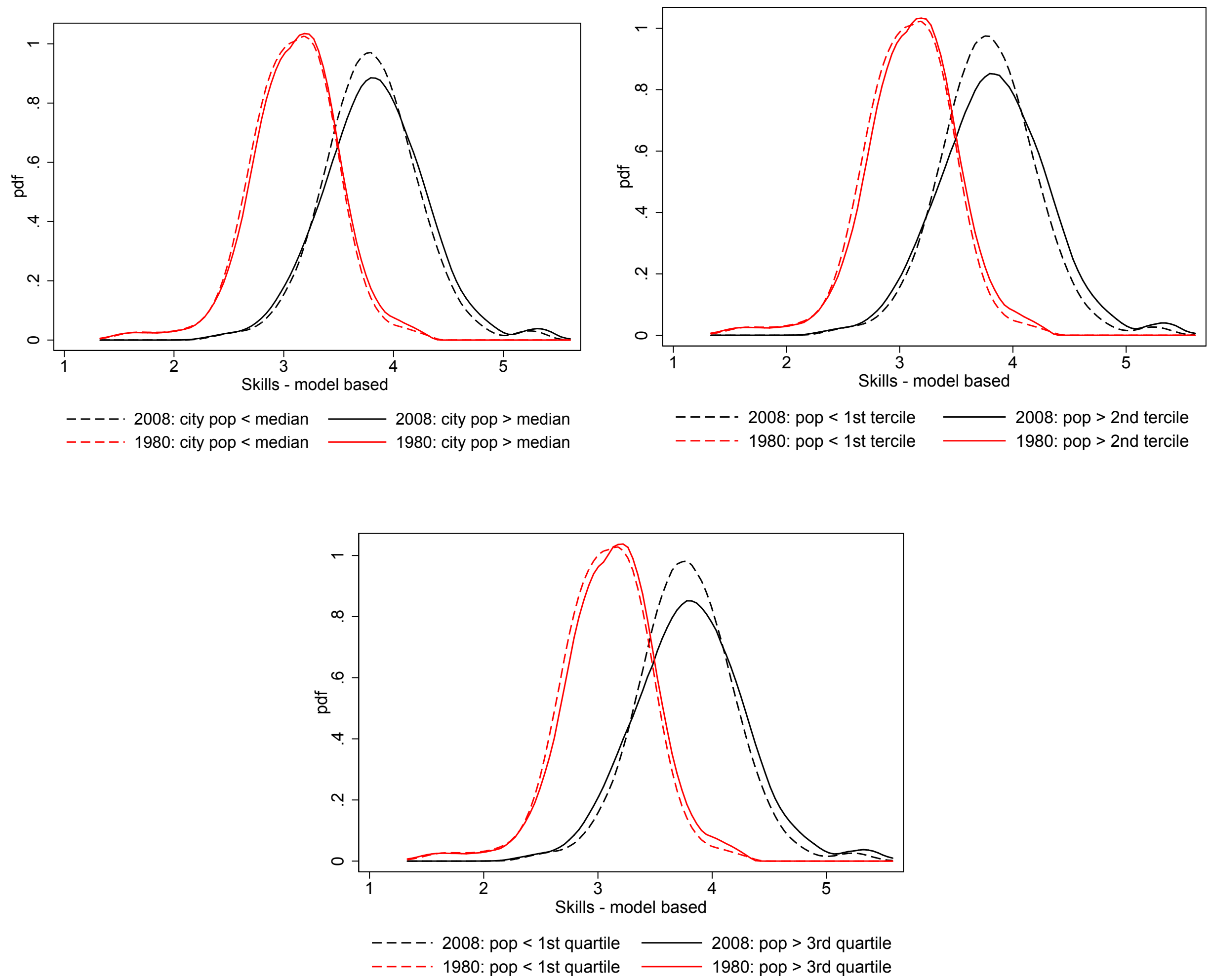

Figure 3: Skill distribution in 1980 (red) and 2008 (black) in small (dashed lines) and large (thick lines) cities. The Left panel compares metropolitan areas with population above and below the median in the reference year while right panel compares metropolitan areas with population above the 3rd quartile and below the 1st quartile in the reference year.

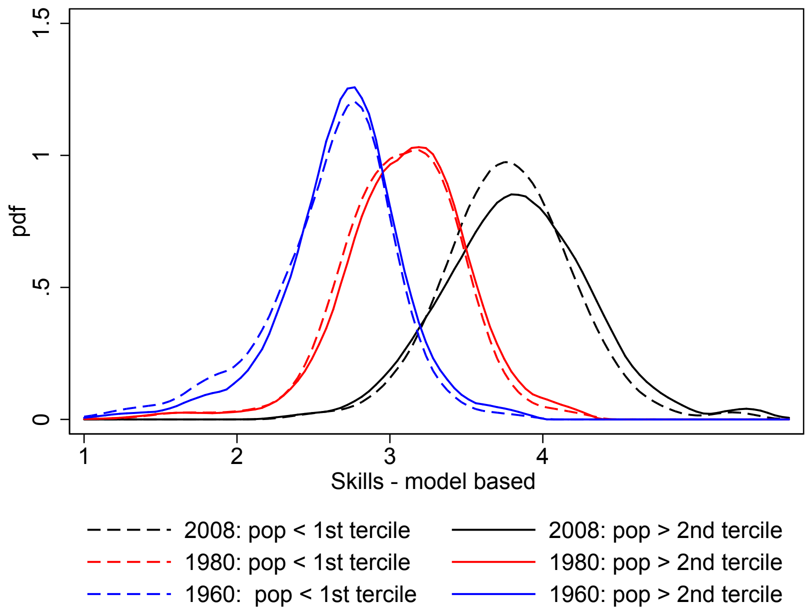

Figure 4: Skill distribution in 1960 (blue), 1980 (red) and 2008 (black) in small (dashed lines) and large (thick lines) cities. The figure compares metropolitan areas with population above the 2nd tercile and below the 1st tercile in the reference year.

2.1.2 Quantile regressions

To provide a quantitative assessment on the dynamics of the wage and the skill distribution in large and small cities we perform a set of quantile regressions. More precisely, we want to analyse how the effect of city size on both wages and skills changes at different points of the distribution. Formally, assuming a linear relation between individual wages (w_{ik}) and population (N_k) in location k, we estimate the following specification for each quantile τ:

$$Q_\tau(w_{ik}|N_k) = \alpha(\tau) + \beta(\tau)N_k,$$

where consistent estimators of $\alpha(\tau)$ and $\beta(\tau)$ are obtained by minimizing an asymmetrically weighted sum of absolute errors. We perform this exercise for both the wage and skill distribution in 1980 and 2008. Each of these four exercises is represented in a figure with two panels: on the left one we plot five quantiles of the distribution (the 10th, the 25th, the median, the 75th and the 90th) against city size, while in the right panel we plot the coefficient of each quantile against its rank. This procedure allows to observe how the effect of city size on the shape of the wage and skill distributions changes from 1980 to 2008.

Wage distribution in 1980. Figure 5 shows that in 1980 the quantiles values increase with city size (i.e. city-size wage premium). Coefficients are all positive and homogeneous along the skill distribution. This suggests that in 1980 the wage distributions shifts to the right with city size, without a change in its shape.

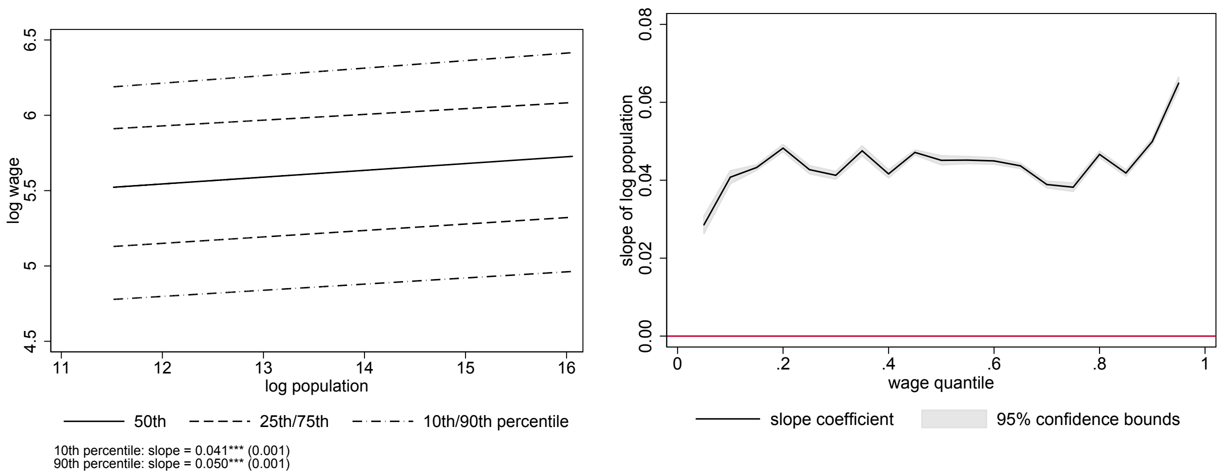

Figure 5: Quantile regression of wage on population in 1980: left, five selected quantiles; right, estimated slope for all quantiles

Wage distribution in 2008. Figure 6 shows that each quantile of the wage distribution increases with city size (left panel) except the bottom one. The whole distribution shifts to the left (city-size wage premium) so that, like for 1980, coefficients of the relationships between quantiles and city size are positive (right panel). In this case, however, the distribution is also expanding, as coefficients are increasing in quantiles (right panel). This confirms results in Eeckhout et al. (2014), who report similar coefficients for the 2000.

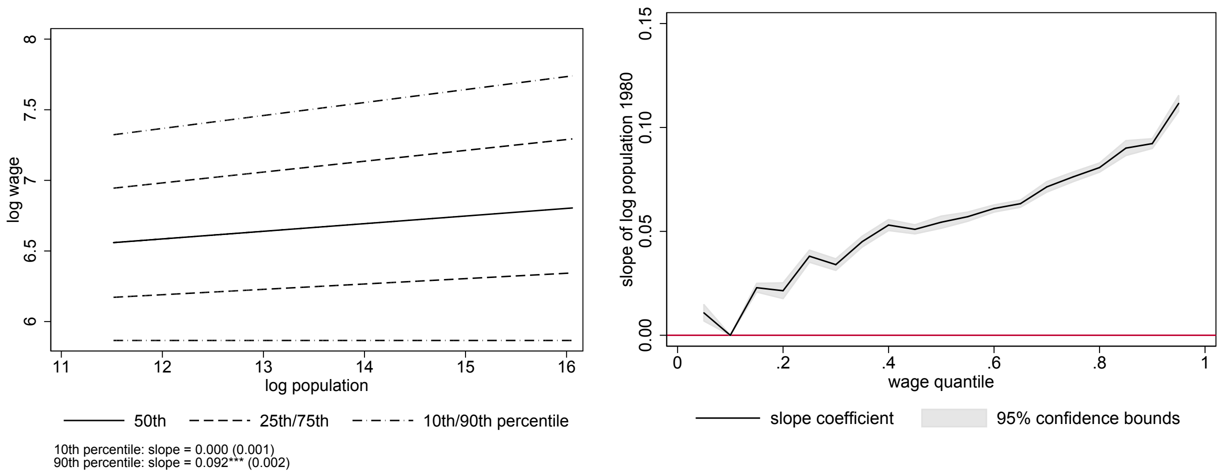

Figure 6: Quantile regression of wage 2008 on population in 1980: left, five selected quantiles; right, estimated slope for all quantiles

Skill distribution in 1980. Figure 7 reports a result for 1980 similar to that for the wage distribution. There is no divergence across city size in 1980. Coefficients of the quantile regressions are slightly positive and similar for each quantile (except the very last quantiles). So the quantile regression confirm that in 1980 there is no evidence of fatter tails for larger cities.

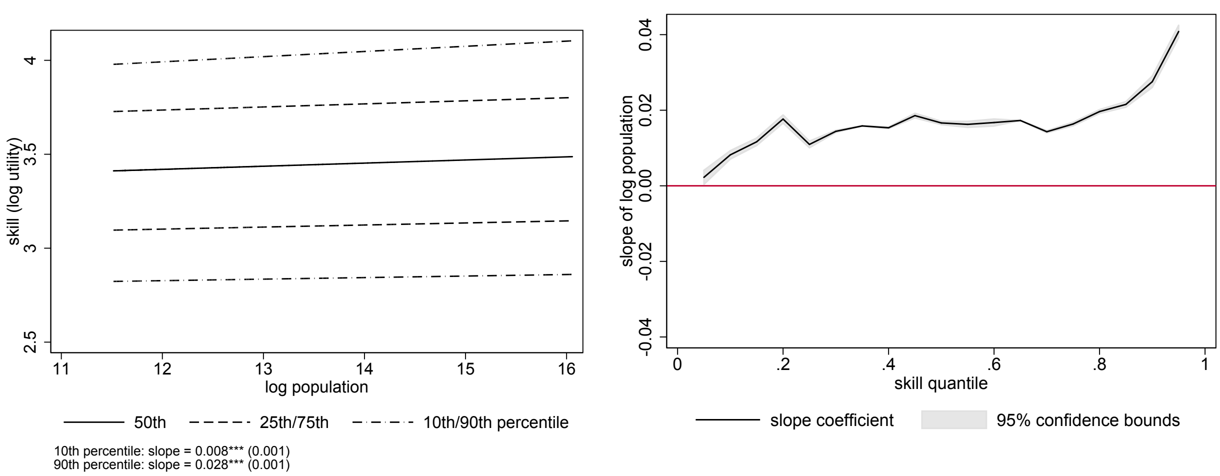

Figure 7: Quantile regression of utility on population in 1980 (i.e. model-based skill measure): left, five selected quantiles; right, estimated slope for all quantiles

Skill distribution in 2008. Figure 8 reports the results of quantile regressions for the skill distribution in 2008. The right panel shows that slopes are increasing with the quantile rank, being negative up to the 30th percentile and positive otherwise. This confirms results for 2008, which broadly confirms Eeckhout et al. (2014) results: lower quantiles decrease with city size while the opposite happens for higher quantiles (left panel). This represent evidence of fatter tails in the skill distribution for larger cities relative to smaller ones.

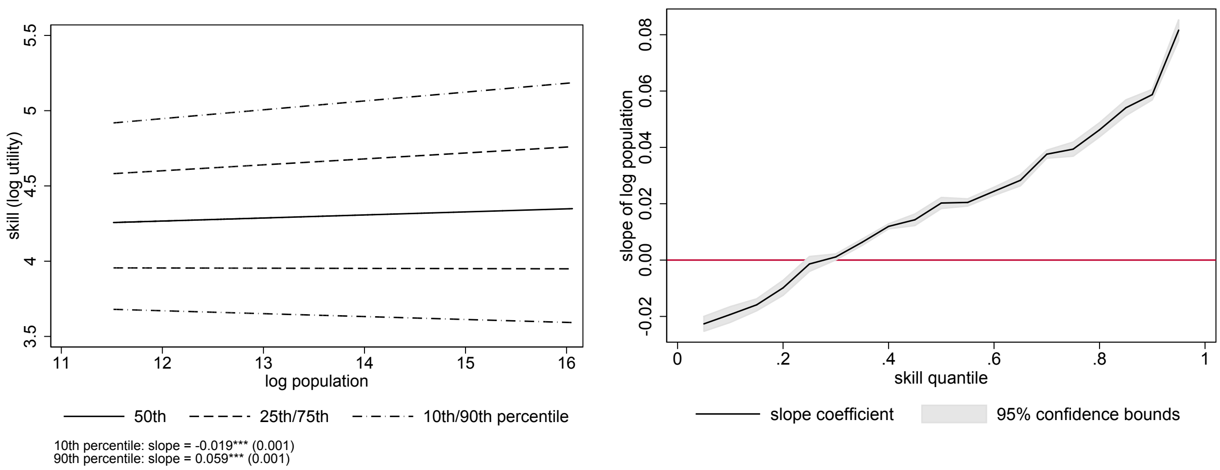

Figure 8: Quantile regression of utility in 2008 (i.e. model-based skill measure) on population in 1980: left, five selected quantiles; right, estimated slope for all quantiles

2.2 Employment polarization and city size

In this section we provide evidence that the phenomenon of employment polarization is more pronounced in larger cities. To do this, we use Dorn (2009) occupations classification and divide occupations into three broad skill groups, according to their wage in 1980. The group of low skilled occupations is that of Services (codes 405-472) which, as documented in Autor and Dorn (2013) are responsible for the increase of employment shares at the aggregate level in the U.S. between 1980 and 2008. On the other spectrum of the skill distribution, we define as high-skilled all Managerial and Professional Specialty Occupations (codes 004-199). All remaining occupations are in the middle-skilled group (codes 203-889 except 405-472). As in the previous subsection we consider three different groupings for city size: i) cities above the median city size and those below, ii) cities below the first tercile and that above the second tercile of city size and iii) cities below the first quartile and above third quartile of city size. Results are reported in 1. Consider first the *Median* grouping. In this case the increase of employment shares of low-skilled occupations and that of high-skilled ones between 1980 and 2008 is bigger in large cities than in small ones (3.05% versus 2.92% and 9.38% versus 8.34% respectively). For middle-skilled occupations, we have that the decline

Median			Share 1980		Change 2008		Growth Rate	
Group	**Occs**	**Wage 80**	**Small**	**Large**	**Small**	**Large**	**Small**	**Large**
Low-skill	33	4,95	10,65%	10,48%	+2,92%	+3,05%	0.27	0.29
Medium-skill	196	6,89	63,48%	60,59%	-11,27%	-12,43%	-0.18	-0.21
High-skill	88	9,69	25,88%	28,93%	+8,34%	+9,38%	0.32	0.32

Terciles			Share 1980		Change 2008		Growth Rate	
Group	**Occs**	**Wage 80**	**Small**	**Large**	**Small**	**Large**	**Small**	**Large**
Low-skill	33	4,95	10,96%	10,50%	+2,96%	+3,52%	0.27	0.34
Medium-skill	196	6,89	64,00%	60,32%	-11,18%	-13,31%	-0.17	-0.22
High-skill	88	9,69	25,04%	29,18%	+8,23%	+9,79%	0.33	0.34

Quartiles			Share 1980		Change 2008		Growth Rate	
Group	**Occs**	**Wage 80**	**Small**	**Large**	**Small**	**Large**	**Small**	**Large**
Low-skill	33	4,95	11,29%	10,67%	+2,99%	+3,99%	0.27	0.37
Medium-skill	196	6,89	63,57%	61,30%	-10,40%	-13,26%	-0.16	-0.22
High-skill	88	9,69	25,14%	28,03%	+7,40%	+9,28	0.29	0.33

Table 1: Employment polarization for three different grouping of city size. In the *Median* grouping Small refers to the group of cities below the median of the distribution of size and Large to those above the median. In the *Terciles* grouping Small refers to the group of cities below the first tercile and Large above the third tercile. A similar categorization applies to the grouping *Quartiles.*

of employment shares is bigger in large cities than in small ones. Note that, although it is not a measure commonly used to report employment polarization, we also report, in the last two columns the percentage change in employment shares, that is the difference between the two periods divided by the initial value of the share. Also in this case large cities display a more pronounced employment polarization, although the increase of employment shares of high skilled is similar in the two groups of cities.

As in the case of spatial sorting, the divergence between small and large cities increases when considering the extreme terciles and quartiles of the distribution of city size. In the case of terciles, we have a percentage increase of employment shares of low-skilled occupations of 27% in small cities compared to a 34% in large ones. For middle-skilled occupations we have -22% of large cities versus -17% of small ones and for high-skilled occupations we have 34% of large cities versus 33% of small ones. In the case of quartiles, the percentage increase of employment shares of low-skilled occupations is 27% in small cities compared to a 37% in large ones. In middle-skilled occupations we have -22% of large cities versus -16% of small ones and for high-skilled occupations we have 33% of large cities versus 29% of small ones.

The results for broad occupation categories confirm the existence of employment polarization at the aggregate level, but suggest the the phenomenon is more pronounced in large cities than in small ones. To provide further evidence on this distinction by city size, we compute, for each group of city (i.e. small and large) the change in the employment share of each occupation between 1980 and 2008, and then sort all occupations by mean wage in 1980.[4] With this information we fit a quadratic relationship for the group of large cities and one for that of small cities. Results are reported in Figure 9. Consistent with the analysis above, we report the results for the city split according to the median, terciles and quartiles. As for broad occupation categories, employment polarization is more pronounced in large cities than in small ones.

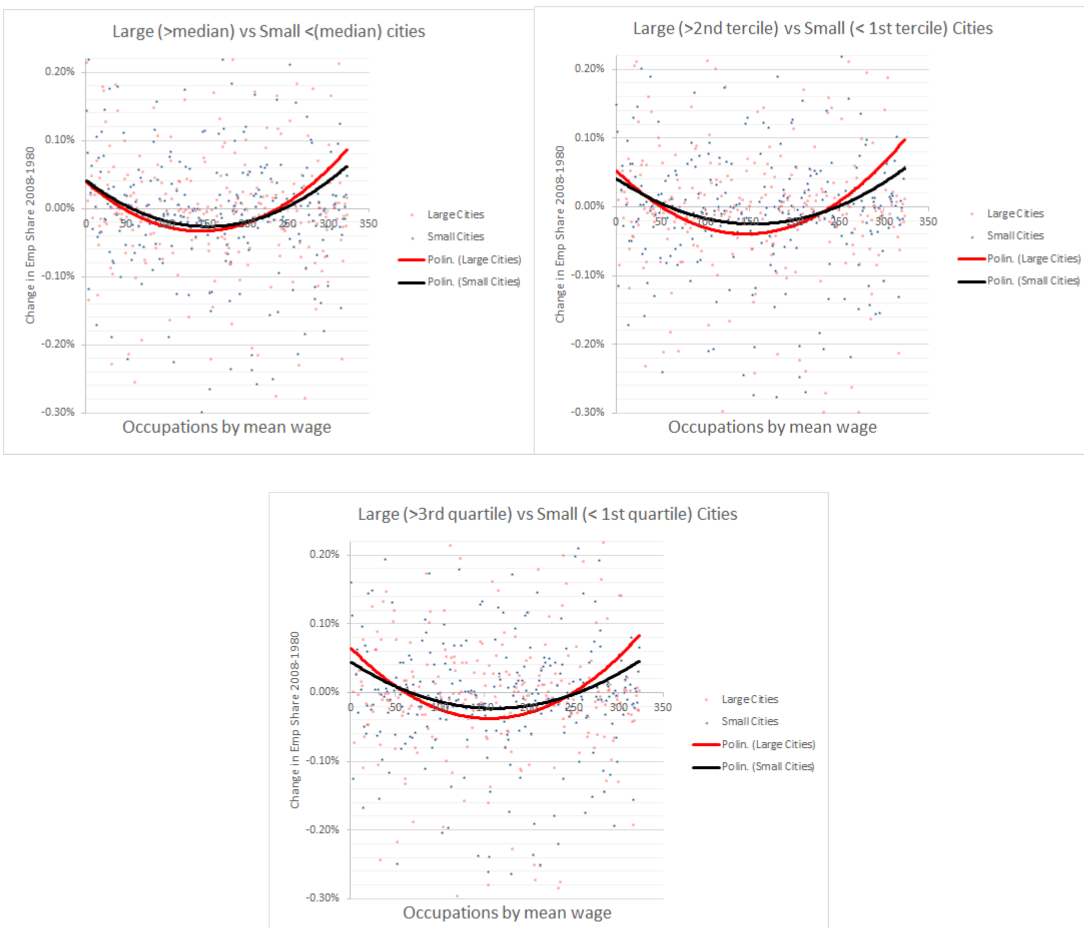

Figure 9: Employment polarization by city size. The Left panel compares metropolitan areas with population above and below the median in the reference year while right panel compares metropolitan areas with population above the 2nd tercile and below the 1st tercile in the reference year.

[4] We use Autor and Dorn (2013) ranking of occupations.

3 Theoretical framework

In this section we develop a general equilibrium model that allows to jointly study the spatial sorting of workers with heterogeneous skills and employment polarization. In the model, the main source of technological progress is of the skill-biased type, and different locations can display a different amount of it. Workers make a location decision based on their skill level, the amount of skill-biased technological change across cities, and the congestion costs, represented by a fixed amount of housing in each location. In equilibrium, the utility of two workers with the same skill level but living in two different cities is equalized.

3.1 The environment

The economy consists of K locations (cities) indexed by $k \in (1, 2, ...K)$. In each location there is a fixed amount of housing H^k whose unit-price is location-specific and defined by p_H^k. As in Eeckhout et al. (2014) we think of the expenditure on housing as the flow value that compensates for the depreciation and interest on capital. In a competitive rental market, the flow payment equals the rental price. To highlight the main mechanisms at work we restrict the number of cities to $K = 2$. For the same reason, we assume that both locations have an equal amount of land, $H^k = H$ for $k = 1, 2$.

Both cities are populated by workers with heterogeneous skills indexed by $i \in (1, 2....I)$ and associated with this skill order is a level of productivity a^{ik}. We focus on the case of three skills, $i = h,\ m,\ l$. At the economy wide level, there is a fixed amount of workers for each skill N^i for $i = h,\ m,\ l$.

There are two market sectors producing goods $j = g,\ s$. The good, g, broadly interpreted as manufacturing, is tradable across location while the second, s, interpreted as services, is non-tradable and can only be consumed in the same location where it is produced. Also, there exists a non-marketable service h which is produced within the household and interpreted as home production.

By n_j^{ik} we define the number of workers of skill i working in sector $j = g,\ s$ in location k. Hence $S_k = \sum_i^I n^{ik} = \sum_i^I \sum_i^J n_j^{ik}$ is the population size of city k. Similarly to Eeckhout et al. (2014) workers of each skill move towards the city where their utility is higher so that the size of city k is an endogenous equilibrium outcome pinned down by the equalization of utilities across cities for the same skill. Total population of the economy is then exogenously given by $S = \sum_k^K S^k = \sum_k^K \sum_i^I n^{ik}$.

3.2 Demand

Citizens of skill type i who live in city k have preferences over consumption of the tradable good c_g^{ik}, the amount of housing H^{ik} and consumption of services c_n^{ik}. We assume the latter is a CES bundle of home home services c_h and market services c_s, which are assumed to be imperfect substitutes with elasticity of substitution equal to $\gamma > 1$. More precisely, a worker of skill i living in city k has the following preferences

$$\begin{aligned} U^{ik} &= \left(H^{ik}\right)^{\alpha}\left(c_g^{ik}\right)^{\omega}\left(c_n^{ik}\right)^{1-\omega-\alpha} \\ c_n^{ik} &= \left(\psi\left(c_s^{ik}\right)^{\frac{\gamma-1}{\gamma}}+(1-\psi)\left(c_h^{ik}\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{1}$$

where c_j, with $j = g,\ n,\ s,\ h$, represents consumption of goods, services, market services and home services, respectively. We impose $\alpha + \omega < 1$ and $\psi \in (0, 1)$.

Home services are produced within the household according to the technology

$$c_h^{ik} = A_h l^{ik} \tag{2}$$

where $l^{ik} \in (0, 1)$ is the fraction of time an agent of skill i in city k devotes to work at home, thus being $1 - l^{ik}$ the fraction of time dedicated to work in the firm. We assume that home productivity is invariant across skills and locations, as allowing for heterogeneity in home productivity. The budget constraint for workers of ability i living in city k is

$$p_g c_g^{ik} + p_s^k c_s^{ik} + p_H^k H^{ik} = w^{ik}(1 - l^{ik}) \tag{3}$$

where p_s^k and p_H^k are, respectively, the price of market services and housing, which are both location-specific and therefore indexed by k. Instead, the price of the tradable good, p_g, is equal in the whole economy. In what follows, we choose good g as the numeraire and, therefore, we set $p_g = 1$. We also assume workers are perfectly mobile across sectors so that, in a given location and for a given skill i, the wage rate is equal across sectors and therefore $w_g^{ik} = w_s^{ik} = w^{ik}$ holds. Workers of skill i living in city k solve the following problem

$$\begin{aligned} \max_{c_g^{ik},c_s^{ik},c_h^{ik},l^{ik}} U^{ik} &= \left(H^{ik}\right)^{\alpha}\left(c^{ik}\right)^{\omega}\left(\left(\psi\left(c_s^{ik}\right)^{\frac{\gamma-1}{\gamma}}+(1-\psi)\left(c_h^{ik}\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right)^{1-\omega-\alpha} \\ s.t. &: \ c_g^{ik} + p_s^k c_s^{ik} + p_H^k H^{ik} = w^{ik}(1 - l^{ik}) \\ c_h^{ik} &= A_h l^{ik} \end{aligned}$$

Which leads to the following demand functions

$$
\begin{aligned}
l^{ik}\left(\frac{w^{ik}}{\underset{-}{p_s^k}}\right) &= \frac{1-\omega-\alpha}{1+\left(\frac{w^{ik}}{A_h p_s^k}\right)^{\gamma-1}\left(\frac{\psi}{1-\psi}\right)^{\gamma}} && (4)\\
c_h^{ik}\left(\frac{w^{ik}}{\underset{-}{p_s^k}}\right) &= A_h\frac{1-\omega-\alpha}{1+\left(\frac{w^{ik}}{A_h^k p_s^k}\right)^{\gamma-1}\left(\frac{\psi}{1-\psi}\right)^{\gamma}} && (5)\\
c_s\left(\frac{w^{ik}}{\underset{+}{p_s^k}}\right) &= A_h\frac{(1-\omega-\alpha)\left(\frac{w^{ik}}{A_h^k p_s^k}\frac{\psi}{1-\psi}\right)^{\gamma}}{1+\left(\frac{w^{ik}}{A_h^k p_s^k}\right)^{\gamma-1}\left(\frac{\psi}{1-\psi}\right)^{\gamma}} && (6)\\
c_g\left(\underset{+}{w^{ik}}\right) &= \omega w^{ik} && (7)\\
H\left(\frac{w^{ik}}{p_H^k}\right) &= \frac{\alpha w^{ik}}{p_H^k} && (8)
\end{aligned}
$$

Labor supply at home is a negative function of $\frac{w^{ik}}{A_h p_s^k}$, which can be interpreted as the relative price between home services and market services: in cities in which wages are higher relative to the market price of services, a worker will devote less time to home production and, as (6) shows, she/he will increase the demand of market services. This element is the main source of demand spillovers which generate fat tails and employment polarization in the model.

3.3 Production

3.3.1 The tradable sector

There is a representative firmn each location which employs three kinds of labor, h, m and l. The production function of the representative firm in city k in the g sector is

$$Y_g^k = A_g^k F\left(e_g^{hk}, e_g^{mk}, e_g^{lk}\right)$$

where e_g^i is the amount of hours worked by a worker of skill i. In equilibrium, this amount of time is the product of an intensive margin - the individual labor supply $1-l^{ik}$, and an extensive margin - the number of workers employed by the firm, n_g^{ik}. Since the individual labor supply is chosen by the individual worker who maximizes utility, the equilibrium number of workers of each skill employed by the firm is then pinned-down by the relationship $n_g^{ik} = e_g^{ik}/1-l^{ik}$. A_g^k is the location-specific TFP in the tradable sector. The production function of the representative firm has the following functional form:

$$Y_g^k = A_g^k \left[\left(\phi^h \left(a^{hk} e_g^{hk} \right)^\eta + \phi^l \left(a^{lk} e_g^{lk} \right)^\eta \right)^\lambda + \phi^m \left(a^{mk} e_g^{mk} \right)^\eta \right].$$

We assume $\eta < 1$ so that there are decreasing returns to scale.[5] We also assume that the firm is owned by absentee capitalists, such that the profits of the firm do not enter the budget constraint of the workers. As in Eeckhout et al. (2014), we allow $\lambda > 0$ to be potentially different from one. With $\lambda > 1$ there is extreme-skill complementarity and when $\lambda < 1$ there is extreme-skill substitutability. The parameters ϕ^h, ϕ^l and ϕ^m are weights common across cities. Instead the a's are city specific productivities of the three skill types. We interpret changes in a^{hk} as skill-biased technological change.Without loss of generality we normalize $a^{lk} = 1$.

The representative firm solves the following problem

$$max_{\{e_g^{hk}, e_g^{mk}, e_g^{lk}\}} \pi^k = Y_g^k - w^{hk} e_g^{hk} - w^{mk} e_g^{mk} - w^{lk} e_g^{lk}$$

where w^{ik} is wage per unit of time worked by a worker of skill i in location k. Note that, despite workers' perfect spatial mobility, wages are not equalized across cities because workers decide their location according to their utility, which depends both on wages and on local prices of housing and services. Also, note that wages are not indexed by sector because workers are also mobile across sectors and therefore wages of the same type of workers are equalized.

3.3.2 The non-tradable service sector

The representative firm in the non-tradable service sector operates with the following production function

$$Y_s^k = A_s^k e_s^{lk}$$

Profit maximization implies equality between prices and marginal costs.

$$p_s^k = \frac{w^{lk}}{A_s^k} \tag{9}$$

We assume that only low-skilled workers are employed in the services sector as in the data the share of this type of workers (i.e. individuals employed in service occupations, as defined in section 2) in the non-tradable service sectors (47,8% in 1980 and 48,6% in 2008) is

[5] It is possible to show that with constant returns to scale the spatial equilibrium would be indeterminate.

substantially larger than in the overall economy (10,6% in 1980 and 13,5% in 2008)[6]. Also, conditional of being employed in a service occupation, the probability of working in the non-tradable sector is substantially larger (33,8% in 1980 and 37,8% in 2008) than the same probability computed for the overall economy (7,4% in 1980 and 10,5% in 2008). These data suggests the intimate link between low-skill workers and the non-tradable sector. .

3.4 Equilibrium

3.4.1 Spatial mobility of workers

Workers of each skill can choose at zero cost the location which ensures higher utility. By using (4), (5), (6), (7) and (8) into (1), the indirect utility for a worker of skill i in city k is given by

$$U^{ik} = \Omega \left(p_H^k\right)^{-\alpha} \left(w^{ik}\right)^{\alpha+\omega} \left(1 + \left(\frac{\psi}{1-\psi}\right)^{\gamma} \left(\frac{A_s^k}{A_h} \frac{w^{ik}}{w^{lk}}\right)^{\gamma-1}\right)^{\frac{1-\omega-\alpha}{\gamma-1}} \tag{10}$$

where

$$\Omega = \alpha^{\alpha}\omega^{\omega} \left(1-\omega-\alpha\right)^{(1-\omega-\alpha)} \left(1-\psi\right)^{\frac{\gamma(1-\omega-\alpha)}{\gamma-1}} \left(A_h\right)^{(1-\omega-\alpha)}$$

The assumption of workers mobility ensures that utility of two workers of the same type is the same across locations ($U^{i1} = U^{i2}$). Thus, there is one-to-one mapping between equilibrium utility and skill level for the worker of type i in city k. Thus, as in Eeckhout et al. (2014), we interpret (10) as the measure of skill implied by the model and we use it to construct the model-based distribution of skills presented in the previous sections. Note that if $\alpha + \omega = 1$ our setting coincides with that of Eeckhout et al. (2014), in which there is no home production and no market production of services.

3.4.2 Market clearing

There is labor markets clearing at the city level

$$\left(n_s^{lk} + n_g^{lk}\right) = N^{lk} \tag{11}$$

$$n_g^{mk} = N^{mk} \tag{12}$$

$$n_g^{hk} = N^{hk} \tag{13}$$

where n_j^{ik} is the number of workers of skill $i = h\ , m,\ l$ employed in sector $j = g,\ s$ in location $k = 1,\ 2$ and N^{ik} is the (exogenous) supply of workers of skill $i = h,\ m,\ l$ in city $k = 1,\ 2$.

[6] In the quantitative analysis below the list of sectors included in low-skilled services is the same as in Moro et al. (2017). See the appendix for details.

Clearing markets at the city level imply clearing markets at the economy wide level as

$$N^{l1} + N^{l2} = N^{l} \tag{14}$$

$$N^{m1} + N^{m2} = N^{m} \tag{15}$$

$$N^{h1} + N^{h2} = N^{h} \tag{16}$$

and N^i is the (exogenous) supply of workers of skill $i = h,\ m,\ l$ in the economy.

In equilibrium, the amount e_j^{ik} of labor time units required by firms is equal, in each sector $j = g,\ s$, in each location $k = 1, 2$ and for each skill $i = h,\ m,\ l$, to the amount of labor units $\left(1 - l^{ik}\right)$ supplied by each worker of skill i, living in location k and employed in sector j (i.e. the intensive margin) times the number of workers n_j^{ik} of that type (i.e. the extensive margin)

$$e_j^{ik} = \left(1 - l^{ik}\right) n_j^{ik}. \tag{17}$$

The housing market clears in each city, so the following holds for $k = 1, 2$:

$$(n_g^{lk} + n_g^{lk})H^{lk} + n^{mk}H^{mk} + n^{hk}H^{hk} = H^k \tag{18}$$

where H^{ik} is given by (8) and H^k is the exogenous amount of local land which as in Eeckhout et al. (2014) we assume to be equalized across cities and equal to one. The market for services also clears in each city so, for $k = 1,\ 2$

$$(n_g^{lk} + n_g^{lk})c_s^{lk} + n^{mk}c_s^{mk} + n^{hk}c_s^{hk} = A_s^k e_s^{lk} \tag{19}$$

Finally, by Walras law, the market for tradable goods must also clear in equilibrium at the economy wide level. The equilibrium can then be defined as follows.

Definition 1. An equilibrium for this economy is:

- a vector of prices and wages $\left(p_s^1, p_s^2, p_H^1, p_H^2, w^{l1}, w^{l2}, w^{m1}, w^{m2}, w^{h1}, w^{h2}\right)$;
- a vector of labor units employed in market $\left(e_g^{l1}, e_g^{l2}, e_g^{m1}, e_g^{m2}, e_g^{h1}, e_g^{h2}, e_s^{l1}, e_s^{l2}\right)$;
- a vector of market time allocations for each type of worker $\left(l^{l1}, l^{l2}, l^{m1}, l^{m2}, l^{h1}, l^{h2}\right)$;
- a vector of workers employed in the g and in the s sector $\left(n_g^{l1}, n_g^{l2}, n_g^{m1}, n_g^{m2}, n_g^{h1}, n_g^{h2}, n_s^{l1}, n_s^{l2}\right)$;
- a consumption vector for goods $\left(c_g^{l1}, c_g^{l2}, c_g^{m1}, c_g^{m2}, c_g^{h1}, c_g^{h2}\right)$;
- a consumption vector for services $\left(c_s^{l1}, c_s^{l2}, c_s^{m1}, c_s^{m2}, c_s^{h1}, c_s^{h2}\right)$;

- a consumption vector for home services $\left(c_h^{l1}, c_h^{l2}, c_h^{m1}, c_h^{m2}, c_h^{h1}, c_h^{h2}\right)$;
- a consumption vector for housing $\left(H^{l1}, H^{l2}, H^{m1}, H^{m2}, H^{h1}, H^{h2}\right)$,
- a location choice $k = 1,\ 2$ for each agent,

such that:

1. given wages, prices, and location choice, agents maximize utility;
2. given wages and prices, firms maximize their profits;
3. there is market clearing for housing and non-tradable services in each city;
4. there is market clearing in the labor market in each city;for each skill level, indirect utility is equalized across locations.

4 Quantitative analysis

4.1 Calibration

A number of parameters, $\{\alpha,\ \omega,\ \gamma\}$, are set from previous studies based on empirical evidence. Following the discussion in Ngai and Pissarides (2008) and Moro et al. (2017) we set the elasticity of substitution between home production and substitutable services to $\gamma = 2.3$. Given the values of α and ω calibrated in Eeckhout et al. (2014), we re-scale them to take into account that we also have services in the utility function. This procedure gives a value of ω equal to 0.45, and of α equal to 0.18. Productivities by sector, including the home sector, are normalized to one and do not vary over time, $A_{j,t} = 1$ and $A_{h,t} = 1$. The relative supply of skills (i.e. the aggregate skill distribution) in 1980 and 2008 is taken from the US Census data. The definition of low-, middle- and high-skilled is the same as in section 2.2. Lastly, producitivities of workers in small cities in 1980 are also normalized, $\{a_{1980}^{j1}\}_{j=m,h} = 1$.

The remaining 9 parameters: (1) weights in production and preferences $\{\phi^m, \phi^h, \psi\}$, (2) market productivity $\{\{a_{1980}^{j2}\}_{j=m,h},\ \{a_{2008}^{hk}\}_{k=1,2}\}$, and (3) production parameters $\{\eta,\ \lambda\}$ are calibrated to match a number of moments.[7] While the calibration procedure matches all 9 parameters to 12 moments concurrently, by minimizing the distance between data targets and model moments, some targets are more informative for certain parameters than others. Below we outline the general strategy:

- Weight on market purchased services, $\{\psi\}$ (1 target): aggregate home hours in 1980.

[7] Note that $\phi^l = 1 - \phi^m - \phi^h$.

Table 2: Calibrated Parameters

Preferences						Technology					
α	ω	γ	ψ	η	λ	a^{h1}_{2008}	a^{m2}_{1980}	a^{h2}_{1980}	a^{h2}_{2008}	ϕ^{m}	ϕ^{h}
0.18	0.45	2.3	0.19	0.77	1.07	1.0	1.0	1.04	1.63	0.36	0.47

- Production parameters, $\{\phi^{jk},\ a^{h2}_{1980}, \eta,\ \lambda\}$ (7 targets): low-skilled tradable share by city in 1980, middle- and high-skilled tradable share in small cities in 1980, aggregate wage premia of high- and middle-skilled relative to low-skilled in 1980, relative large to small city size in 1980.

- Skill-biased technical change $\{a^{hk}_{2008}\}_{k=1,2}$ (3 targets): aggregate wage premia of the high- and middle-skilled relative to low-skilled and relative large to small city size in 2008.

All targets are computed using the 1980 Census and the 2008 American Community Survey unless noted. Table 2 reports the parameter values.

4.2 Results

The calibration described in the previous section pins down parameter values by using *aggregate* targets for the year 2008. Given the difference across cities in skill-biased technological change, which is faster in city 2, the model endogenously produces heterogeneous changes across cities over time. In this section we report such performance of the model, and use the data to assess the quantitative properties of our theory. In comparing the two equilibria, we focus on the (change in the) share of hours worked by each skill group of workers in both locations: $\frac{e^{ik}}{\sum_i e^{ik}} = \frac{n^{ik}\left(1-l^{ik}\right)}{\sum_i n^{ik}\left(1-l^{ik}\right)}$ (i.e. employment polarization). Table 3 reports the results for the two cities and the aggregate economy. The table also reports the corresponding figures for the data.

By calibration, the model reproduces the aggregate pattern of employment polarization observed in the data, with an increase in employment shares at the bottom and at the top of the skill distribution. Thus, as in Cerina et al. (2017), skill-biased technological change alone is able to generate a u-shape of changes in employment shares along the skill distribution in the economy. We now turn to the effect of skill-biased technological change on the spatial equilibrium. As in the data, the model produces employment polarization in the large city (city 2), the one experiencing a faster pace of skill-biased technological change. Quantitatively, while the increase in low-skilled is similar to that in the data (+3.14% vs +3.05%), the increase in high-skilled (+22.67% vs +9.37%) and the decrease in middle-skilled

Table 3: Spatial Polarization

		City 2 (large)			City 1 (small)			Whole Economy		
		$\frac{e^{lk}}{\sum_i e^{ik}}$	$\frac{e^{mk}}{\sum_i e^{ik}}$	$\frac{e^{hk}}{\sum_i e^{ik}}$	$\frac{e^{lk}}{\sum_i e^{ik}}$	$\frac{e^{mk}}{\sum_i e^{ik}}$	$\frac{e^{hk}}{\sum_i e^{ik}}$	$\frac{e^{lk}}{\sum_i e^{ik}}$	$\frac{e^{mk}}{\sum_i e^{ik}}$	$\frac{e^{hk}}{\sum_i e^{ik}}$
Model	1980	10.51%	57.18%	32.31%	10.37%	66.71%	22.93%	10.44%	61.87%	27.70%
	2008	13.65%	31.37%	54.98%	12.87%	71.97%	15.17%	13.27%	51.34%	35.40%
Data	1980	10.48%	60.59%	28.93%	10.65%	63.48%	25.88%	10.56%	62.01%	27.43%
	2008	13.53%	48.16%	38.30%	13.57%	52.21%	34.22%	13.55%	50.15%	36.30%

(-25.81% vs -12.43%) are both larger than in the data. In the small city (city 1), in contrast with the data, the model does not produce employment polarization. As shown in section 2.2 employment polarization is present both in large and small cities, although it is more pronounced in the former. The model instead produces an increase of employment shares both at the bottom and the middle of the skill distribution, with a decline a the top.

5 Conclusion

[TBD]

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Data Appendix

This section briefly discusses the data used in this paper and especially how we document the evolution of wage and skill distributions over time and accross locations. One important challenge is to deal with comparability issues, knowing that spatial boundaries of geographical statistical areas have changed over time.

Individual data

To construct information about workers of different skills and show empirical evidence of job polarization, we use the national 5 percent public-use micro data samples for the, 1960, 1980 and 2008 Censuses of Population (IPUMS). We use data for all individuals who report positive wages and salary income and work at least 40 hours per week and 40 weeks per year working in services and manufacturing in contracting wage information.[8] When constructing employment figures we also include all individuals with at least 1 hour of work per week. We drop the lowest 0.5 percent of wages as a simple way of eliminating likely misreported wages close to zero. Instead of using the IPUMS version of the 1990 Census Bureau occupational classification scheme, we chose to work with a balanced set of occupations for 1980 and 2008 used in Autor and Dorn (2013).

In addition to wages, we construct a price-theoretic measure of skills, following Eeckhout et al. (2014) but based on our model. For this, we need to compute location-specific housing price indexes by proceeding with a hedonic regression model. Indeed, if housing is modeled as a homogeneous good, in practice housing differs in many characteristics, that may affect prices. So by relating the log of rent against a number of housing characteristics (number of rooms, age and size of the structre, etc.) and with *city-specific fixed effects*, we isolate the location-specific component of housing prices that can be used to index the difference in housing values across cities and to correct our measure of skills, according to our model. Data on dwelling features comes from the American Community Survey (ACS) and are reported in the IPUMS database at the public use metropolitan area level (PUMA codes) after 2000 and at the metropolitan area level (METAREA) before 1990. Metro areas are “regions consisting of a large urban core together with surrounding communities that have a high degree of economic and social integration with the urban core”. The main issue is that variable reports a combination of metropolitan area codes (MSA, primary MSA, central city or county), which has evolved considerably over time and leads to important difficulties in matching with PUMA codes or any other harmonized classification of cities.

[8] Farmers activities and military have been excluded.

Spatial boundaries

To analyse how the patterns of the distributions differ accross city size, we need to match census micro data to metropolitan areas. As explained above, one issue is to define spatial boundaries of locations which are consistent over time and which allow us to identify a "constant" city size effect. The most common way to proceed is to use allocation factors between PUMA (or CBSA) codes in 2008 and metro areas in 1980. But this step requires special attention and some manual correction when the county composition of each metro area has changed between 1980 and 2008. For this, population data at the county level is useful in order to check the consistency of geographical composition. Once this consolidation of spatial boundaries is done, it is possible to merge individual data with population data coming from the 1960, 1980 and 2008 National Censuses. We obtain a subset of 218 metro areas, which represent 63% of the 1980 US population and 71% of the 2008 US population. To construct information about workers of different city size, we split these 218 areas into two groups "small" and "large" cities, according to median, terciles or quartiles of the population distribution in 1980.

Additional Evidence

In this section we provide some additional evidence of divergence between small and large cities overtime, based on some observable measures of skills.

Changes in the spatial distribution of educational attainments

Table 4 shows how the distribution of educational attainments evolved differently in large and small cities between 1980 and 2008. We observe that while in 1980 the relative frequencies of the three different categories considered (less than high-school, less than college, college or more) were similar across city size, in 2008 larger cities display a relative increase in both low-skilled workers (less than high school) and high-skill workers (with a college degree or more) and a relative decrease in middle-skilled workers (less than college). We also observe how the relative increase in high-skilled workers and the relative decrease in medium-skilled workers increases with more extreme definitions of large and small cities (i.e. when we compare cities belonging to the 3rd and 1st quartile). We conclude that this evidence on observable skill measure corroborates the evidence presented above.

Table 4: Overtime changes in education in large and small cities

		Group	1980	2008	Change	Ch. L-S
Median	Small	Less than HS	18,50%	5,77%	-12,73%	
		Less than College	61,84%	58,79%	-3,05%	
		College or more	19,66%	35,44%	15,78%	
	Large	Less than HS	18,60%	6,65%	-11,94%	0,79%
		Less than College	57,90%	50,01%	-7,89%	-4,84%
		College or more	23,51%	43,34%	19,83%	4,05%
Quartiles	Small	Less than HS	18,90%	6,20%	-12,70%	
		Less than College	62,45%	61,53%	-0,92%	
		College or more	18,65%	32,76%	13,61%	
	Large	Less than HS	20,39%	8,09%	-12,30%	0,40%
		Less than College	57,18%	49,26%	-7,92%	-7,01%
		College or more	22,43%	42,65%	20,22%	6,61%

Employment shares non-tradables across cities and overtime

Non-tradable sectors, namely substitutes for home services[9], employ a share of low-skilled workers (i.e. workers employed in low-skilled - service - occupations) which is about 5 times larger than the rest of the economy (47,82% vs. 10.5% in 1980 and 48,52% vs, 13,5% in 2008). An implication and a supportive evidence of our main idea would be that the employment shares of non-tradable sectors increase more in large rather than in small cities. This is exactly what we can observe in table 5: market services which are good substitute to household production increase overtime both in small and large cities but such increase is stronger in the latter. Moreover, once again, the relative increase in large cities is larger when we compare more extrem definitions of large and small cities.

[9] We identify as homes services the following industries, from the 1990 classification: Bakery products Miscellaneous, personal services, beauty shops, laundry, cleaning, and garment services, taxicab service, food stores, n.e.c., private households, child day care services, retail bakeries, nursing and personal care facilities, miscellaneous repair services, educational services, n.e.c., residential care facilities, without nursing, eating and drinking places, liquor stores and barber shops.

Table 5: Employment shares of the non-tradables across cities and overtime

		1980	2008	Change	Ch. L-S
Median	Small	7,24%	10,03%	2,79%	
	Large	7,64%	11,07%	3,43%	0,64%
Quartiles	Small	7,36%	10,19%	2,83%	
	Large	7,99%	12,08%	4,09%	1,26%

Model Appendix

[TBD]

Results Appendix

[TBD]