Dynamic fiscal limits and monetary-fiscal policy interactions[*]

Niccolò Battistini[†]   Giovanni Callegari[‡]   Luca Zavalloni[§]

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Abstract

This paper analyzes the impact of monetary policy constraints and environment on public debt sustainability through the lens of a general equilibrium model of fiscal limits *à la* Bi (2012). We find that the mere possibility of a binding ZLB has a detrimental impact on debt sustainability, as it changes the shape of the Laffer curve and moves its peak to the right. This peak entails lower tax revenues and higher interest rates, inducing a dead-weight loss in the present discounted value of future primary surpluses, compared to an unconstrained economy. Moreover, we find that debt sustainability increases with monetary policy activeness, that is, with the elasticity of the interest rate to changes in inflation and the output gap. This result is mainly due to the lower fiscal costs stemming from mitigated inflation fluctuations. On this basis, we assess the trade-off between economic stabilization and debt sustainability faced by the government depending on the monetary policy environment. In normal times, large public spending shocks may engender perverse debt dynamics and eventually cause economic contractions. At the ZLB, a muted trade-off between stabilization and sustainability instead expands the fiscal margin for such shocks, especially if coupled with a commitment to a more active monetary policy during normal times.

Keywords: fiscal limits, sovereign default, monetary-fiscal policy interactions, zero lower bound

JEL Classification: E52, E61, E63

Non-technical summary

Risk-free interest rates are the main instruments used by central banks in normal times to fulfill their objectives of preserving price stability and fostering macroeconomic performance. The interest rate setting behavior of central banks may also affect the sustainability of public debt by influencing both the current and the expected financing conditions of governments. In this way, central banks can further play a key role in determining the capacity of governments to pursue stabilization policies.

The interaction between monetary policy and debt sustainability became especially relevant in the wake of the Great Recession, when the zero lower bound (ZLB) constraint on monetary policy enhanced the role of fiscal policy as a stabilization tool. Despite its relevance in the current policy debate, the interaction between debt sustainability and the interest rate setting behavior of the central bank – as well as its propagation channels – remained relatively unexplored.

To fill this gap, our analysis focuses on the following three questions:

1. How is the interaction between monetary policy and debt sustainability altered by the possibility of a binding ZLB constraint?

2. How does the interest rate setting behavior of the central bank affect debt sustainability in normal times?

3. How do these interactions affect the capacity of public spending shocks to support economic activity, at and away from the ZLB?

In order to answer these questions, we build a dynamic stochastic general equilibrium model in which we interpret debt sustainability as the probability that the sovereign borrower will be able to service its debt obligations. Central to this class of models is the concept of *fiscal limit* distributions, defined as the present discounted value of future maximum primary surpluses conditional on the initial state of the economy and the processes governing the exogenous shocks. Compared to the existing literature, we make fiscal limit distributions endogenous to monetary policy assuming that the monetary authority sets the risk-free interest rate according to a standard Taylor rule subject to a ZLB constraint.

Regarding the first two questions, we find that the possibility of a binding ZLB has a detrimental impact on debt sustainability. Besides keeping interest rates above those prevailing in the absence of the limit, a binding ZLB entails lower tax revenues, thus inducing a dead-weight loss in the present discounted value of future primary surpluses. Moreover, we find that debt sustainability improves with monetary policy activeness, that

is, with the degree of reactiveness of the interest rate to changes in inflation and the output gap. This result is mainly due to the lower fiscal costs stemming from mitigated inflation fluctuations.

Regarding our last question, we conclude that, in normal times, large public spending shocks may engender perverse debt dynamics and eventually cause economic contractions. At the ZLB, however, a muted trade-off between stabilization and sustainability instead expands the available space for such shocks, especially if coupled with a commitment to a more active monetary policy during normal times.

1   Introduction

There is little doubt that central banks can play a key role in determining the capacity of governments to pursue stabilization policies while preserving public debt sustainability (see, for instance, Reis, 2016). Their interest rate setting behavior affects the government's capacity to service its debt both directly – through interest payments – and indirectly – through growth, inflation and, thus, tax revenues (Leeper and Leith, 2016, among others).

The interaction between monetary policy and debt sustainability became especially relevant in the wake of the Great Recession, when the zero lower bound (ZLB) constraint on the risk-free rate enhanced the role of fiscal policy as a stabilization tool (see, e.g., Coenen, Straub and Trabandt, 2012, and Fernández-Villaverde et al., 2015).[1] However, such an active role for fiscal policy rekindled concerns on the sustainability of public finances and its second round effects on financial stability and economic activity.[2] Looking forward, the perspective of a normalization of monetary policy sheds further doubts on the sustainability of the large stocks of public debt piled up in many advanced countries during the crisis (Beck and Wieland, 2017).

Despite its relevance in the current policy debate, the interaction between debt sustainability and the interest rate setting behavior of the central bank – as well as its propagation channels – remained relatively unexplored, especially from a general equilibrium point of view. In fact, the existing literature has abstracted from the direct impact of the ordinary conduct of monetary policy and its constraints on the sustainability of public debt (see, e.g., Uribe, 2006, Davig, Leeper and Walker, 2011, and Bi, Leeper and Leith, 2018).

To fill this gap, our analysis focuses on the following three questions:

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3. How do these interactions affect the capacity of public spending shocks to support economic activity, at and away from the ZLB?

In order to answer these questions, we build on the model of fiscal limits pioneered

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[1]Unconventional monetary policy measures can further extend the reach of these channels beyond the limits set by the ZLB constraint (Wu and Xia, 2016).

[2]Among many others, see Attinasi, Checherita-Westphal and Nickel (2009), Favero and Missale (2012), Fatás and Mihov (2012), Battistini, Pagano and Simonelli (2014), Carnot (2014), Cimadomo, Claeys and Poplawski-Ribeiro (2016), Ricco, Callegari and Cimadomo (2016) and Bi, Leeper and Leith (2018).

by Bi (2012). In this context, we interpret debt sustainability as the probability that the sovereign borrower will be *able* to service its debt obligations. Central to this class of models are *fiscal limit distributions*, defined as the present discounted value of future maximum primary surpluses conditional on the initial state of the economy and the processes governing the exogenous shocks. A fiscal limit distribution thus represents the probability associated with the maximum amount of debt that a government can service at a specific point in time given the possibility of external shocks.

Compared to the existing literature, our main contribution is to make the fiscal limit distributions endogenous to monetary policy in a new Keynesian DSGE model with Rotemberg pricing and distortionary taxation similar to those proposed by Bi, Leeper and Leith (2013, 2018). Differently from these studies, we derive the fiscal limit distributions assuming that the monetary authority sets the risk-free interest rate according to a standard Taylor rule subject to a ZLB constraint.[3]

In relation to the first question, we find that introducing the possibility of a binding ZLB constraint has a detrimental impact on debt sustainability. With a constrained monetary policy at the ZLB, output, inflation and, thus, tax revenues remain below the levels prevailing in the unconstrained case because of the well-known adverse real interest rate effect. By changing the shape of the Laffer curve, the ZLB forces the revenue-maximizing fiscal authority to raise the tax rate as to push inflation closer to its target and act as a substitute for the constrained monetary authority. However, the peak of the Laffer curve is associated with a higher tax rate, which shrinks the tax base and impairs revenues, and a higher interest rate, which further stymies economic activity and depresses the present value of future revenues, compared to an unconstrained economy. As a result, this imperfect substitution of fiscal policy for a constrained monetary policy at the ZLB induces a dead-weight loss in the present discounted value of future primary surpluses and, then, a deterioration in debt sustainability.[4]

Second, a more aggressive monetary policy – that is, a higher elasticity of the risk-free rate to changes in inflation and the output gap – leads to an improvement in debt sustainability, with a shift of the fiscal limit distributions to the right. This result –

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[3]Notice that, to reach the ZLB, we introduce a consumption preference shock as in Erceg and Lindé (2014), in order to mimic the economic developments that made the ZLB binding during the recent crisis. The introduction of a demand-side shock is another novelty compared to the literature on fiscal limits, typically focused on macroeconomic supply-side shocks.

[4]Our analysis also shows the importance of distinguishing the debt sustainability impact of the ZLB itself with that of the shock driving the economy to the ZLB (which has beneficial effects thanks to the reduction in interest rates).

answering our second question above – is mainly due to the lower volatility in inflation, which reduces the associated distortions and strengthens the tax base, increasing the fiscal primary balance. Importantly, an increase in the inflation target also reduces sustainability concerns, but it has a more limited impact – and may thus be considered as a second-best policy in terms of debt sustainability – compared to an increase in the degree of activeness of monetary policy.

On the basis of our analysis of fiscal limits, and in order to answer our last question, we investigate the capacity of spending shocks to support economic activity while preserving debt sustainability, conditional on different monetary policy environments. The trade-off between debt sustainability and economic stabilization faced by the government in normal times (in line with Erceg and Lindé, 2014) is amplified by the presence of default risks, as a debt-financed increase in public spending implies a leftward shift of the fiscal limit distributions and, thus, a rise in spreads with further detrimental effects on debt, taxes and economic activity.

However, this trade-off shrinks once the economy is at the ZLB. In this case, the larger effect of public spending on output already identified in the literature (see, e.g., Erceg and Lindé, 2014, and Fernández-Villaverde et al., 2015), coupled with the low rate environment, expands the government's borrowing opportunities – even after accounting for the costs associated to the ZLB – and with them the fiscal margin available for economic stabilization. Importantly, the systematic conduct of monetary policy outside of the ZLB – that is, the elasticity of the risk-free rate to inflation and output gap – plays a role in determining sustainability also when the ZLB constraint is binding, with a more active monetary policy expanding the margin for stabilization purposes.

Looking at the latest related literature, the closest works to ours are Reis (2016), Bi, Leeper and Leith (2013, 2018) and Uribe (2006). Within the taxonomy introduced by Reis (2016), we focus on the first channel through which monetary policy affects the fiscal burden, namely the control of inflation through its interest rate setting behavior. Differently from his study, however, we conduct our analysis in a general equilibrium model. Moreover, the explicit role of monetary policy in the definition of fiscal limits and the analysis of the ZLB are the main differences between our model and those presented in Bi, Leeper and Leith (2013, 2018). Further, we abstract from the possibility of non-Ricardian fiscal policy and optimal default as a way to restore the intertemporal government budget constraint, as instead studied by Uribe (2006), and we focus on the case of *passive* fiscal policies, in which the tax rate adjusts to bring debt to a specific target.

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Our analysis is also linked to the literature studying the implications of the fiscal theory of the price level on inflation determination, and the different combination of active/passive fiscal and monetary policy (see, among others, Davig, Leeper and Walker, 2011, and Leeper and Leith, 2016). Differently from them, however, we study how *active* monetary policy regimes or temporary suspension of monetary policy rules (i.e. periods of binding ZLB) affect fiscal limits.

In the spirit of Bi (2012), we model sovereign default as a random event whose likelihood increases with the level of debt and where the haircut is exogenous and not dependent on the policy mix. Moreover, our paper focuses on the *capacity* to pay, rather than the *willingness* to pay, instead a crucial element of the study by Arellano (2008). We also abstract from considerations of self-fulfilling dynamics and multiple equilibria as in Lorenzoni and Werning (2013) or on the capacity of monetary policy to prevent them as in Corsetti and Dedola (2016).

The rest of the paper is structured as follows. Section 2 outlines the general equilibrium model. Section 3 presents the methodology for its numerical solution alongside the calibration parameters. Section 4 assesses the role of monetary policy and the ZLB in the determination of the fiscal limit distributions. In section 5, we evaluate the impact of spending shocks on debt sustainability with different scenarios for the degree of monetary policy activeness, including the analysis of periods of binding ZLB. Section 6 concludes.

2   The model

The model builds on the works by Bi (2012) and Bi, Leeper and Leith (2013, 2018) by introducing preference shocks *à la* Erceg and Lindé (2014) and allowing for the presence of a ZLB on the nominal risk-free interest rate. Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The closed economy is populated by a representative household who consumes, works, owns monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing a homogeneous final good, and invests in two types of state-noncontingent assets, namely risk-free bonds and risky (i.e., defaultable) government bonds. The government finances public consumption and debt service through distortionary taxation on labor income and profits. The central bank sets the risk-free interest rate to reduce deviations of inflation from its target and output from its potential through a Taylor rule truncated at the ZLB.

The household. The representative household maximizes the following initial utility function, based on preferences à la Greenwood, Hercowitz and Huffman (1988):

$$\max_{\left\{C_t, N_t, B_t, B_t^F\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - C\nu_t - \frac{\kappa}{1+\frac{1}{\chi}} N_t^{1+\frac{1}{\chi}}\right)^{1-\gamma}}{1-\gamma}, \tag{1}$$

subject to the flow budget constraint (in real terms):

$$C_t + \frac{B_t}{R_t} + \frac{B_t^F}{R_t^F} = (1-\tau_t)(W_t N_t + \Upsilon_t) + Z + \frac{B_t^d}{\Pi_t} + \frac{B_{t-1}^F}{\Pi_t}, \tag{2}$$

where E_0 denotes the expectations operator, β the household's discount factor, γ its relative risk aversion, χ its Frisch elasticity and κ the relative weight of its disutility of labor.[5] C_t denotes private consumption, N_t hours of labour, τ_t the tax rate on wage income and profits, W_t the wage rate, Υ_t the representative firm's profits, Π_t (gross) inflation, Z (constant) transfers from the government to the households, B_t risky (i.e. defaultable) government bonds, with associated (gross) nominal interest rate R_t, and B_t^F risk-free bonds, with associated (gross) nominal interest rate R_t^F, at time t. Notice that $B_t^d \equiv (1-\Delta_t)B_{t-1}$ denotes the part of real outstanding debt actually repaid and Δ_t is the haircut on outstanding debt in case of default. As in Erceg and Lindé (2014), the utility function depends on the household's current consumption C_t as deviation from a reference level $C\nu_t$. The exogenous consumption taste shock ν_t lowers the reference level and increases the marginal utility of consumption and follows an $AR(1)$ process

$$\nu_t = (1-\rho_v)\nu + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_t, \qquad \epsilon_t \sim N(0,1). \tag{3}$$

The first order conditions of the household problem define the following labor supply schedule:

$$N_t = \left[\frac{(1-\tau_t)W_t}{\kappa}\right]^{\chi}, \tag{4}$$

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[5]GHH preferences have been often used in the literature on sovereign default to avoid a counterfactual rise in labor upon a sharp drop in consumption, as is the case in default episodes, by removing the wealth effect on labor supply ((see, e.g., Mendoza and Yue, 2012). In our model, GHH preferences also help us define a more realistic Laffer curve compared to that implied by CES separable preferences in consumption and labor.

the standard Euler equation for the riskless bonds:

$$\frac{1}{R_t^F} = \beta E_t \left[\frac{mrs_{t,t+1}}{\Pi_{t+1}} \right], \tag{5}$$

where we have defined

$$mrs_{t,t+1} \equiv \left(\frac{C_{t+1} - C\nu_{t+1} - \frac{\kappa}{1+\frac{1}{\chi}} N_{t+1}^{1+\frac{1}{\chi}}}{C_t - C\nu_t - \frac{\kappa}{1+\frac{1}{\chi}} N_t^{1+\frac{1}{\chi}}} \right)^{-\gamma}$$

as the household's marginal rate of substitution between consumption at t and $t+1$, and the Euler equation for the (risky) government bonds:

$$\frac{1}{R_t} = \beta E_t \left[(1 - \Delta_{t+1}) \frac{mrs_{t,t+1}}{\Pi_{t+1}} \right], \tag{6}$$

where

$$\Delta_t = \begin{cases} 0 & \text{if } B_{t-1} < B_t^* \\ \delta & \text{if } B_{t-1} \geqslant B_t^*, \end{cases} \tag{7}$$

where δ is the size of the haircut if default. As in Bi (2012), the default scheme depends on the effective fiscal limit B_t^*. Each period B_t^* is drawn from the state-contingent distribution of fiscal limits, see section 2.1 below.

Final goods producers. The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(i)$. Competitive final good firms buy the differentiated goods produced by intermediate goods producers and combine them according to an aggregate function which has the CES (constant elasticity of substitution) form:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

Cost minimization for final good producers results in the demand curve for the generic intermediate good i:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \tag{9}$$

and an associated price index for the final good:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \tag{10}$$

where θ is the elasticity of substitution between goods.

Intermediate goods producers. A continuum of intermediate goods $Y_t(i)$ for $i \in [0,1]$ is produced by monopolistically competitive firms each producing a single differentiated good. Intermediate goods firms are subject to Rotemberg adjustment costs that penalize large price changes in excess of (deterministic) steady-state inflation rates. Producer i's maximization problem reads:

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \beta^t mrs_{0,t} \frac{P_0}{P_t} \left[P_t(i)Y_t(i) - mc_t P_t Y_t(i) - \frac{\phi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)\Pi} - 1\right)^2 P_t Y_t \right] \tag{11}$$

$$s.t. \quad Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t, \tag{12}$$

where $P_t(i)$ is the price chosen by firm i and P_t is the nominal aggregated price level. Intermediate good producers are endowed with a linear production function:

$$Y_t(i) = A_t N_t(i), \tag{13}$$

where A_t is total factor productivity which follows an exogenous AR(1) process of the form:

$$\ln A_t = (1-\rho_a)\ln A + \rho_A \ln A_{t-1} + \sigma_A \varepsilon_t^A, \qquad \varepsilon_t^A \sim N(0,1) \tag{14}$$

which, in equilibrium, implies the real marginal cost $mc_t = W_t/A_t$. In a symmetric equilibrium, the first-order condition gives the non-linear New Keynesian Phillips curve under

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Rotemberg pricing:

$$(1 - \theta) + \theta mc_t - \phi \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right) + \phi \beta E_t \left[mrs_{t,t+1} \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right] = 0, \quad (15)$$

where ϕ controls Rotemberg (quadratic) price adjustment costs. Intermediate goods producers' monopolistic real profits are:

$$\Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t. \quad (16)$$

Fiscal and monetary policy. The government's budget constraint (in real terms) is determined by the following equation:

$$\frac{B_t}{R_t} + T_t = \frac{B_t^d}{\Pi_t} + G_t + Z, \quad (17)$$

where T_t is tax revenue, G_t is government consumption and Z denotes constant fiscal transfers to the household. Public consumption follows an exogenous AR(1) process:

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_G \varepsilon_t, \quad \varepsilon_t \sim N(0, 1), \quad (18)$$

while the tax rate responds to a standard debt-targeting rule:

$$\tau_t = \tau + \mu_\tau (B_t^d - B), \quad (19)$$

where μ_τ is the elasticity of the tax rate to debt in deviation from its steady-state level B, and tax revenue is then given by:

$$T_t = \tau_t (W_t N_t + \Upsilon_t). \quad (20)$$

The central bank is assumed to follow a truncated Taylor rule subject to the ZLB:

$$R_t^F = \max \left\{ R^F \left(\frac{\Pi_t}{\Pi} \right)^{\alpha_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\alpha_y} e^{\eta_t^R}, 1 \right\} \quad (21)$$

where α_π and α_y are the elasticities of the nominal risk-free interest rate to inflation and output in deviations from their intended levels, respectively, while Π is the target inflation rate (assumed to be equal to the steady-state inflation rate), Y_t^* potential output and η_t^R

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a monetary policy shock:

$$\eta_t^R = \rho^\tau \eta_{t-1}^R + \sigma^R \varepsilon^R, \quad \varepsilon_t^R \sim N(0,1), \tag{22}$$

with zero long-run mean.

Aggregate resource constraint. Closing the model economy, the aggregate resource constraint is given by:

$$C_t + G_t = Y_t \left[1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right], \tag{23}$$

whereby transfers Z and net proceeds from government bonds $\frac{B_t}{R_t} - \frac{B_t^d}{\Pi_t}$ cancel out as they simply redistribute resources between the household and the government. Finally, as the representative household is a pool of identical units in a closed economy trying to hedge aggregate risk, risk-free bonds are in zero net supply. Hence, provided that the Euler conditions for risk-free and risky bonds hold, only the latter are traded to satisfy the government's budget constraint. This final condition closes our economy.

2.1 Distribution of fiscal limits

Following Bi (2012), we quantify the risk of sovereign default on the basis of fiscal limits, defined as the state-contingent present-discounted value of all the maximum present and future primary surpluses. The government maximizes each period's surpluses by setting the tax rate at the peak of the Laffer curve. The Laffer curve, in turn, is generated conditional on the level of total primary spending and the current state of the exogenous shocks. Revenues, expenditures and consumption's preferences vary with the shocks hitting the economy, generating a distribution for the maximum debt level.

Differently from Bi, Leeper and Leith (2013), who assumes that the monetary authority is always able to peg inflation to the target when computing the fiscal limits, we assume that the monetary authority follows a truncated Taylor rule and we allow inflation to vary around its steady state. Inflation dynamics then affect the fiscal limit distributions that become endogenous to monetary policy. This allows us to study how the fiscal limits depend on the monetary policy activeness and respond to the presence of an occasionally-binding ZLB constraint.

Formally, the stochastic processes governing the exogenous states induce stochastic

processes for both the tax rate τ_t^{max} and the associated maximum tax revenue $T_t^{\max}$. Conditional on the exogenous state of the economy, the revenue-maximizing government solves the following problem:

$$T^{\max}(A_t, \nu_t, G_t, \eta_t^R) = \max_{\{\tau_t\}_{t=0}^{\infty}} \tau_t(W_t N_t + \Upsilon_t), \tag{24}$$

subject to the other equilibrium conditions.[6] Hence, the function T^{max} maps the exogenous current state of the economy into the tax revenue at the peak of the Laffer curve. As stated above, the fiscal limit is defined as the discounted sum of expected maximum primary surpluses in all future periods as of time t:

$$B_t^* = E_t \sum_{s=t}^{\infty} \beta^{s-t} \beta_t^p \, mrs_{t,s}^{\max}(A_s, \nu_s, G_s, \eta_s^R) \left[T^{\max}(A_s, \nu_s, G_s, \eta_s^R) - G_s - Z \right], \tag{25}$$

which essentially consists of two components, namely the household's marginal rate of substitution $mrs_{t,s}^{\max}$ between consumption at t and $s \geq t$ and the primary surplus $s_t^{\max} \equiv T_t^{\max} - G_t - Z$ evaluated when the tax rate is at the peak of the Laffer curve. These two components reflect two interrelated, but distinct, channels affecting the fiscal limit. On the one hand, the *tax revenue channel* affects $s_t^{\max}$ and mainly works through changes in the tax base and tax rates; on the other hand, the *interest rate channel* affects $mrs_{t,s}^{\max}$ and thus the way the stream of future primary surpluses is discounted back to present. Similarly to Bi, Leeper and Leith (2018), we include a regime-switching political risk parameter $\beta_t^p \in \{\beta_L^P, \beta_H^p\}$ with transition probabilities from state i to state j $p(j|i)$:

$$\begin{bmatrix} p(L|L) & p(H|L) \\ p(L|H) & p(H|H) \end{bmatrix} = \begin{bmatrix} p_L & 1 - p_L \\ 1 - p_H & p_H \end{bmatrix} \tag{26}$$

This parameter may be interpreted in several ways. First, the political risk factor may reflect the government's planning horizon, so that a more myopic policymaker would feature a lower β_t^p, as shown by Cuadra and Sapriza (2008), among others. A second interpretation posits that the political risk factor may reflect the weight placed by investors on the government's ability to reach the peak of the Laffer curve, given all its political constraints. To see this, simply rewrite the stream of future primary surpluses as $\beta_t^p \cdot s_t^{\max} + (1 - \beta_t^p) \cdot 0$. Heuristically, the political risk factor and the transition probabilities serve as location and shape parameters of the fiscal limit distribution, which we use to produce realistic spreads

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[6]See Appendix A.

and debt levels. Ultimately, the distribution of fiscal limits is then computed using Markov Chain Monte Carlo simulations.

3   Numerical solution and calibration

As in the standard framework pioneered by Bi (2012), the model is solved in two stages. In the first stage, in order to simulate the state-contingent fiscal limit distributions, the model is solved conditional on the government maximizing the primary surplus as in problem (24). In the second stage, in order to explore the impact of shocks on the full general equilibrium with risky debt, the model is solved conditional on the government following the debt-targeting rule (19). The fiscal limit distributions obtained through (25) in the first stage are used to compute the state-contingent probability of default according to (7). From a computational perspective, since the revenue-maximizing tax rate does not depend on debt but only on the exogenous states (and default is ruled out in the first-stage model by construction), the solution of the first-stage model at the peak of the Laffer curve is typically less demanding compared to the solution of the full second-stage model.

Notice that the assumption of a constant inflation rate (as in Bi, Leeper and Leith, 2013, 2018) would transform the first-stage model into a real economy, requiring a relatively simple closed-form solution (as in Bi, 2012). In contrast, our assumption of a time-varying inflation rate preserves also the first-stage model as a nominal economy. Due to the forward-looking nature of the Phillips curve and the risk-free Euler equation, no closed-form solution is available to determine the equilibrium levels of the tax rate, the real wage and the inflation rate, so that a numerical solution is necessary. We use global methods (detailed in Appendix A) to deal explicitly with the non-linearities associated with the ZLB constraint and, in the full model, with different levels of outstanding debt and binary default choices.[7]

The calibration is presented in Table 1. The model is calibrated at quarterly frequency. For the sake of comparability with previous literature, where possible, we refer to the calibration used by Bi, Leeper and Leith (2013), who set parameters to match average EU-14 data from 1971 to 2007. The discount factor β is set to 0.99, the relative risk aversion γ to 1 and the elasticity of substitution between goods θ to 11. In steady state,

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[7]Notice that, if risky government bonds were the only available assets in the economy, agents' optimal solutions would also be contingent on debt, even in the first-stage model. The introduction of a risk-free bond in zero net supply, alongside a risky government bond, is then pivotal to pin down the path of consumption without an additional endogenous state variable in the first-stage model. Hence, when we solve the first-stage model for the maximum tax rate, the real wage and the inflation rate, we need not consider government debt, with considerable benefits in terms of computational time.

Table 1: Calibration

Parameters		Value/Target	Source
Discount factor	β	0.99	
Risk aversion	γ	1	
Elasticity of substitution	θ	11	
Public consumption/GDP s.s.	G/Y	21%	Bi, Leeper and Leith (2013)
Transfers/GDP s.s.	Z/Y	18%	
Tax rate s.s.	τ	41%	
Inflation s.s.	Π	3% (annual)	
Disutility of labour	κ	Labor s.s. $N = 1/4$	
Political factor regimes	$\beta_L^p - \beta_H^p$	$0.6 - 1$	Bi, Leeper and Leith (2018)
Political factor probabilities	$p_L - p_H$	$0.99 - 0.99$	
Haircut	δ	9.47%	Bi (2012)
Tax rule	μ_τ	0.25	
Taylor rule inflation elasticity	α_π	3	
Taylor rule output elasticity	α_y	1.5	
Frisch elasticity	χ	0.7	
TFP s.s.	A	1	
TFP shock persistence	ρ_a	0.85	
TFP shock variance	σ_a^2	0.003^2	Standard
Preference shock weight	C	1	
Preference shock s.s.	ν	0	
Preference shock persistence	ρ_ν	0.85	
Preference shock variance	σ_ν^2	0.0025^2	
Public consumption shock persistence	ρ_g	0.65	
Public consumption shock variance	σ_g^2	0.03^2	
Monetary policy shock persistence	ρ_r	0.85	
Monetary policy shock variance	σ_r^2	0.01^2	

government purchases (public consumption) are 21 percent of GDP and lump-sum transfers are 18 percent of GDP. The steady-state tax rate is set to 41 percent, which implies a debt-to-GDP ratio of 50.38 percent on an annual basis in steady state. Steady-state inflation Π is set to 3 percent on annual basis. The weight on the disutility of labor κ targets a steady-state labor supply N of $1/4$, implying a value of about 3.9. The political risk factor β_t^p switches between a low regime of 0.6 and a high regime of 1, in line with the constant value of 0.85 used by Bi, Leeper and Leith (2013), with equal transition probabilities of 0.99 (implying an expected duration of 25 years) for each regime, higher than the transfer regime transition probabilities of 0.975 used by Bi (2012) in order to produce distributions with thicker tails and higher spreads for lower debt levels. We set the Frisch elasticity to 0.7, which is in the range of the various estimates used in the literature, varying from 0.4

(Lindé and Trabandt, 2018) to 1 (Schmitt-Grohé and Uribe, 2007). Given our assumption of GHH preferences, this value for the Frisch elasticity implies a peak of the Laffer curve at a tax rate on wage income and profits below 70 percent, in the middle of the range of peak labor tax rates (54 percent) and peak capital tax rates (79 percent) found by Trabandt and Uhlig (2011) for a sample of EU-14 countries. Upon default, we assume a constant haircut δ of 9.47 percent, which is the expected value of the empirical distribution presented in Bi (2012) and within the range of estimates provided by Cruces and Trebesch (2013). As discussed in Bi, Leeper and Leith (2018), such a sizable default rate negatively affects convergence and thus requires particularly strong stabilization parameters. In our model, these parameters refer to the "policy" parameters, namely the debt elasticity of the tax rate μ_τ, set to 0.25 (rather than 0.125 as in Bi, Leeper and Leith, 2013) as well as the inflation and output elasticities of the interest rate α_π and α_y, set to 3 and 1.5 (rather than 1.5 and 0 as in Bi, Leeper and Leith, 2013), respectively. The parameters for exogenous processes take on values commonly found in the quantitative macroeconomic literature (see, e.g., Schmitt-Grohé and Uribe, 2007).

# 4   Fiscal limits and monetary policy

This section studies how fiscal limit distributions change depending on the monetary policy environment. In a first step, we explore the debt sustainability implications of introducing the possibility of a binding ZLB constraint on monetary policy, driven by a negative preference shock. In a second step, we turn to the investigation of how the conduct of monetary policy influences the shape of fiscal limit distributions $\mathcal{B}_t^*$.

4.1   Constrained monetary policy: fiscal limits at the ZLB

To explore the debt sustainability implications of a binding ZLB constraint on monetary policy, we first need to evaluate the effects of the macroeconomic shock leading the economy to the ZLB.

Figure 1 distinguishes the impact of the negative taste shock ν_t on fiscal limit distributions into two components: the interest rate channel (showing the impact of the increase in the marginal rate of substitution $mrs_{t,t+1}^{\max}$) and the tax revenue channel (presenting the impact of the reduction in the primary surplus $s_t^{\max}$). This is done by running counterfactual simulations in which the consumption taste shock ν_t is reduced from its steady-state level (thin lines) to its lowest level in our grid (thick lines) both in a situation where the

Figure 1: Impact of a negative consumption preference shock on fiscal limits
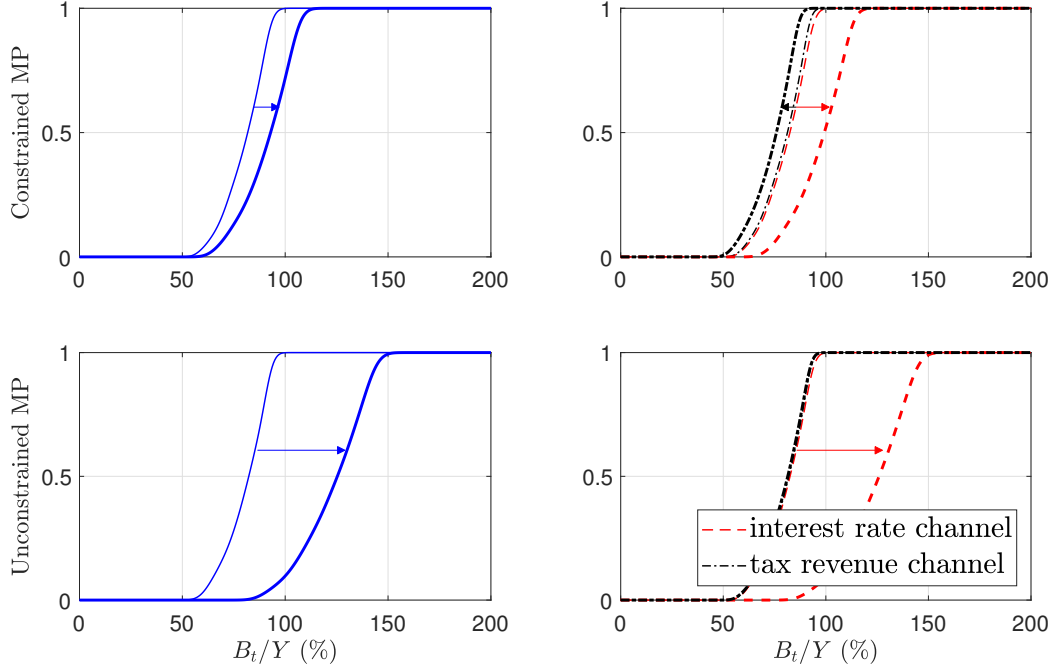
Note: In each chart, thick (thin) lines refer to the CDF of fiscal limits with the low (steady-state) level for the consumption preference shock (ν_t). The charts on the left side show the total impact of the negative demand shock while the charts on the right side show the impact of its components. The upper charts present the case in which interest rates cannot drop below zero, the lower charts when they can. In all charts, the negative demand shock has the same magnitude and is such that, when monetary policy is constrained, the economy is at the ZLB.

ZLB constraint is applied (upper charts) and in one where it is not (lower charts).[8] In the counterfactual simulations used to identify the interest rate and the tax revenue channels, the distributions are simulated by holding $E_t[mrs_{t,t+1}]$ and T_t, respectively, constant at their steady-state values and allowing the other component to freely move.[9]

When monetary policy is unconstrained, a large gain from low interest rates and a small (negligible, in this calibration) loss in tax revenues significantly contribute to the improvement of the fiscal limit distribution $\mathcal{B}_t^*$. Hence, the adverse consumption prefer- ence shock has an overall significant positive impact on debt sustainability, increasing the

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[8]To clearly evaluate the impact of the ZLB on the fiscal limit, the consumption preference shock ν_t is set to a value 10 standard deviations away from its steady-state value, implying an expected duration of the ZLB of 11 quarters.

[9]For this reason, the shape of the fiscal limit distribution when the consumption shock is at steady state differs from the shape of the standard distribution shown in the left panels.

Figure 2: Laffer curve components with constrained and unconstrained monetary policy
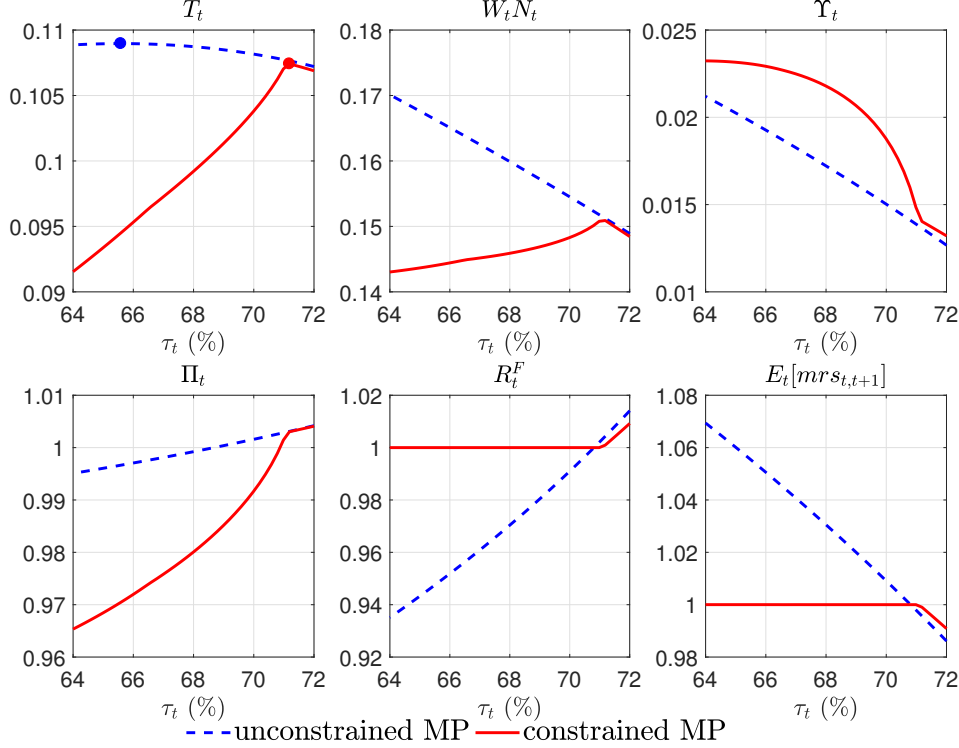
Note: The chart shows how changes in the tax rate τ_t affects tax revenues T_t, wage income $W_t N_t$, profits Υ_t, (gross) inflation Π_t, (gross) risk-free interest rate R_t^F and the marginal rate of substitution $E_t[mrs_{t,t+1}]$ after a negative consumption preference shock ν_t with constrained (solid red lines) and unconstrained (dashed blue lines) monetary policy. In both charts, the negative aggregate demand shock has the same magnitude and is such that, when monetary policy is constrained, the economy is pushed to the ZLB.

government's borrowing opportunities. In contrast, when monetary policy is constrained, the smaller reduction in interest rates triggers a larger loss in tax revenues, reducing considerably the right shift in the fiscal limits' distribution.

To better understand how the ZLB constraint affects the fiscal limit distribution, Figure 2 inspects the way the Laffer curve and its determinants in both the unconstrained and constrained case. When monetary policy is constrained (solid red lines), output remains below the level prevailing in the unconstrained case (dashed blue lines) because of the adverse real interest rate effect, which depresses aggregate demand and, thus, tax revenues (see Christiano, Eichenbaum and Rebelo, 2011, and Erceg and Lindé, 2014). Then, the revenue-maximizing government raises the tax rate until inflation is so high as to lift the interest rate away from the zero floor. In correspondence to this point, the Laffer curve reaches its peak, as the marginal increase in tax revenues due to lower inflation costs (as

inflation approaches its target) is compensated by the marginal reduction in tax revenues due to the combination of the contractionary effect of the reactivation of monetary policy, given that further tax rate hikes would increase the interest rate above zero, and the marginal distortionary effect of the tax rate on labor supply. Compared to the typical bell-shaped curve of the unconstrained case, the presence of the ZLB introduces a kink in the Laffer curve corresponding to its peak. For higher values of the tax rate, the curve coincides with the unconstrained Laffer curve as the ZLB is not binding anymore.

Notice that the peak in the constrained case is associated with a higher tax rate, which shrinks the tax base and impairs revenues, and a higher interest rate, which further stymies economic activity and depresses the present value of future revenues compared to an unconstrained economy. Hence, when fiscal policy seeks to substitute for a constrained monetary policy, it can do so only imperfectly, as both components of the fiscal limit distribution, tax revenues and marginal rate of substitution, are negatively affected. As a result, this imperfect substitution of fiscal policy for a constrained monetary policy at the ZLB induces a dead-weight loss in the present discounted value of future primary surpluses and, then, a deterioration in debt sustainability.[10]

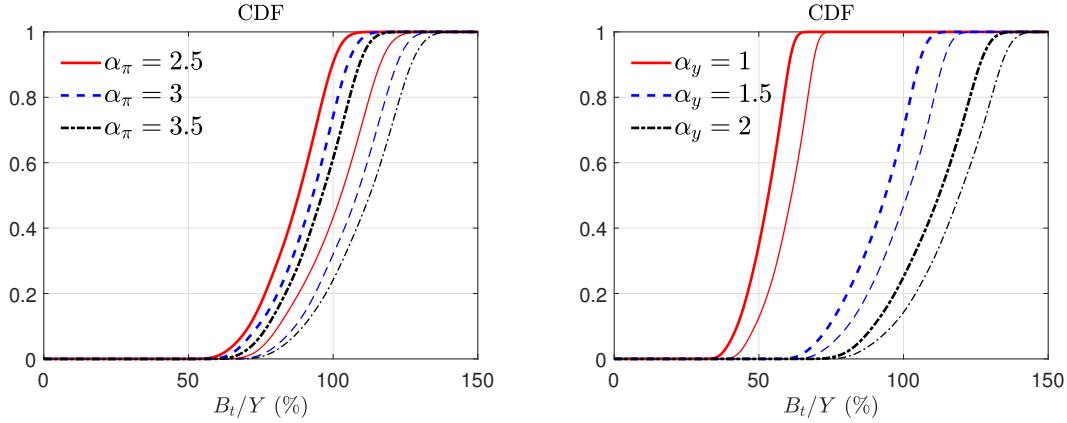## 4.2 Monetary policy conduct and fiscal limit distributions

We now turn to the investigation of how the conduct of monetary policy influences the shape of fiscal limit distributions $\mathcal{B}_t^*$. Looking at the Taylor rule in Equation (21), monetary policy can influence debt sustainability in both the systematic component, that is (i) its activeness in responding to deviations of inflation from its target and output from its potential level, represented by the Taylor coefficients α_π and α_y and (ii) the selected inflation target Π; as well as the unexpected component, that is the unanticipated shock η_t^R to the risk-free interest rate.

Figure 3 shows how a different monetary policy activeness affects $\mathcal{B}_t^*$. To emphasize the difference between the ZLB constrained and unconstrained cases, an adverse consumption preference shock 4 standard deviations below its steady state is introduced.[11] Fiscal

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[10]Notice that, when monetary policy is constrained at the ZLB, firms' profits Υ_t increase above the level implied by an unconstrained monetary policy. This result is due to the decline in firms' marginal costs mc_t reflecting – through the Phillips curve – the decrease in inflation Π_t due to the decrease in aggregate demand. Despite the increase in profits, total tax revenues T_t drop because both wages W_t – mirroring marginal costs – and labor N_t – through the labor supply equation – decline more than the increase in profits, thus shrinking the total tax base.

[11]The impact of changes in α_π and $_y$ on $\mathcal{B}_t^*$ is qualitatively similar at steady state, as shown in Figure 12 in Appendix B.

Figure 3: Impact of monetary policy activeness on fiscal limit distributions with constrained and unconstrained monetary policy
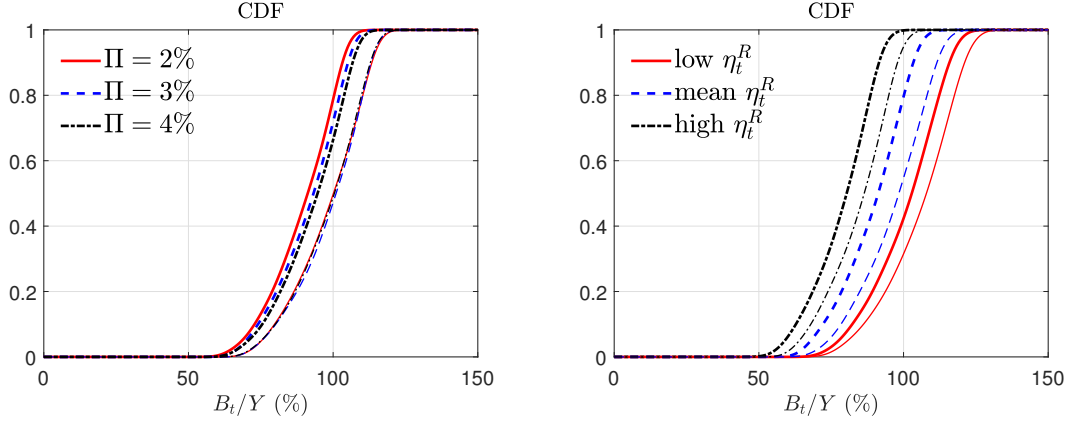

Note: In both charts, to highlight the differences between constrained and unconstrained monetary policy, consumption preference shock is initially set at 4 standard deviations below its steady state, so that the ZLB constraint is more frequently binding. In both charts, the thick (thin) lines report the fiscal limit distribution with constrained (unconstrained) monetary policy, i.e. imposing (not imposing) the ZLB constraint.

limit distributions are derived for α_π equal to 2.5 (weak), 3 (baseline) and 3.5 (strong) in the left-hand chart and α_y equal to 1 (weak), 1.5 (baseline) and 2 (strong) in the right-hand chart. The sustainability of debt clearly improves with a more active monetary policy, since inflation volatility – and the associated costs in terms of firms' profits and, thus, tax collection – falls with a higher α_π. The benefits from stronger monetary policy activeness in terms of sustainability are considerably amplified by an increase of α_y, as a higher α_y limits the fluctuations of potential output and of the output gap, thus further limiting the variability of inflation.

Notice, however, that the magnitude of the shift in the fiscal limit distribution $\mathcal{B}_t^*$ is decreasing in both α_π and α_y. Indeed, as the α's increase, $\mathcal{B}_t^*$ converges to the fiscal limit distribution that would emerge if interest rates could freely adjust to maintain inflation pegged to its target and output at its potential (which is the case studied by Bi, Leeper and Leith, 2013).

In Figure 4, the left-hand chart runs a similar exercise as in Figure 3, but now changing the inflation target rather than the degree of activeness. This policy change has a minimal impact on the fiscal limit distributions, visible only in the case with the constrained monetary policy (thick lines). It can thus be considered as a second-best policy in terms of debt sustainability effects compared to an increase in the degree of

Figure 4: Impact of changes to the inflation targets and of monetary policy shocks on fiscal limit distributions with constrained and unconstrained monetary policy

Note: In both charts, to highlight the differences between constrained and unconstrained monetary policy, consumption preference shock is initially set at 4 standard deviations below its steady state, so that the ZLB constraint is more frequently binding. In both charts, the thick (thin) lines report the fiscal limit distribution with constrained (unconstrained) monetary policy, i.e. imposing (not imposing) the ZLB constraint.

activeness of monetary policy. The right-hand chart of Figure 4 shows the impact on the fiscal limit distribution of both contractionary and expansionary monetary policy shocks η_t^R. An expansionary shock, that is a low η_t^R, is associated with a reduction in interest rates and an increase in output. Hence, both the interest rate and the tax revenue channels move debt limits in the same direction. This increases debt sustainability, as it shifts the fiscal limit distributions to the right.[12]

5 Trading off stabilization and sustainability: the case of spending shocks

In this section, we focus on how the nature and constraints surrounding monetary policy affects the trade-off between stabilization and sustainability in the use of fiscal instruments (see Carnot, 2014, Furceri and Mourougane, 2010, and Kamps et al., 2017).[13]

In order to operationalize our analysis, we introduce an index to measure the max-

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[12]Notice that, however, when the economy is initially at steady state, the gap between $\mathcal{B}_t^*$ with constrained and unconstrained monetary policy is much smaller, almost negligible, in all the simulations presented in this section. See Figure 13 in Appendix B.

[13]Differently from Leith and Wren-Lewis (2012), we assume perfect commitment, thus ignoring the problem of time inconsistency.

imum size of a government spending shock that is consistent with fiscal sustainability. Differently from the existing literature (see Ghosh et al., 2013, and Leeper and Walker, 2011), our index is forward-looking and it explicitly considers the whole path of current and future debt levels and fiscal limits. We label this index as "fiscal margin".

As a second step, we check how the transmission of spending shocks consistent with different calibration of the fiscal margin changes depending on the monetary policy environment and the degree of activeness of the central bank.

5.1 Forward-looking index of fiscal margin

Our index for the fiscal margin represents the public spending paths that can be implemented while keeping the default probability below a pre-specified threshold over a given time horizon. In other words, the fiscal margin index is a "reverse Value-at-Risk", where the default probability is our measure of risk. While our analysis focuses on spending, the fiscal margin can be defined for every fiscal instrument.

Let F_t be the fiscal instrument in relation to which we want to measure the fiscal margin. Let $\mathcal{B}_t^*(\phi)$ denote the debt limit corresponding to a given probability threshold ϕ conditional on the realization of the exogenous states. Moreover, let the fiscal signal $FS_t(\phi) \in \{0,1\}$ indicate whether debt B_t exceeds $\mathcal{B}_t^*(\phi)$ at t. Formally, it is defined as follows:

$$FS_t(\phi) = \begin{cases} 0 & \text{if } \mathcal{B}_t^*(\phi) - B_t \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

Then, the fiscal margin $FM_{t,T}(\phi)$, quantifying the maximum leeway for the government without receiving the fiscal signal corresponding to the set probability of default ϕ given a certain path for fiscal instrument $(F_s)_{s=t}^T$ at any point between t and $T \geq t$, can be computed as:[14]

$$FM_{t,T}(\phi) = \sup\{(F_s)_{s=t}^T : (FS_s(\phi) = 0)_{s=t}^T\}.$$

This index of fiscal margin is fully state-contingent and forward-looking, as it takes into

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[14]Given our focus on government spending shocks in a recessionary period, we assume that the fiscal authority aims at *increasing* its instrument to the maximum extent possible before breaching the conditions for the fiscal margin. Naturally, if the considered fiscal instrument were the tax rate, the sup operator should be replaced by the inf operator, as the government would seek to *decrease* its instrument during a recession.

account the general equilibrium implications of changes in the state of the economy and of the fiscal instrument itself F_t.[15]

The forward-looking nature of the fiscal margin is in stark contrast with a commonly used framework for debt sustainability in the corporate, policy and academic literature, in which the space for possible policy shocks is calculated based on the distance between current debt and a unique, backward-looking and (sometimes) deterministic debt limit (see, for instance European Commission, 2016, International Monetary Fund, 2010, Moody's, 2016, Ghosh et al., 2013 and Collard, Habib and Rochet, 2015).[16]

In our framework, the fiscal margin tends to be much smaller than what would be implied by a simple difference between the fiscal limit associated to a specific default probability threshold – regardless of the considered threshold – and the initial level of debt, since the entire future paths of the debt levels (taking into account the implementation of the shock) and fiscal limits are considered.

As a first application of our measure of the fiscal margin $FM_{t,T}(\phi)$, Figure 5 shows the impact of two four-quarter sequences of unanticipated government spending shocks corresponding to 1.5% and 3% of steady-state output (i.e., corresponding to shocks of the same magnitude in annual terms). The responses are aligned with what is generated by a standard new Keynesian model, but amplified by the detrimental consequences of higher spreads, debt and taxes. The deficit-financed spending shock increases debt and pushes up domestic demand, raising the output and consumption gaps. The magnitude of the output multiplier of the spending shock seems to be proportional to the magnitude of the shocks. In contrast, the response of the spread is highly non-linear as the large G_t shock implies a spread more than four times the size of the spread implied by the small G_t shock. Finally, the debt-financed spending shock brings the output level above its steady state only in the short term, whereas it has a persistent negative impact in the long term.[17]

The assessment of debt sustainability in terms of fiscal margin crucially depends on the size of the fiscal shock G_t as well as the target probability of default ϕ and the relevant

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[15]This definition implies that multiple (potentially, infinite) combinations of shocks over the considered horizon may correspond to the fiscal margin available to the government. Moreover, the fiscal margin is inherently uncertain, as it is linked to a default *probability* ϕ, rather than a point estimate.

[16]Because of the lack of a closed-form solution, we can only simulate $FM_{t,T}(\phi)$ based on a deterministic path of macroeconomic and policy shocks as well as the current outstanding level of debt after solving the fully non-linear stochastic model.

[17]Compared to a standard model without default, the inclusion of a fiscal limit leads to a higher increase in debt and a lower output multiplier: first, the spread rises in response to both the higher debt level and the lower fiscal limit; then, the higher servicing costs lead a stronger response of the tax rate, which increases inflation (which in turn amplifies the risk-free response) and reduces output. For these additional results, see Figure 14 in Appendix B.

Figure 5: Impact of different government spending shocks in normal times
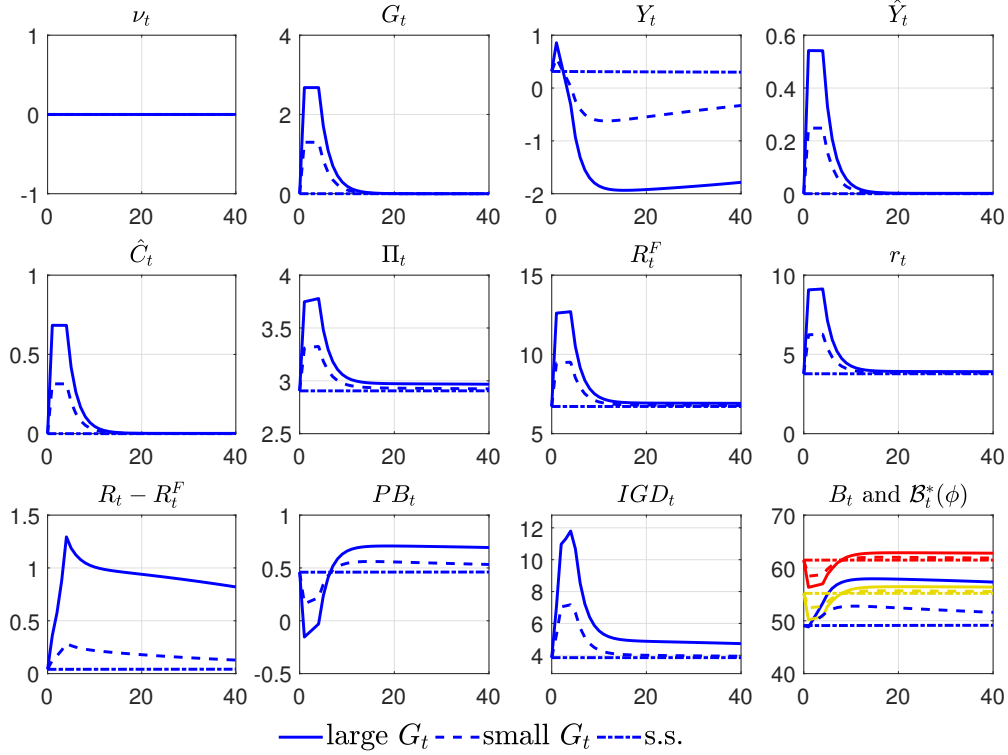
Note: In the charts, the x-axis time is expressed in quarters, while in the y-axis ν_t, G_t and Y_t are expressed in deviations from their steady-state levels as percentage of, respectively, Y, $\hat{Y}_t$ and $\hat{C}_t$ (output and consumption gaps) in percentage deviations of Y_t and C_t from their potential levels (obtained setting the coefficient of price-adjustment costs $\phi = 0$), Π_t, R_t^F, r_t (real interest rate $R_t - E_t[\Pi_t]$), $R_t - R_t^F$ (spread) and IGD_t (interest-growth differential $R_t - \Delta Y_t/Y_{t-1}$) in annualized percentage points, PB_t (primary balance) as a percentage of Y_t, B_t, $\mathcal{B}_t^*(\phi)$ in percentage of annual Y_t. In two right-most chart of the bottom row, the yellow and the red lines indicate the evolution of the fiscal limit corresponding to an annualized default probability ϕ equal to 5% and 25%, respectively.

horizon T. Over a horizon of 10 years ($T = 40$ quarters), the small G_t shock of figure 5 corresponds to the fiscal margin leading the government to default with only a 3.2 percent (annualized) probability ϕ or, formally, $FM_{t,T}(3.2\%) = \{G_s = 1.5\%Y\}_{s=t}^{t+4}$. The large G_t shock would instead be the fiscal margin associated to a 15.8% default probability or, formally, $FM_{t,T}(15.8\%) = \{G_s = 3\%Y\}_{s=t}^{t+4}$. From a policy-maker perspective seeking to keep default probability below the more conservative sustainability threshold of 5% (yellow line), the small G_t shock would thus be consistent with debt sustainability, whereas a spending shock twice as large would breach the set threshold. It is important to note, moreover, that a fiscal margin associated with excessive probability thresholds may not be

compatible with stable debt dynamics, as defined, for instance, by Ghosh et al. (2013).[18]

As a consequence, during normal times, fiscal authorities need to carefully assess the trade-off between sustainability and support to economic activity when deciding whether to implement spending shocks. This trade-off becomes particularly relevant in the case of countries with large stocks of debt, in which the increase in spreads has a bigger impact on debt dynamics.

5.2   The transmission of public spending shocks in different monetary policy environments

In this section, we first analyze how a binding ZLB affects the transmission mechanism of spending shocks with endogenous fiscal limits. This will allow us to assess how the ZLB alters the trade-off between sustainability and stabilization, with a look also at the role of the timing of the shock . In a second step, we analyze how different degrees of activeness of monetary policy affect the results from the first exercise, both during normal periods and periods of binding ZLB.

Figure 6 presents the baseline scenario, in which a four-quarter sequence of unanticipated negative shocks to consumption preferences ν_t brings the economy to the ZLB for more than two years.[19] The series of shocks ν_t depresses demand for consumption goods – as shown by the low consumption gap $\hat{C}_t$ – and inflation Π_t, with detrimental consequences for economic activity – as shown by the low output gap $\hat{Y}_t$.[20]

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[18]In a partial-equilibrium backward-looking framework, Ghosh et al. (2013) define the debt limit as the debt level beyond which debt dynamics become explosive. In a sample of 23 advanced countries between 1970 and 2007, their estimated debt limits have a mean, median and minimum of 195.7 percent, 191.6 percent and 152.3 percent of GDP, respectively. In our calibration based on EU-14 data from 1971 to 2007, our debt may already become unsustainable at a debt-to-GDP ratio above 75 percent even in the absence of shocks (not shown in charts).

[19]The negative taste shock corresponds to a value for ν_t aboubt 7 standard deviations below the steady-state value. Importantly, nothe studied case is within the grid of points used for the numerical solution.

[20]Notice that the large drop in output gap (with a trough of -6.7 percent) is smaller than the output gap loss (with a trough of -10 percent) after an 8-quarter liquidity trap in the non-linear model presented by Lindé and Trabandt (2018) in their Figure 2. In contrast, in their model, the deflation ensuing the negative demand shock has a smaller magnitude (an annualized inflation rate about -2 percent) compared to the one in this model (-10 percent). Although both models are solved with global solution methods and embed a similar structure of the economy, Lindé and Trabandt (2018) use a Kimball aggregator with Calvo pricing to generate a flat Phillips curve, which tends to mitigate inflation fluctuations. However, in the case of Rotember pricing (as in our model), using a Kimball aggregator would not help, as the additional parameter associated with the Kimball aggregator would not show up separately from the elasticity of substitution between goods in the Phillips curve. Moreover, shifting to Calvo pricing would require adding a state variable with detrimental implications on computational time. Further sources of different quantitative impact compared to other related studies (Bi, Leeper and Leith, 2013, 2018), such

Figure 6: Impact of a preference shock leading the economy to the ZLB


ZLB ——— s.s. - - -
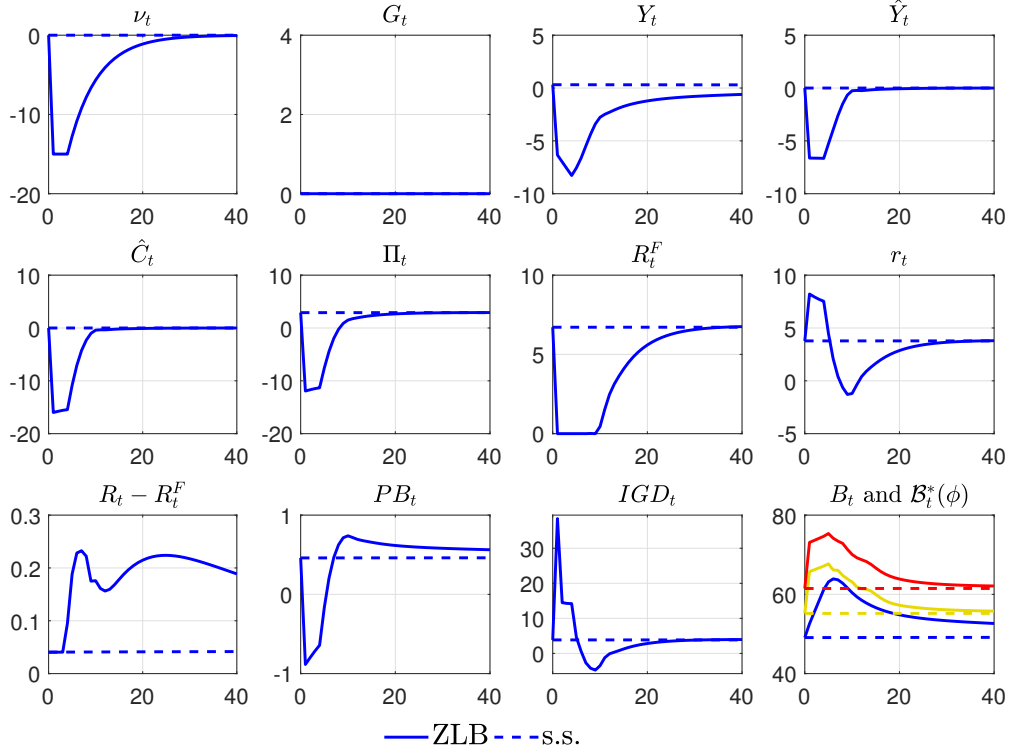
Note: See note of Figure 5.

The output contraction leads to an increase in the debt ratio driven by both a deterioration of the primary balance and an increase in the interest-growth differential. The latter, however, is relatively short-lived, because of the sudden monetary policy reaction that follows the contraction in inflation. The spread increases during the first five quarters to a peak of about 20 basis points (corresponding to an annualized default probability of 2.8 percent), where it returns as debt enters the same area of the risky zone again after 20 quarters. The spread then gradually declines as the debt-targeting tax rule increases the primary balance and brings the debt ratio back to its steady-state level.

Figure 7 shows the impact of the two aforementioned spending shocks of 1.5% (small G_t) and 3% (large G_t) of steady-state output at the ZLB, corresponding to probability thresholds of 2.2% and 1.7%, respectively, or, formally, $FM_{t,T}(2.2\%) = \{G_s = 1.5\%Y\}_{s=t}^{t+4}$ and $FM_{t,T}(1.7\%) = \{G_s = 3\%Y\}_{s=t}^{t+4}$. The lower probability thresholds for the fiscal margin

as our choice of large values for the policy parameters, do not however affect the qualitative implications of our model. Results with different calibrations are available from the authors upon request.

Figure 7: Impact of different government spending shocks at the ZLB
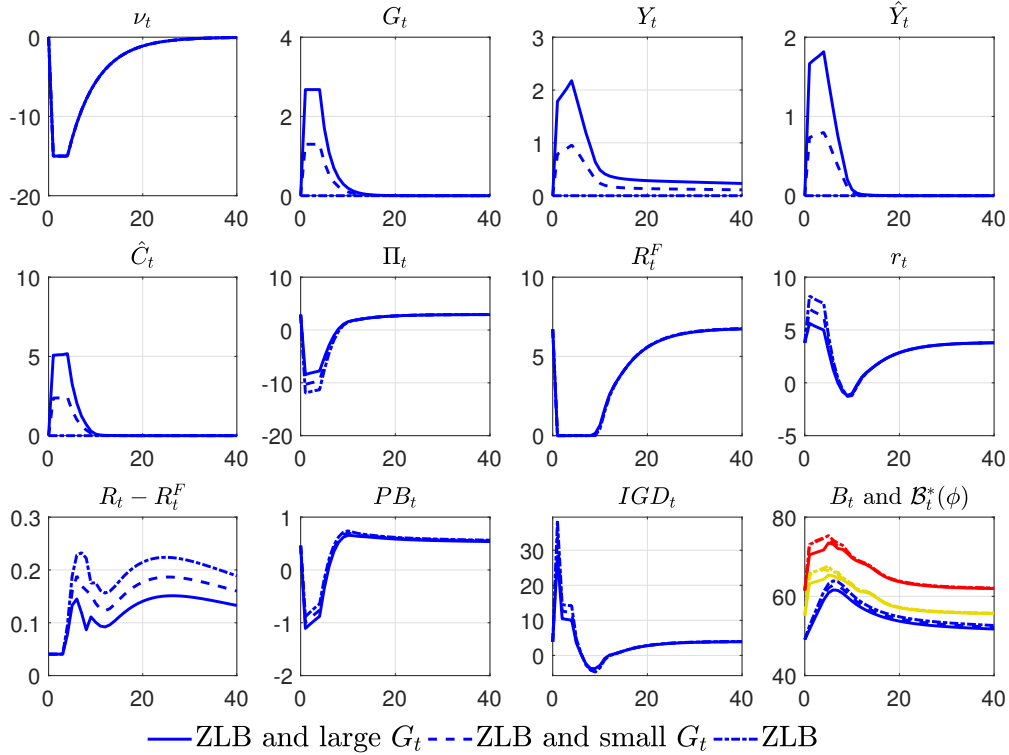
Note: See note of Figure 5. However, Y_t is reported in percentage deviation from its level given only the taste shock ν_t, while $\hat{Y}_t$ and $\hat{C}_t$ (output and consumption gaps) are reported in differences of from the same metrics obtained given only the taste shock ν_t (i.e. without the spending shock G_t).

compared to what implied by the shocks in normal time (see figure 5) confirm that the low-rate environment provides the government with additional leeway to stimulate the economy through public spending.

Compared to normal times, the large G_t shock at the ZLB has almost four times as large a positive effect on the output gap, as it limits deflation and depresses the real interest rate to a greater extent.[21] Hence, despite the shift of the fiscal limit distribution to the left triggered by the shock, the boost in demand more than offsets the rise of the spread, thus limiting the rise in the interest-growth differential and the decline in the primary balance, with a final net positive impact both on output and debt dynamics. These positive developments are more pronounced with a larger G_t shock.

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[21]The large economic impact of fiscal policy shocks at the ZLB is in line with the empirical literature (see, among others, Christiano, Eichenbaum and Rebelo, 2011) and theoretical literature (see, among others, Woodford, 2011).

Figure 8: Impact of government spending shocks with different timing at the ZLB

——ZLB and large delayed G_t - - -ZLB and large G_t ----ZLB
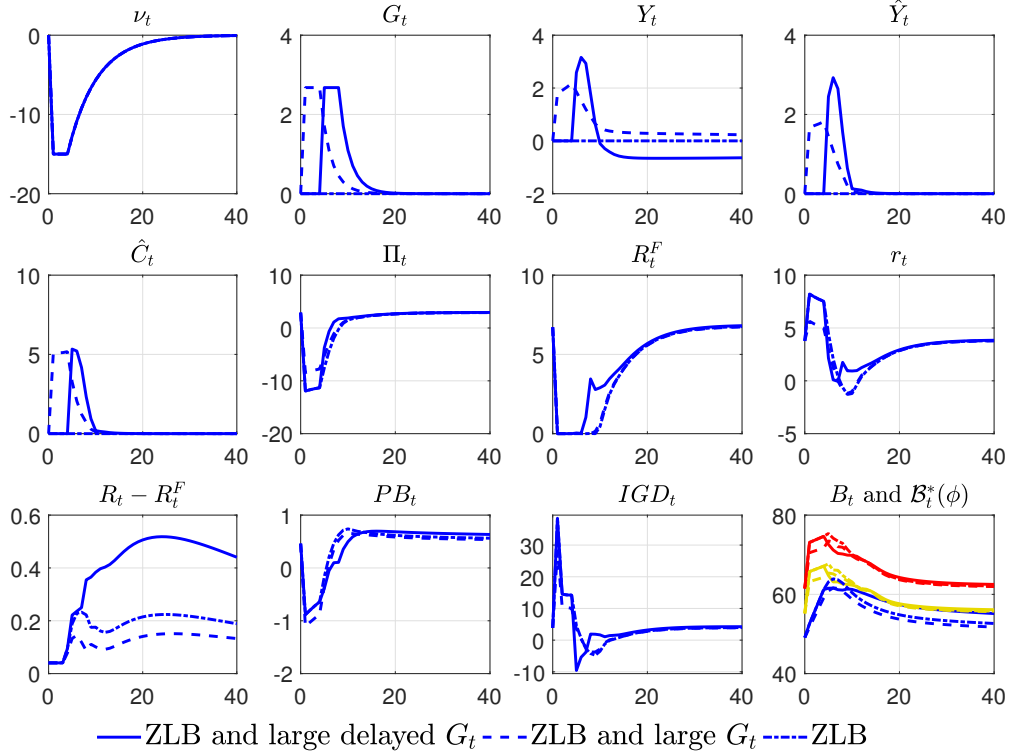
Note: See note of Figure 7.

The dynamics shown in figure 7 point to a sizable fiscal margin available to the government to stabilize the economy at the ZLB, as the spending shock may offset the dead-weight loss generated by the constrained monetary policy. Importantly, in our calibration, the extent of the trade-off between stabilization and sustainability in the use of government spending does not hold during periods of binding ZLB, as our simulated spending shocks bring benefits under both dimensions.

However, the reduction in the extent of the trade-off between stabilization and sustainability depends crucially on the timing of fiscal policy shocks. As shown in Figure 8, a large G_t shock delayed by one year may still have positive implications in terms of stabilization, possibly even accelerating the exit from the ZLB. However, this shock would imply a stronger deterioration in terms of debt sustainability, due to a more pronounced rise in spreads, associated with a higher probability of default (4.3 percent) and hence a smaller fiscal margin $FM_{t,T}(4.3\%) = \{G_s = 3\%Y\}_{s=t+4}^{t+8}$, compared to a timely shock of similar magnitude.

28

Figure 9: Impact of a government spending shock with different degrees of monetary policy activeness in inflation targeting
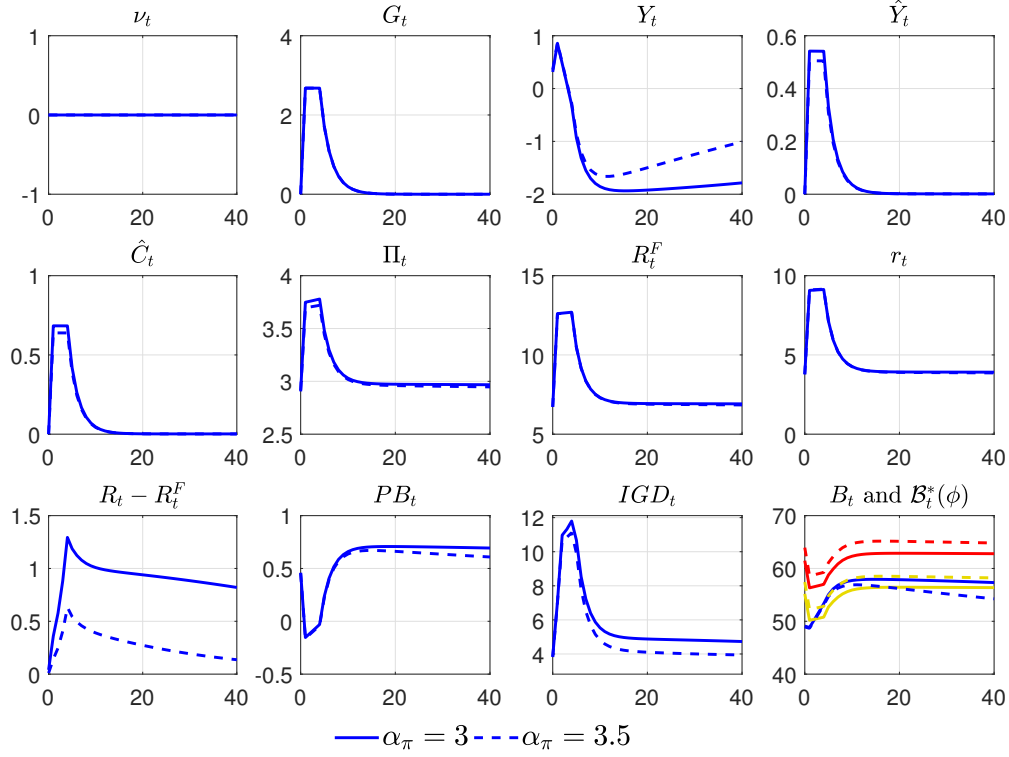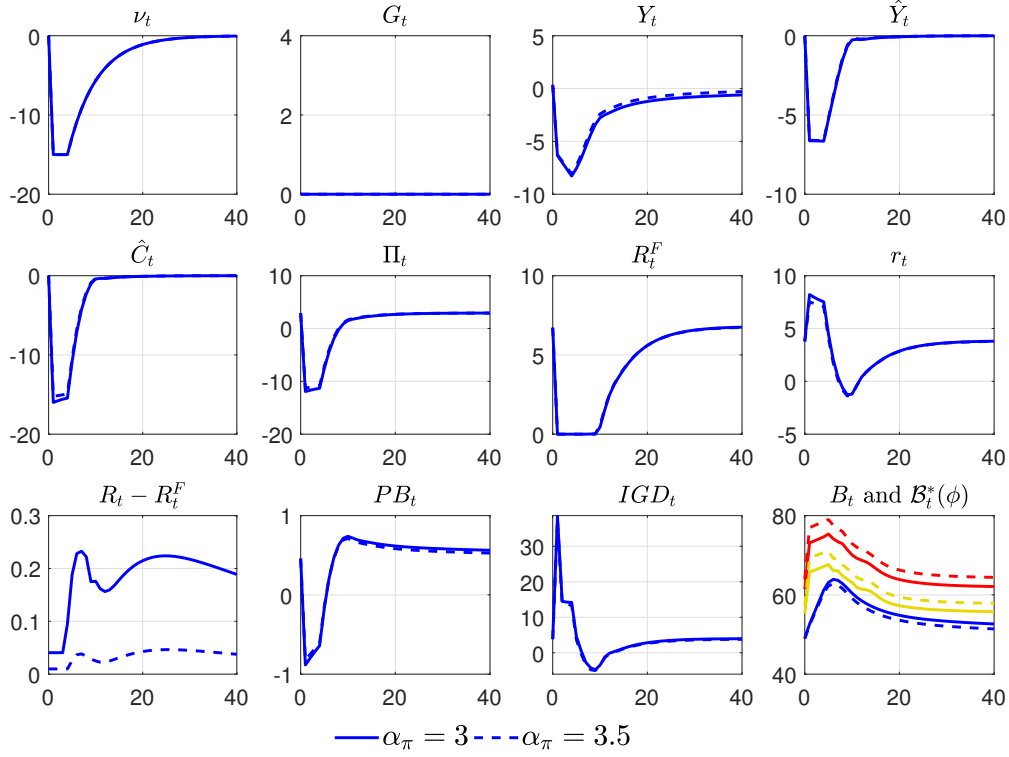

Note: See note of Figure 5.

As a next step, we analyze the effect of the monetary policy activeness on sustainability during periods of binding ZLB and on the transmission of spending shocks. This analysis shows how the conduct of monetary policy outside the ZLB can actually affect the impact of policy interventions within ZLB periods by changing the way the sustainability risks are assessed. In line with the previous analysis and differently from the existing literature focusing on out-of-ZLB inflation targets (see Eggertsson and Woodford, 2003, Christiano, Eichenbaum and Rebelo, 2011, Coibion, Gorodnichenko and Wieland, 2012, and Fernández-Villaverde et al., 2015), we focus on the degree of responsiveness of monetary policy to changes in inflation α_π.[22]

Figure 9 shows how the transmission mechanism of spending shocks changes under our baseline calibration ($\alpha_\pi = 3$) and a more active monetary policy ($\alpha_\pi = 3.5$). The shock occurs in normal times. In line with the results of section 4.2, the fiscal limits' distribution

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[22]Results are qualitatively similar considering government spending shocks and changes in the degree of responsiveness of monetary policy to output gap α_y. See Figures 9, 15 and 16 in Appendix B.

Figure 10: Impact of a preference shock leading the economy to the ZLB with different monetary policy activeness in inflation targeting

Note: See note of Figure 5.

shifts to the right with a higher α_π. This leads to lower spreads and to a higher output dynamics (although still with lower output levels compared to the steady-state), and a slightly lower profile of debt, mostly due to the reduced interest-growth differential.

Figure 10 shows the impact of the negative consumption preference shock ν_t leading the economy to the ZLB. While the output dynamics are similar, the impact on sustainability changes considerably. With the higher α_π, the right shift of the fiscal limit distribution tames the response of the spread to ν_t. As shown in Figure 3, this is the result of the lower real losses due to the smaller inflation fluctuations. Overall, confirming our results from Section 4, a more active monetary policy activeness outside the ZLB changes the way risk is evaluated also within periods of binding ZLB, further thwarting debt sustainability concerns. This, in turn, creates additional space for fiscal policy to stabilize output.

6 Conclusions

In this paper, we analyzed fiscal sustainability and monetary-fiscal policy interactions through the lens of a general equilibrium model with risky sovereign debt à la Bi (2012). We showed that, when a negative demand shock drives the economy to the ZLB, fiscal policy incurs a dead-weight loss as it seeks to imperfectly substitute for a constrained monetary policy, thus impairing debt sustainability. However, a higher degree of activeness in the ordinary conduct of monetary policy, as well as unexpected declines in the risk-free rate, may significantly enhance the sustainability of public debt.

Moreover, we put forward a novel measure to assess the leeway for the government to stabilize the economy while preserving debt sustainability at a specific threshold for the default probability. This measure – labelled as " fiscal margin" – is fully consistent with the probabilistic notion of fiscal limits and takes into account the forward-looking nature of agents' decisions in general equilibrium. We apply the notion of fiscal margin in two exercises. First, we assess the sustainability of government spending shocks in normal times and at the ZLB. In normal times a large spending shock leading to high spreads and debt may bring about a recession more than offsetting the initial expansion. At the ZLB may stimulate the economy and, at the same time, reduce debt sustainability concerns. Second, we compare the effect of different degrees of activeness of the monetary policy following a government consumption shock in normal times and at the ZLB. In line with our previous results, a more active monetary policy is more effective at stabilizing inflation, reducing economic fluctuations and mitigating debt sustainability concerns in normal times and, even more so, during periods of binding ZLB, by affecting forward-looking agents' expectations.

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Appendix A   Solution algorithm

Before calculating the fiscal limit distribution via Monte Carlo simulations, the absence of functional forms for all the endogenous variables requires the solution of part of the non-linear model (i.e., for the real wage, the inflation rate and the maximum tax rate), with numerical methods.

The solutions of both the first and the second stage are based on the monotone mapping method, developed by Coleman (1991) and Davig (2004), which discretizes the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. The decision rules for the real wage $W_t^* = f^w(\psi_t^j)$, the inflation rate $\Pi_t^* = f^\pi(\psi_t^j)$ and, in the first stage, the maximum tax rate $\tau_t^* = f^\tau(\psi_t^1)$ or, in the second stage, the optimal debt level $B_t^* = f^b(\psi_t^2)$ are solved in the following steps, for stage $j = 1, 2$.

1. Discretize the state space $\psi_t^1 = \{A_t, \nu_t, G_t, \eta^R\}$ and $\psi_t^2 = \{B_t^d, A_t, \nu_t, G_t, \eta^R\}$. The number of grid points and state variables actually considered can vary depending on the problem at hand, in order to deal with curse of dimensionality. In the first stage, in general $n_\nu = 19$ (except when A_t is also considered, in which case $n_\nu = n_A$), while – when needed – $n_A = 11$, $n_G = 7$ and $n_{\eta^R} = 7$. In the second stage, the model is solved with $n_B = 40$, $n_\nu = 19$ and $n_G = 7$. In each chart, reported simulations do not show results based on extrapolations outside the considered grids.

2. For $i = 1, 2, ...$, make a guess for the decision rules $(f_g^w, f_g^\pi, f_g^\tau, f_g^b)$ over the state space. If $i = 1$, set $(f_g^w, f_g^\pi, f_g^\tau, f_g^b)$ to their steady state values; if $i > 1$, set $(f_g^w, f_g^\pi, f_g^\tau, f_g^b)$ to the solutions in the previous iteration $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau, f_{i-1}^b)$.

3. At each grid point, solve the model and obtain the updated rule $(f_i^w, f_i^\pi, f_i^\tau, f_i^b)$ using the given rule $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau, f_{i-1}^b)$ as a guess. Given equations (4), (16), (21), (23) and, in the second stage, (6) to pin down $(n_t, \Upsilon_t, R_t^F, c_t, R_t)$, respectively, use the model equations (5), (15) and (20) in the first stage or (17) in the second stage to solve the non-linear model and determine the decision rules $(f_i^w, f_i^\pi, f_i^\tau, f_i^b)$. In the first stage, in particular, maximize (20) subject to the other equilibrium conditions and non-negativity constraints on endogenous variables as appropriate. The integrals implied by the expectation terms are evaluated using numerical quadrature. The exogenous AR(1) processes are approximated as first-order Markov processes according to the quadrature approach by Tauchen and Hussey (1991).

4. Notice that $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau, f_{i-1}^b)$ are assumed to be decision rules at t+1 when eval-

uating expectations, as they provide a set of intra-temporally consistent solutions for the optimizing agents. To ensure that the solution is also inter-temporally consistent, establish a rule to check convergence of the decision rules $(f_i^w, f_i^\pi, f_i^\tau, f_i^b)$ and $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau, f_{i-1}^b)$ as follows:

(a) if $\max\{|(f_i^w, f_i^\pi, f_i^\tau, f_i^b) - (f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau, f_{i-1}^b)| > tol\}$, where the tolerance level tol is $1e-5$ in the first stage and $1e-4$ in the second stage, go back to step 2;

(b) otherwise, $(f_i^w, f_i^\pi, f_i^\tau, , f_i^b)$ are the decision rules.

At the end of the first stage, run 2,000 simulations starting at every point in the grid of exogenous state variables ψ_t^1 for 200 periods to derive the conditional distribution of fiscal limits to be used in the second stage.

Finally, notice that a "solution of the model" includes a set (i.e., matrix) of one-to-one relationships between a specific value for the vector of state variables and a specific value for the vector of control variables. So, the model needs to be solved only once before the simulation of the fiscal limit distribution. When we calculate the fiscal limit distributions all the control variables are readily available as a function of the state variables either through functional forms or through the one-to-one relationships between state and control variables established in our first step.

Appendix B Additional results on fiscal limits, monetary policy and spending shocks

Figure 11: Impact of TFP shocks and consumption preference shocks on fiscal limit distributions with constrained and unconstrained monetary policy
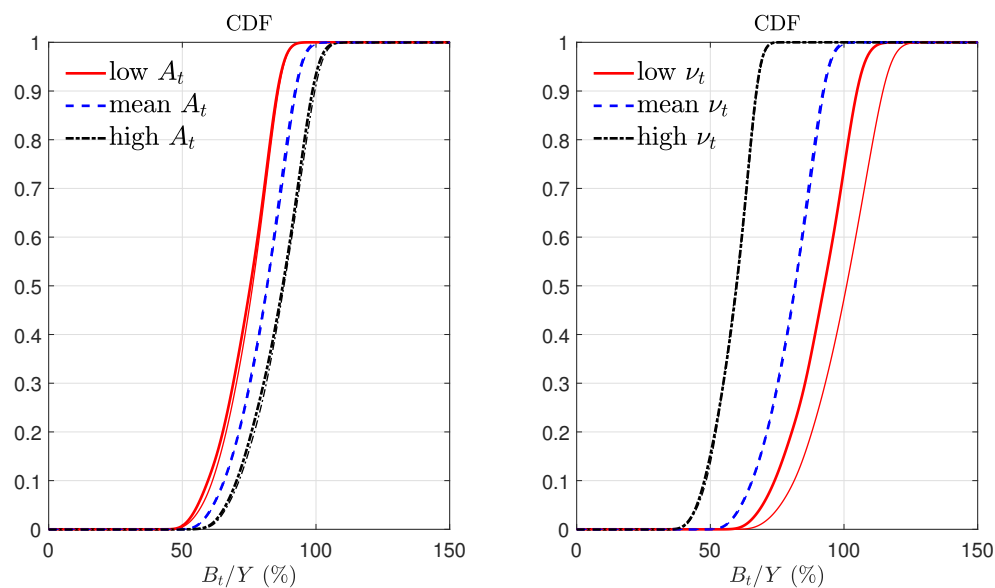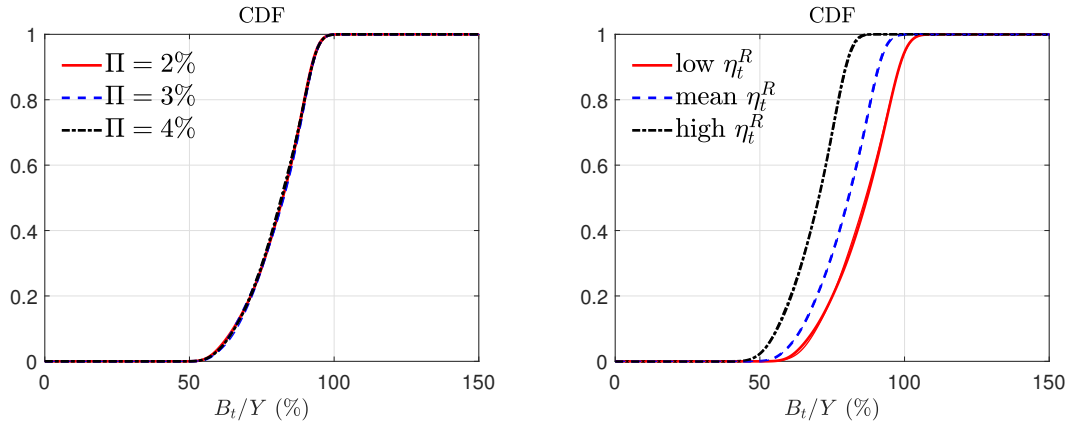

Note: In both charts, the thick (thin) lines report the fiscal limit distribution with constrained (unconstrained) monetary policy, i.e. imposing (not imposing) the ZLB constraint.

Figure 12: Impact of monetary policy activeness on fiscal limit distributions with constrained and unconstrained monetary policy
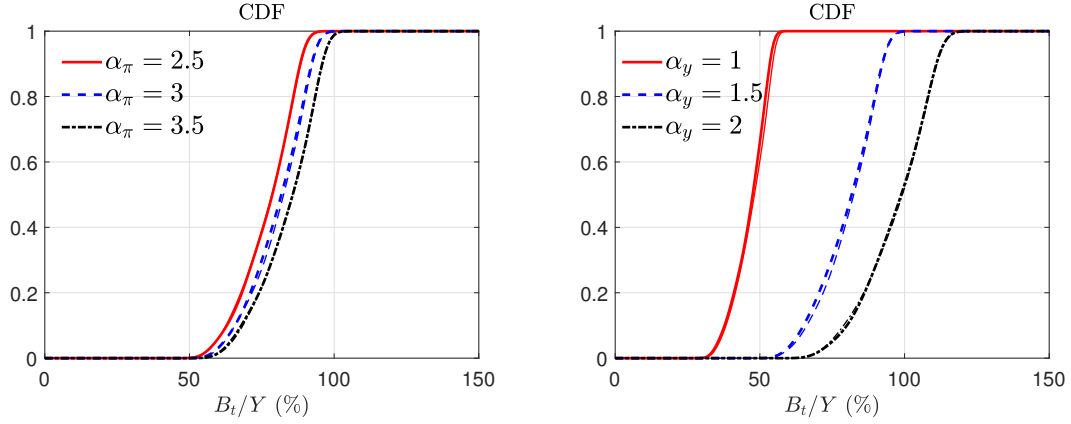

Note: In both charts, consumption preference shock is initially set at its steady state. In both charts, the thick (thin) lines report the fiscal limit distribution with constrained (unconstrained) monetary policy, i.e. imposing (not imposing) the ZLB constraint.

Figure 13: Impact of changes to the inflation targets and monetary policy shocks on fiscal limit distributions with constrained and unconstrained monetary policy

Note: In both charts, consumption preference shock is initially set at its steady state. In both charts, the thick (thin) lines report the fiscal limit distribution with constrained (unconstrained) monetary policy, i.e. imposing (not imposing) the ZLB constraint.

Figure 14: Impact of different government spending shocks in normal times in models with and without default
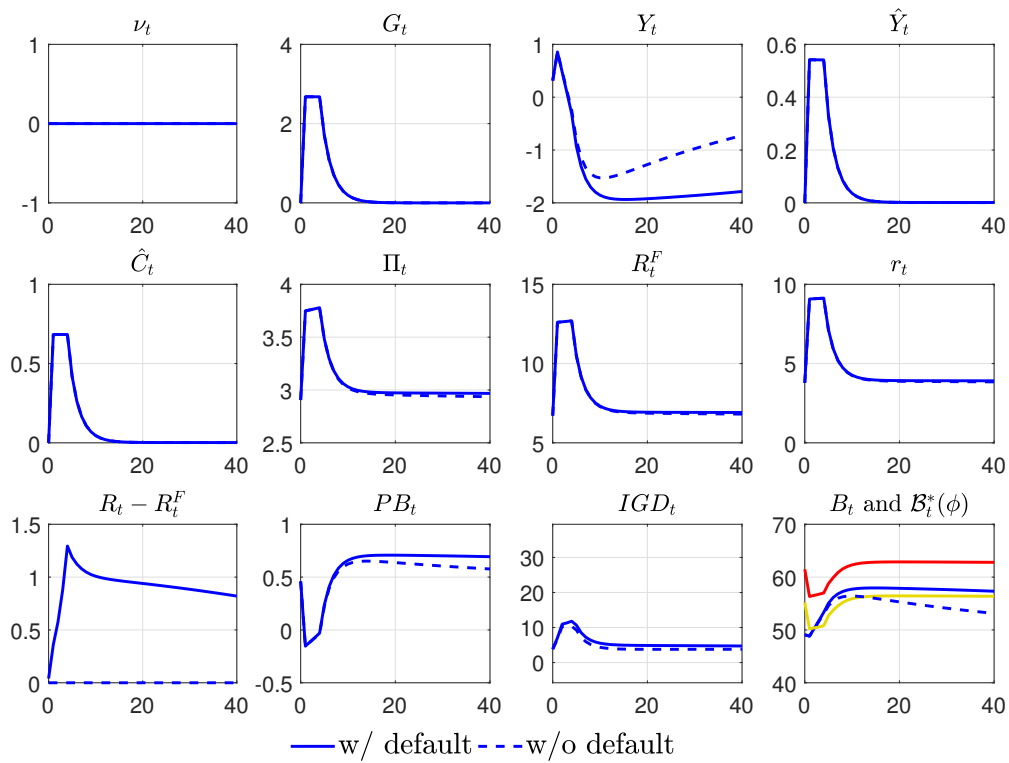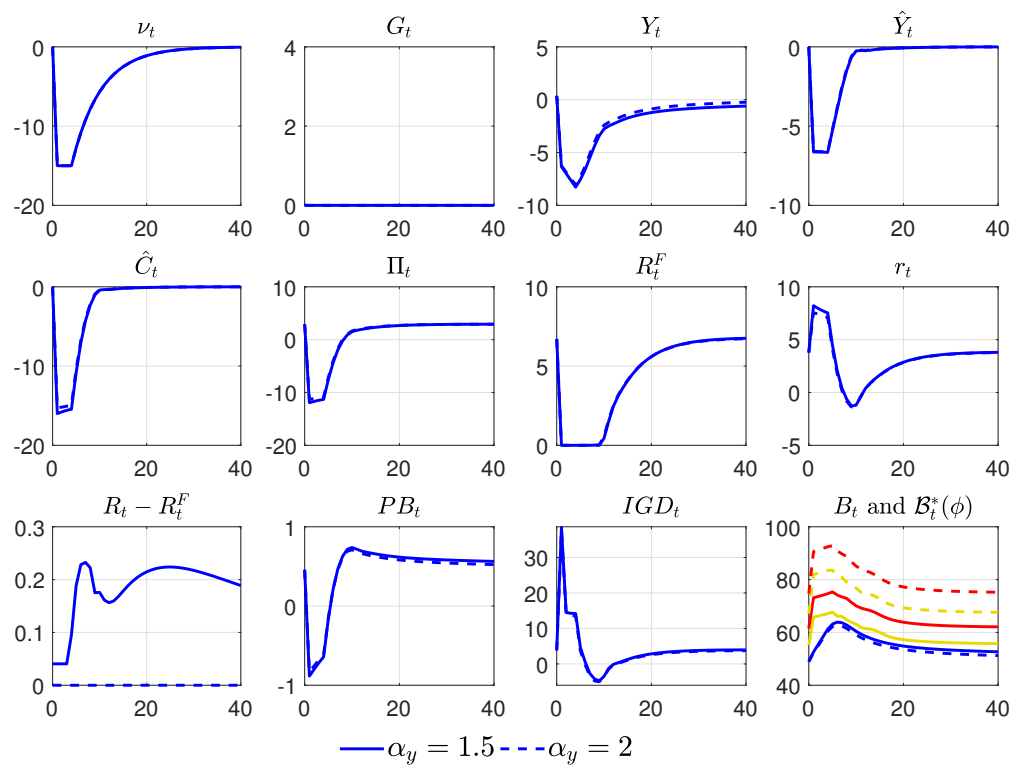

Note: See note of Figure 5. To obtain the response of variables without default, the model was solved assuming a haircut of 0 percent.

Figure 15: Impact of a government spending shock leading the economy to the ZLB with different monetary policy activeness in output gap targeting
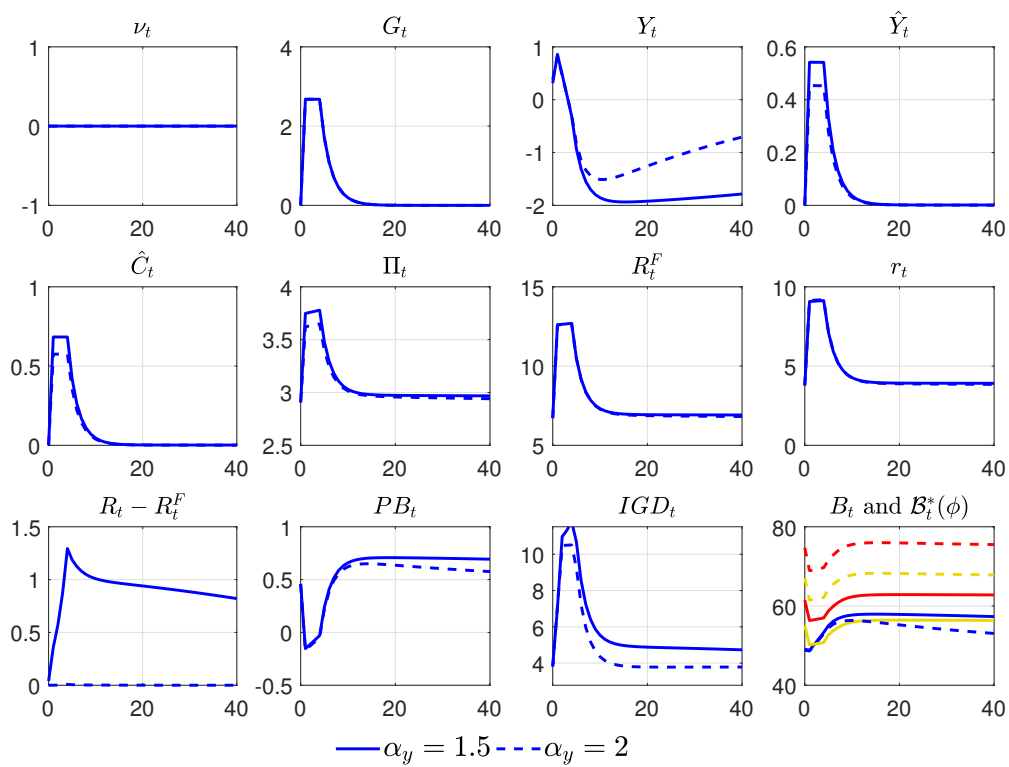
Note: See note of Figure 5.

Figure 16: Impact of a preference shock leading the economy to the ZLB with different monetary policy activeness in output gap targeting

Note: See note of Figure 5.