Stable Marriage and Children: Child Custody and Intrahousehold Sharing*

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Abstract

We present a revealed preference framework to study sharing of resources in households with children. Children's consumption is often treated as a public good in the household. However, it is different from other public goods (like housing or transportation) in a sense that it may not be internalized by a new partner (in case of remarriage). Moreover, the "technology" used to produce children's consumption after divorce is determined by the custody legislation in place. We explicitly model the impact of presence of children in the context of stable marriage markets under three types of custody arrangement (no legislation, single person custody and joint custody). The models deliver testable revealed preference conditions and allow for the identification of intrahousehold allocation of resources. An empirical application of these conditions to a survey data of US households shows that children and related custody laws are important factors affecting intrahousehold sharing. Incorporating these factors support many empirical findings that couples with children are more stable than couples without children. In addition, ignoring these factors leads to biased intrahousehold allocations. Specifically, our analysis shows that taking these factors into account increases the recovered women's share, on average, by 7%.

1 Introduction

Presence of children can have a substantial impact on the divorce (and remarriage) of married individuals. When the potential utility from outside options in the marriage market is seen as a threat point in intrahousehold bargaining analysis, the associated welfare effects from having children can be substantial. In this paper, we propose a structural method to account for the impact of children on couple's intrahousehold bargaining analysis when the threat points are the outside marital options. An important aspect of our methodology is that it is nonparametric in nature, as such it does not impose any parametric/functional form assumption on individual utilities,

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threat points or the decision making process. We study three scenarios of post divorce custody arrangements: no legislation, single person custody and joint custody. Through an empirical application to a household data taken from the Panel Study of Income Dynamics (PSID), we show that accounting for children and related custody laws are important factors affecting intrahousehold resource allocation. Incorporating these factors in the analysis support the empirical findings that couples with children are more stable as compared to childless couples. Further, the recovered share of resources for women is, on average, 7% higher as compared to the analysis when these factors are ignored.

Households consist of multiple decision makers with potentially different preferences. The cooperative bargaining model, also referred to as the collective model (Chiappori, 1988, 1992) has now become the go-to model for analyzing household decisions. The only requirement of this framework is that the outcome of the intrahousehold bargaining process be Pareto efficient. Even with this generality, the collective model generates implications which can be tested on the data (see Chiappori and Ekeland (2006, 2009)). The need to develop this framework arises because almost all expenditure surveys collect information about household (not individual) consumption.¹ In the absence of direct information on 'who gets what' in the household, one has to rely on economic theories to estimate the intrahousehold resource allocation (sharing rule) (see Chiappori and Ekeland, 2006, 2009). While empirical studies have shown that the sharing rule depends on individual's outside options, wages, prices and other distribution factors, the collective model itself does not explicitly model how these factors affect the intrahousehold allocation.² A natural way to account for the outside options as a bargaining instrument is to merge the collective models with the implications of marriage markets (stable marriage model).³ The basic intuition behind stable marriage models is that marriage generates utility and individuals act rationally when choosing their partners. In this paper, we use the collective models and stability of marriages as the key identifying assumption for household consumption analysis. Our main focus is on incorporating the impact of children and custody laws in the stable marriage market framework.

Most collective models treat children's consumption as a public consumption within the household (see e.g. Blundell, Chiappori, and Meghir, 2005; Cherchye, De Rock, and Vermeulen, 2012). Well being of children, characterized via expenditures on children, is assumed to give utility to both the parents. However, children's consumption is crucially different from other public goods such as housing or transportation. While public goods remain public no matter whom the individual is married to, children's consumption delivers utility only to the parents of the children. Hence, while children's consumption can be considered a public good for the parents, it is not guaranteed

¹ Some notable exceptions are Danish household survey used by Browning and Gørtz (2012), Dutch household data used by Cherchye, De Rock, and Vermeulen (2012) and Japanese data used by Lise and Yamada (2014).

 $^{^{2}}$ A distribution factor is a variable which affects the intrahousehold distribution of resources by shifting the bargaining power (or Pareto weight) but does not affect individual preferences. For instance, (Chiappori, Fortin, and Lacroix, 2002) use sex ratio and divorce laws as distribution factors. Other examples are wage ratio and age difference (Cherchye, De Rock, and Vermeulen, 2012), wealth at marriage (Thomas, Contreras, and Frankenberg, 1997), property division laws and grounds for divorce (Voena, 2015).

³ See Gale and Shapley (1962); Becker (1973, 1974); Becker, Landes, and Michael (1977) for the early contributions on stable matching and economic analysis of marriage markets.

that, upon remarriage, the new partner (who has no biological relation to the child) would receive any utility from the step-children's consumption. Thus, children's consumption can be seen as "marital-specific capital" which is likely to increase the attractiveness of current marriage and decrease individual's attractiveness for new partners (see White and Booth (1985); Waite and Lillard (1991); Buckle, Gallup Jr, and Rodd (1996)). In addition, while children's consumption decisions can be made cooperatively when the parents are living together, the mechanism through which the parents provide children's consumption after divorce depends on the child custody law. Since a significant share of household's budget is spent on children, it is crucially important to incorporate these factors while analyzing household consumption decisions from the perspective of marital stability.⁴

To take into account the impact of children, we assume that only parents (but not potential partners) internalize children's consumption.⁵ Further, we consider three scenarios through which child custody legislation can affect post-divorce children's consumption: (1) no legislation (2) single-person custody and (3) joint custody. Under no legislation, we consider a setting in which there is no legal enforcement governing couples behavior after divorce. Next, under single-person custody, one of the parents (custodian) is assigned legal and physical custody over children and the non-custodian parent is supposed to make child support payments to the custodian. Lastly, under joint custody, we consider a scenario where both parents have legal and physical custody over children. In such a setting, children spend some time with both the parents and the big decisions related to children are decided jointly by both the parents.

We make two main contributions to the literature. First, from a methodological perspective, we extend the model of Cherchye, Demuynck, De Rock, and Vermeulen (2017a) (henceforth referred to as CDDV) to incorporate children's consumption and child custody law in modeling household consumption decisions. A key feature of our models is that we use revealed preference framework in the spirit of Samuelson (1938); Afriat (1967); Varian (1982) which avoids erroneous conclusions due to parametric misspecification of the utility functions and allows for fully heterogeneous individual preferences. Recent work by Cherchye, De Rock, and Vermeulen (2007, 2009, 2011a) have extended the revealed preference framework to a collective household setting. Our models also connect to different extensions of the methodology presented in CDDV. In particular, Cherchye, Demuynck, De Rock, and Vermeulen (2017b) show that the revealed preference conditions presented in CDDV are also sufficient for stable matching. Cherchye, De Rock, Surana, and Vermeulen (2016a) use

⁴ On average, children's consumption accounts for 26% of the total household expenditure for one child families to 49% for three children families (see Lino et al. (2017)).

⁵ Similar assumption has been used to model children's consumption, e.g. by Chiappori and Weiss (2007). Moreover, it has indirect empirical support. Bramlett and Mosher (2002) show that children in step-families are equally likely to be adversely affected as children from a single-parent family. Children from such households do worse than children living with both biological parents. Ermisch and Pronzato (2008) look at the remarried fathers with kids in the previous marriage and find that the child support amount transfered for the children is positively correlated with the share of income father brings to the new couple. Had the preference for welfare of stepchildren internalized by the new partner, the transfers should not be correlated with the share of income father brings. Further, this is consistent with the idea of kin selection (introduced by Hamilton, 1964a,b), a theory from evolutionary biology which can be used to explain unconditional altruism. Kin selection has supporting evidence for humans (see e.g. Smith et al., 1987; Madsen et al., 2007) as well as for other biological species.

stability restrictions to identify intrahousehold economies of scale and Cherchye, De Rock, Walther, and Vermeulen (2016b) extends the framework to include household production.

Second, we show the importance of taking account of children's consumption and child custody law empirically by taking our models to a household data drawn from the 2015 wave of Panel Study of Income Dynamics (PSID). Our methodology allows us to (set) identify intrahousehold allocations through which we compare household resource allocation under three sets of assumption: (1) when we do not account for both children's consumption and custody law (CDDV), (2) when we account for children's consumption but ignore custody law (no legislation) and (3) when we account for both children's consumption and child custody law (single person and joint custody). Our empirical application shows that children's consumption and related custody laws are important factors affecting intrahousehold sharing and ignoring these factors would lead to biased intrahousehold allocations. In particular, the results show that couples with children are more stable as compared to couples without children and the recovered share of resources for women is, on average, 7% higher when these factors are taken into the analysis. These findings are also empirically supported by various studies that show that a) the probability of remarriage or new cohabitation is lower among individuals with children from prior marriage and b) the probability of marital dissolution decreases with childbearing (see Bramlett and Mosher (2002); Sweeney (2010); Fine and Harvey (2013)). This application also contributes to the literature on the connection between divorce laws and household decisions. Several studies have shown that divorce legislation (including child custody legislation) are important distribution factors affecting household decisions. Chiappori, Fortin, and Lacroix (2002) shows that divorce laws affect household decisions and females' labor supply.⁶ Rasul (2005); Stevenson and Wolfers (2006); Voena (2015) study the effects of unilateral divorce on household decisions. Rangel (2006) and Chiappori, Iyigun, Lafortune, and Weiss (2017) study the effect of the alimony (spouse support transfers) on the households' decisions. Finally, Rasul (2006) and Chiappori and Weiss (2007) present theoretical models for the provision of children's consumption under different custodial arrangements and compares welfare of children for different types of custody laws.

The remainder of this paper is organized as follows. Section 2 provides the theoretical models based on the revealed preference characterization of stable marriage markets under different custody arrangements. Section 3 presents the empirical application. Section 4 provides concluding remarks. All proofs are collected in the Appendix.

2 Theoretical Framework

In this section, we lay out three models of stable matching with children's consumption and child custody. In all three models, we assume that individuals take into account the differential nature of children's consumption when considering their outside marriage options. By differential nature we mean that unlike other public goods which can be shared with anyone, children's consumption

 $^{^{6}}$ In particular, Chiappori, Fortin, and Lacroix (2002) summarizes the entire divorce legislation of each state in the single index. This index shows to what extent the divorce custody is favorable towards women.

is always a public good (only) for the biological parents, whether married or separated. The nonbiological parent does not get utility from the step-children's consumption. The difference between the three models comes from the different variants of the child custody legislation in place. In the first case, we assume that individuals do not account for the custody law when considering their outside options (*no legislation*). In both the second and third case, we assume that individuals take into account the child custody legislation. Specifically, in the second case, we focus on a setting where one of the parents is a custodian and another parent is obliged to make transfers to the custodian (*single person custody*) and in the third case we focus on a setting where both parents are custodians and there is no direct transfers between the parents (*joint custody*).

We begin by introducing our household consumption setting and a formal definition of stable matching. Next, we present the revealed preference conditions for a stable matching with children's consumption but without taking into account the child custody law (no legislation). Subsequently, we present the conditions when child custody is also taken into account through single person custody and joint custody. Finally, we discuss how we include match specific preferences when taking these models to the data. Here we also indicate how we can use these models to (set) identify the intrahousehold resource allocation.

2.1 Preliminaries

Structural components. We consider a marriage market with two finite sets of individuals M (males) and W (females). We observe a matching function $\sigma : M \cup W \to M \cup W$ describing who is married to whom and satisfying the following properties:

- $\sigma(m) \in W$ for every $m \in M$;
- $\sigma(w) \in M$ for every $w \in W$;
- $w = \sigma(m)$ if and only if $m = \sigma(w)$.

In what follows, we focus our attention only on married couples (|M| = |W|). Including singles in the theoretical analysis can be done in a similar way. Moreover, we will also include singles in our empirical application to allow for the possibility that a married individual may consider a single of other gender as a potential outside option.

Given a couple (m, w), denote by $q_{m,w} \in \mathbb{R}^n_+$, the vector of total private consumption, by $Q_{m,w} \in \mathbb{R}^N_+$ the vector of public consumption and by $C_{m,w} \in \mathbb{R}_+$ the children's consumption in the household.⁷ The consumption decisions are made under budget constraints formed by prices and income. Let $p_{m,w} \in \mathbb{R}^n_{++}$ be the price vector of private good and $P_{m,w} \in \mathbb{R}^N_{++}$ be the price vector of the public good faced by couple (m, w). Without loss of generality, we will assume that price of children's consumption is normalized to one. Finally, let $y_{m,w}$ be the total income of the household.

 $^{^{7}}$ To make the exposition simple, we treat children's consumption as a Hicksian aggregate market good for children specific consumption. However, all further results can be provided assuming that children's consumption is a vector.

The total private consumption in the household is divided between individual household members. Let $q_{m,w}^m$ and $q_{m,w}^w$ be the private consumption of m and w such that $q_{m,w}^m + q_{m,w}^w = q_{m,w}$. Recall that we would only observe the aggregate private consumption $(q_{m,w})$ and not the individual shares of private consumption $(q_{m,w}^m, q_{m,w}^w)$. Hence, we treat these individual shares as unknown quantities. Denote by $C_{m,w}^m$ the children's consumption that is enjoyed by m and by $C_{m,w}^w$ the children's consumption that w enjoys. We assume that children's consumption is a public good only among couples where the male and the female are the parents of the children $(C_{m,w}^m = C_{m,w}^w = C_{m,w},$ if $w = \sigma(m)$). This implies that a potential partner would not internalize children's consumption of whom he/she is not a parent of. That is, if (m, w) is a potential match, then m would not get any utility from children's consumption of w (that is $C_{m,w}^w$) and vice-versa. This assumption of children's consumption is an important difference between the models developed in this paper and the existing stable matching model (CDDV). To summarize, for a given household aggregate consumption $(q_{m,w}, Q_{m,w}, C_{m,w})$, household's allocation is defined by $(q_{m,w}^m, q_{m,w}^w, Q_{m,w}, C_{m,w}^w)$.

Two remarks are in order. First, $(q_{m,w}, Q_{m,w}, C_{m,w})$ denote the household consumption of match (m, w). In the data, we will only observe the consumption bundle of matched couples (i.e. when $w = \sigma(m)$). We will treat the consumption bundle of all potential matches as unknown quantities with the restriction that they are feasible under the budget set of the potential couple (m, w) (i.e., when facing prices $p_{m,w}, P_{m,w}$ and having income $y_{m,w}$). Further, the budget set will also depend on the child custody law in place. We will describe this in more details later while talking about particular models of children's consumption. Second, let \emptyset denote the stay alone option of an individual. We will refer to (m, \emptyset) and (\emptyset, w) as outside option of $m \in M$ and $w \in W$ as singles.

Each individual is assumed to get utility from the consumption of n private goods, N public goods and own children's consumption. The preferences of individual i can be described by a continuous, concave and strictly monotone utility function $u_i : \mathbb{R}^n_+ \times \mathbb{R}^N_+ \times \mathbb{R}_+ \to \mathbb{R}^8$

Stable Matching. A matching is called stable if it satisfies individual rationality and has no blocking pairs.⁹ In other words, a matching is stable if it belongs to the core of all matching allocations. We use stability of marriage markets as our key identifying assumption. In particular, stability requires that a) all individuals prefer being in the current match rather than staying alone

⁸ Note that we have restricted individual preferences to be egoistic in nature. However, we do allow for the fact that individual utilities depend on public consumption in the household. Public consumption can be either interpreted as pure public consumption in the household or as private consumption with externalities.

⁹ CDDV also includes Pareto efficiency (collective rationality) as an additional requirement for a stable marriage market. However, they also note that in a cross sectional setting, Pareto efficiency alone does not generate additional testable implication. That is, the identification of intrahousehold allocation is mainly driven by the requirements of individual rationality and no blocking pairs condition. In this paper, we will also maintain the requirement of collective rationality within every couple (current and potential matches). As we will explain later, when formally introducing our models, we assume that children's consumption are always produced by the current spouses (even in case of divorce or remarriage of one of the spouse). We deviate from the assumption of Pareto efficiency for this type of consumption. That is, unlike CDDV, when individuals consider outside options, we allow for inefficiency in public good (children's consumption) to be produced by ex-partners. Thus, if (m, w) is a potential match, we will assume that the consumption decision of (m, w) is Pareto efficient but own children's consumption of m and w, which are produced with ex-partners ($\sigma(m)$ and $\sigma(w)$), may be inefficient.

and b) there is no $m \in M$ and $w \in W$ (with $w \neq \sigma(m)$) such that each of them would be better off by marrying each other rather than in the current match. More formally, we have the following definitions.

Individual rationality. A matching σ is individually rational if all individuals prefer the allocation obtained in the current matching to any of the allocations the individual can get staying alone. Formally, individual rationality (for $m \in M$ and $w \in W$) looks as follows,

$$u_{m,\emptyset} \le u_m(q_{m,\sigma(m)}^m, Q_{m,\sigma(m)}, C_{m,\sigma(m)}^m)$$
$$u_{\emptyset,w} \le u_w(q_{\sigma(w),w}^w, Q_{\sigma(w),w}, C_{\sigma(w),w}^w)$$

where $u_{m,\emptyset}$ and $u_{\emptyset,w}$ are the maximum achievable utilities of m and w when single. The right hand side of the two inequalities are the utilities obtained in the current matching.

No blocking pairs. No blocking pairs condition requires that there are no pairs of male and female who find it preferable to marry each other rather than being with their current matches. That is, (m, w) is a blocking pair if there exists a consumption vector $(q_{m,w}^m, q_{m,w}^w, Q_{m,w}, C_{m,w}^m, C_{m,w}^w)$ achievable for the couple (m, w) such that

$$u_m(q_{m,\sigma(m)}^m, Q_{m,\sigma(m)}, C_{m,\sigma(m)}^m) \le u_m(q_{m,w}^m, Q_{m,w}, C_{m,w}^m) \text{ and,} u_w(q_{\sigma(w),w}^w, Q_{\sigma(w),w}, C_{\sigma(w),w}^w) \le u_w(q_{m,w}^w, Q_{m,w}, C_{m,w}^w)$$

with at least one inequality being strict. The left hand side of these inequalities are the utilities of m and w in their current matches.

Definition 1 A matching σ is stable if it is individually rational and there are no blocking pairs.

Rationalizability. Having defined the concept of stable matching, now we discuss the type of dataset that we will study and the meaning of rationalizability of a dataset by stable matching. We consider a dataset \mathcal{D} with the following information,

- a matching function σ ,
- one household consumption $(q_{m,\sigma(m)}, Q_{m,\sigma(m)}, C_{m,\sigma(m)})$ for all matched couples $(m, \sigma(m))$,
- prices $(p_{m,w}, P_{m,w})$ and income $(y_{m,w})$ faced by all possible pairs (m, w) for $m \in M \cup \{\emptyset\}$ and $w \in W \cup \{\emptyset\}$.

Given a dataset \mathcal{D} , we say that it is rationalizable by a stable matching if there exists at least one allocation for all matched couples such that the matching σ is stable. In what follows, we present the revealed preference conditions for a stable matching under three cases of child custody: no legislation, single person custody and joint custody. These restrictions are necessary conditions that are linear in unknown quantities. The linear nature of these conditions makes them easy to be used for the (set) identification of intrahousehold resource allocations.

2.2 No Legislation

As mentioned above, in this setting, individuals do not take into account the child custody law while evaluating their outside marriage options but they do account for the differential nature of children's consumption. Note that ignoring both the differential nature of children's consumption and child custody would take us back to the basic matching model of CDDV. Differential nature of children's consumption essentially means that it gives utility only to the biological parents of the children. Hence, in the observed matchings, where both spouses are the parents, children's consumption is a public good. Upon divorce (and remarriage), own children's consumption will still be public between the parents (who are now ex-partners). However, step-children's consumption would not generate any utility for the new partner.

In particular, we formulate the differential nature of children's consumption in the following way. Upon divorce, each of the ex-partners will independently contribute to own children's consumption.¹⁰ Since own children's consumption is still a public good for ex-partners, both of them receive utility from the sum of individual contributions to the children's consumption. We assume that the consumption decisions of $(m, \sigma(m))$ (upon divorce) would correspond to a Nash equilibrium solution with *voluntary contribution* for own children's consumption. This corresponds to a non-cooperative provision of children's consumption where each of the ex-partners maximizes his/her utility given the consumption decision of the other ex-partner. Non-cooperative public good provision has been considered in several studies to analyze household decisions (see Ulph, 1988; Chen and Wolley, 2001; Browning, Chiappori, and Lechene, 2010; Lechene and Preston, 2011; Cherchye, Demuynck, and De Rock, 2011b). We assume a non-cooperative provision of children's consumption since there is no enforcement mechanism in place. Proposition 1 gives the necessary conditions that the observed data must satisfy to be rationalizable by a stable matching with children's consumption and no child custody.

Proposition 1 If a dataset \mathcal{D} is rationalizable by a stable matching with children's consumption and no legislation, then there exist individual private consumptions $q_{m,\sigma(m)}^m, q_{m,\sigma(m)}^{\sigma(m)}$ for each matched pair $(m, \sigma(m))$ and personalized prices $P_{m,w}^m, P_{m,w}^w$ for each pair (m, w) such that

$$q_{m,\sigma(m)}^m + q_{m,\sigma(m)}^{\sigma(m)} = q_{m,\sigma(m)}$$
 and $P_{m,w}^m + P_{m,w}^w = P_{m,w}$

that simultaneously meet the following constraints:

(i) Individual rationality restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,\emptyset} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + C_{m,\sigma(m)}$$
$$y_{\emptyset,w} \le p_{\emptyset,w} q_{\sigma(w),w}^w + P_{\emptyset,w} Q_{\sigma(w),w} + C_{\sigma(w),w}$$

 $^{^{10}}$ To avoid further abuse of notation, we assume that both ex-partners contribute a strictly positive amount to children's consumption. Conditions can be derived without this assumption, following the lines of Cherchye, Demuynck, and De Rock (2011b).

(ii) No blocking pair restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^w Q_{\sigma(w),m} + C_{m,\sigma(m)} + C_{\sigma(w),w}.$$

The linear conditions presented in Proposition 1 have intuitive interpretations.¹¹ We have two adding up constraints. First specifies that the individual private consumption in the matched couples should add up to the total private consumption in the household. Next adding up constraint specifies that for all potential pairs, the personalized prices for the public good (e.g. housing or transportation) should add up to the market price of the good. These personalized prices quantify individual's willingness to pay for the public good in the household. Having these prices add up to the market price correspond to a Pareto optimal provision of public goods among potential pairs.

Next, consider the individual rationality conditions (i) for $m \in M$ and $w \in W$. These restrictions require that for each individual, the prices and income as single $(p_{m,\emptyset}, P_{m,\emptyset}, y_{m,\emptyset})$ for m and $p_{\emptyset,w}, P_{\emptyset,w}, y_{\emptyset,w}$ for w) should not allow them to buy a bundle that is more expensive than the one consumed in the current match.¹² Lastly, consider the no blocking pairs condition (ii). For any potential match (m, w), the income and prices they face within marriage $(y_{m,w}, p_{m,w}, P_{m,w}^m, P_{m,w}^w)$ should not allow them to buy the bundle both of them are consuming in their current matches $((q_{m,\sigma(m)}^m, Q_{m,\sigma(m)}, C_{m,\sigma(m)}))$ for m and $(q_{\sigma(w),w}^w, Q_{\sigma(w),w}, C_{\sigma(w),w}))$ for w). Indeed, if this condition is not satisfied, then (m, w) can allocate their income such that both of them are better off than in their current matches.

To understand the impact of considering the differential nature of children's consumption, let us focus on the no blocking pairs restriction. Consider a potential pair (m, w) where m has children and w is childless. Since the consumption of m's children's consumption will not giving utility to w, upon remarriage, m has to pay the full price to buy his children's consumption. On the other hand, if we would ignore the differential nature of children's consumption and assume that children's consumption is public between (m, w), then m becomes more attractive to w, as higher children's consumption is now associated with more economies of scale. Intuitively, this is the mechanism behind our finding (in the empirical application) that couples with children are more stable as compared to couples without children, when the differential nature of children's consumption is taken into account.

¹¹ Similar conditions can be obtained from the CDDV model if we would impose that children's consumption is treated as a private good in new matches. Note that this is not a particular weakness of the no legislation model, but rather a feature of voluntary provision of public good assumption. The same argument applies to the result of Cherchye, Demuynck, and De Rock (2011b) if we would assume that all individuals contribute strictly positive amount to the public good.

¹² As mentioned above, upon divorce, the children's consumption is provided by both ex-partners. Therefore, the children's consumption that an individual needs to buy as single would be less than the total children's consumption (e.g., even though m can enjoy $C_{m,\sigma(m)}$, his contribution to children's consumption equals $C_{m,\sigma(m)}$ minus the contribution that $\sigma(m)$ makes). This reflects that individual rationality conditions in (i) are necessary in nature. Hence, if m can alone afford $C_{m,\sigma(m)}$ and other bundles he currently consumes in the marriage, then he can be strictly better of as single. Similar argument applies to the no blocking pairs condition in (ii).

2.3 Single Person Custody

Now we consider a setting where individuals also take into account the child custody law when evaluating their exit options. Specifically, we consider the case when the child custody law requires one of the parents to be in full control of the children and the other parent to support children's well being via transfers to the custodial parent i.e., single person custody. Single-person custody is currently a dominant post-divorce child custody rule being used in the family laws of most countries. In particular, it is implemented as a default rule in most states of the US (see Section 3 for the more detailed discussion), Japan, Russia and several other countries. In this custody setting, after divorce, children stay with one of the parents (custodian) who has both legal and physical control over the children. The other parent (non-custodian) is supposed to make transfers to the custodian for supporting the children. The minimum amount of transfer that the non-custodian needs to pay to the custodian is determined by the legal authorities of the state.

We formulate the single person custody setting as follows. We assume that children's consumption can be produced only by the custodian. The non-custodian parent makes an unconditional transfer to the custodian. After receiving the transfer, the custodian maximizes his/her utility by choosing a consumption bundle that includes the children's consumption. Further, we assume that the non-custodian parent knows the preferences of the custodian and chooses the transfer that maximizes his/her own utility. Recall that children's consumption remains a public good for both parents, therefore, this formulation corresponds to a *delegated provision of public good*. We make few more additional assumptions.

- 1. All children are assigned to the same custodian;¹³
- 2. All custodians are females; 14
- 3. All custodians are compliant. This means that the amount of children's consumption brought by the custodian would be at least as large as the transfer received from the non-custodian;
- 4. Transfer received from the non-custodian is no less than $T_{m,\sigma(m)}$, where $T_{m,\sigma(m)}$ is the amount explicitly defined by the child custody guidelines.

Assumptions 1 and 2 can be relaxed if we have a more precise model or guidelines on how the custodian arrangement is determined by the courts. Assumption 4 comes directly from the child custody law. However, we do maintain that there is a perfect enforcement of the law.¹⁵ The linear

 $^{^{13}}$ Alternatively, there can be a 'split custody' under which different children would be assigned to different parent. This is essentially a mixture of several single-person custody rules. Stability conditions for such a setting can be derived in a similar way as conditions listed in Proposition 2. However, this rule is rarely used in practice. In order to keep the exposition simple, we focus on a setting where all children are assigned to the same custodian.

 $^{^{14}}$ In the US (which is also the setting of our empirical study), it is still the case that in about 80% of child custody cases, the custodian is the mother (see Grall, 2013). The stability conditions without this assumption can be derived in a similar way, if we have a more precise information on who would be the custodian in every couple.

¹⁵ Alternatively, we can allow for imperfect compliances by introducing outside option specific binary variables indicating whether or not the stability restrictions are satisfied with minimum transfers. These variables can then be identified, conditional on minimizing the divorce costs, by e.g. maximizing the sum of compliances in the marriage

conditions in Proposition 2 give the necessary conditions that the observed data must satisfy in order to be rationalizable under this setting.

Proposition 2 If a dataset \mathcal{D} is rationalizable by a stable matching with children's consumption and single-person custody, given the minimum transfers $T_{m,\sigma(m)}$ for every $m \in M$, then there exist individual private consumptions $q_{m,\sigma(m)}^m, q_{m,\sigma(m)}^{\sigma(m)}$ for each matched pair $(m,\sigma(m))$ and personalized prices $P_{m,w}^m, P_{m,w}^w$ for each pair (m, w) such that

$$q_{m,\sigma(m)}^m + q_{m,\sigma(m)}^{\sigma(m)} = q_{m,\sigma(m)} \text{ and } P_{m,w}^m + P_{m,w}^w = P_{m,w}$$

that simultaneously meet the following constraints:

(i) Individual rationality restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,\emptyset} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + C_{m,\sigma(m)}$$
$$y_{\emptyset,w} + T_{\sigma(w),w} \le p_{\emptyset,w} q_{\sigma(w),w}^w + P_{\emptyset,w} Q_{\sigma(w),w} + C_{\sigma(w),w}$$

(ii) No blocking pair restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,w} + T_{\sigma(w),w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^w Q_{\sigma(w),m} + C_{m,\sigma(m)} + C_{\sigma(w),w} +$$

The conditions in Proposition 2 can be given similar intuitive interpretations as the no legislation conditions in Proposition 1. To avoid repetitions, we only discuss the differences between the two sets of conditions. First, note that in this custody setting, m can not directly buy the children's consumption himself. Whatever children's consumption m wants to enjoy after divorce, has to be produced through $\sigma(m)$. Given our assumption that $\sigma(m)$ is compliant, the price that mfaces for the children's consumption does not exceed the market price. Hence, m can (indirectly) buy children's consumption by making an equivalent amount of transfer to $\sigma(m)$. Second, upon divorce, female w receives a transfer of at least $T_{\sigma(w),w}$. This makes the post-divorce income at least $y_{\emptyset,w} + T_{\sigma(w),w}$ (as single) and $y_{m,w} + T_{\sigma(m),m}$ (for potential pair (m,w)). We need to account for this child support transfers while evaluating her outside options.

2.4 Joint Custody

Lastly, we consider a case where individuals evaluate their outside options taking into account a child custody law in which the rights and responsibilities over children is split between the spouses and there are no transfers involved, i.e., joint custody. Under this custody setting, children spend time with both parents (joint physical custody). Moreover, both parents have equal rights when it comes to the major decisions concerned with children, such as education, health-care, etc. (joint legal custody). Joint custody is an uprising custodian arrangement. It has been recently adopted as

markets. However, we choose not to follow this route in our empirical analysis. This, in principle, also allows for testing the empirical validity of the assumption.

a default post-divorce child custody arrangement in Australia, Belgium, France and Germany (only as joint legal custody in the two latter countries). Moreover, more than 20 US states considered laws which would promote the applications of joint custody (including making joint custody a default custodial arrangement) in 2017.

In the following, we refer to joint custody as a setting in which both joint physical and legal custody is in place. We formulate the joint custody setting as follows. We assume that children's consumption consists of two types of expenditures: daily routine (k) and big decisions (K). In our empirical application, we will treat children's consumption on food, transport, clothing and miscellaneous as daily routine consumption. Whereas, consumption on housing, health care, child care and education would form the big decisions consumption. Upon divorce, expenditures of type daily routine would be voluntarily provided by the parent who is in control of children at the moment. Since there is neither monitoring nor enforcement mechanism present in the provision of daily routine consumption, we assume that k is provided non-cooperatively (this is similar to the no legislation case). For this consumption category, each of parents (m and $\sigma(m)$) make voluntary contributions, where each unit of k is bought at the market price. Next, since both parents decide on children's big decision consumption together, we assume that K would be provided cooperatively. Under the cooperative provision of K, there must exist personalized (Lindahl) prices for each of the parents (ex-partners) such that they add up to the market price (Bowen-Lindahl-Samuelson condition of Pareto efficiency). Denote these prices by α_m and $1 - \alpha_m$.¹⁶ Proposition 3 shows the linear conditions that the observed data should satisfy for it to be rationalizable by stable matching with joint custody.

Proposition 3 If a dataset \mathcal{D} is rationalizable by a stable matching with children's consumption and joint custody, then there exist individual private consumptions $q_{m,\sigma(m)}^m, q_{m,\sigma(m)}^{\sigma(m)}$, personalized prices $\alpha_m, 1 - \alpha_m$ for each matched pair $(m, \sigma(m))$ and personalized prices $P_{m,w}^m, P_{m,w}^w$ for each pair (m, w) such that

$$q_{m,\sigma(m)}^m + q_{m,\sigma(m)}^{\sigma(m)} = q_{m,\sigma(m)}; P_{m,w}^m + P_{m,w}^w = P_{m,w} \text{ and } \alpha_m \in [0,1]$$

that simultaneously meet the following constraints:

(i) Individual rationality restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,\emptyset} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + k_{m,\sigma(m)} + \alpha_m K_{m,\sigma(m)}$$
$$y_{\emptyset,w} \le p_{\emptyset,w} q_{\sigma(w),w}^w + P_{\emptyset,w} Q_{\sigma(w),w} + k_{\sigma(w),w} + (1 - \alpha_{\sigma(w)}) K_{\sigma(w),w}$$

¹⁶ Alternatively, these Lindahl prices could depend on particular beliefs about matchings in the marriage market. To make the model tractable, we simplify the belief structures by restricting the Lindahl prices to be (current) match specific. That is, we do not allow the Lindahl price to depend on the outside options of the partners. In our empirical application, we show that even with this restricted amount of degrees of freedom on the post-divorce Lindahl prices, the model is not informative enough to make any welfare conclusions.

(ii) No blocking pair restrictions for all $m \in M$ and all $w \in W$, i.e.

$$y_{m,w} \le p_{m,w}(q_{m,\sigma(m)}^{m} + q_{\sigma(w),w}^{w}) + P_{m,w}^{m}Q_{m,\sigma(m)} + P_{m,w}^{w}Q_{\sigma(w),m} + k_{m,\sigma(m)} + \alpha_{m}K_{m,\sigma(m)} + k_{\sigma(w),w} + (1 - \alpha_{\sigma(w)})K_{\sigma(w),w}.$$

The conditions in Proposition 3 can be interpreted along the lines of the interpretation given in the no legislation and the single-person custody setting. The difference in due to the acknowledgment of different types of children's expenditure (k and K) and the way they are produced by the partners (non-cooperatively and cooperatively) under joint custody. If K = 0 for all couples, then the restrictions coincide with those of the no legislation model. Note that the Lindal prices α_m 's are assumed to be unknown. This brings ann extra degrees of freedom in the model. As soon as $K \neq 0$, the model imposes weaker restriction on the data as compared to the no legislation or the single person custody model. The implication of this is that using this model to identify intrahousehold allocation will usually result in much wider bounds as compared to the conditions in Proposition 1 or 2. This will also be reflected in our empirical application in Section 3.

2.5 Divorce Costs and Instability

The linear conditions shown in Propositions 1-3 assume that the marital decisions of all individuals are solely driven by the utility they get from consumption. In particular, we ignored other factors affecting marital choices like frictions in the marriage markets, costs of divorce and remarriage or other aspects of marriage such as love or companionship. A natural way to account for gains from marriage other than consumption is to introduce match specific preferences. One can easily see that through match specific preferences, any observed matching can be rationalized in a trivial way. Assigning an arbitrary high match-specific utility to observed partners and zero utility for any other potential partner would rationalize any given matching. Thus, an unstructured match specific preferences imply that there is no empirical content behind the stable matching restrictions. Here, we follow the approach of CDDV to account for these unobserved channels deriving marital decisions without losing the empirical content of the stable matching conditions.

Formally, let $d_{m,w} \in [0, 1]$ denote the divorce cost index for $m \in M \cup \{\emptyset\}$ and $w \in W \cup \{\emptyset\}$. The divorce cost index $d_{m,w}$ represents the fraction of post-divorce income that m and w would loose in order to break their current marriages and form a couple (m, w). Given a divorce cost of $d_{m,w}$, the post-divorce potential income of (m, w) is $y_{m,w}(1 - d_{m,w})$. This index can be interpreted as a measure of money metric cost that (m, w) would face in case they marry each other and break their current matchings $(m, \sigma(m))$ and $(\sigma(w), w)$. The higher is the divorce cost index, the more is the cost (m, w) should face to leave their current partners $(\sigma(m) \text{ and } \sigma(w))$. In our empirical application we will specifically focus the divorce costs corresponding to the no blocking pairs restriction. For each couple $(m, \sigma(m))$, we define 'average' index as the average of all divorce costs pertaining to the outside remarriage options of m and $\sigma(m)$. Additionally, we define 'maximum' index as the highest divorce cost among all remarriage options of m or $\sigma(m)$. We will interpret these indices as an

indicator of economic instability. The lower these indices are, the more stable observed matchings would be. We note that there are alternate interpretations possible for these indices. In particular, CDDV use these indices as a proxy for the match quality. In order to distinguish between different unobserved channels that affect marital decisions, a proper structural modeling of these factors is required (see CDDV for further discussion).

We quantify the divorce costs as the minimum post divorce income losses that are necessary such that the observed matchings can be rationalized. These indices can be identified by minimizing the following measure of stability subject to the rationality conditions.

$$\min_{d_{m,w}} \sum_{m \in M \cup \{\emptyset\}} \sum_{w \in W \cup \{\emptyset\}} d_{m,w}$$

If all $d_{m,w}$ are equal to 0, than the observed matchings are perfectly rationalizable by the underlying model. The higher the value of this objective function is, the more is the role of unobserved factors in explaining the observed matchings and the more economically unstable the marriages are. Now, we discuss how the divorce costs can be used in empirical applications. The conditions shown in Propositions 1 – 3 are strict in a sense that a given dataset would either satisfy those conditions or not. In order to identify intrahousehold allocation, we first quantify these divorce costs through the constraint minimization problem described above and then use the observed data together with the divorce costs. Using these divorce costs ensures that the new data set (observed dataset with divorce costs) is rationalizable. This rationalizable dataset allows us to (set) identify the intrahousehold allocation. Identification comes from maximizing (minimizing) a linear objective function subject to the stability restrictions. The objective function can be any relevant parameter which is linear in unknowns, e.g., sharing rule, relative cost of equivalent bundle or share of individual's private consumption

3 Empirical Application

We take the models to a cross sectional data drawn from the 2015 wave of PSID. We consider a setting where households spend their entire potential income on leisure, a Hicksian aggregate market good and children's consumption. In this section, first, we discuss the data and the empirical setting. Next, we focus on subsample of households which belong to states with single person custody law. In order to show the impact of taking into account the differential nature of children's consumption and child custody law, we compare the identification of intrahousehold resource shares under three cases. In the first case, all individuals take into account that the potential partner would not internalize children's consumption but disregard the custody law (no legislation). In the second case, all individuals take into account both that potential partner would not internalize children's of previous two cases with the case when all individuals ignore both the child custody laws and differential nature of children's consumption, we use the model in CDDV as our third model. Our empirical

application shows that ignoring the differential nature of children's consumption and related custody laws leads to biased intrahousehold allocations. Specifically, considering both these factors in the analysis shows that a) couples with children are more stable and b) each additional child leads to a 7% percentage point increase in female's share of private consumption.

Lastly, we focus on households which belong to states with joint custody law. As before, we would compare the bounds on female's share of private consumption under three settings (no legislation, joint custody and CDDV). Result from this subsample shows that using joint custody as the identifying model leads to much wider bounds as compared to those from the no legislation or the CDDV model. Given the uninformative nature of the bounds, we can not claim that, under joint custody, ignoring children's consumption and custody laws would lead to biased intrahousehold allocations.

3.1 Data and Set-up

Sample selection. PSID is a widely used household survey of a nationally representative sample of individuals and families living in the US. The sample collection began in 1968 and has been interviewing the same households since then, making it one of the longest household panel data. The data set contains information at both the individual and the household level on a wide variety of topics like income, education, marriage, fertility, child development etc. Since 1999, the dataset is complemented with additional information on household expenditures on detailed set of consumption categories like food, housing, transport, etc.

For our empirical analysis, we use the 2015 wave of the survey. Our focus is restricted to couples with and without children, where both spouses work at least 10 hours per week on the labor market. We exclude households which contain any household member other than the couple and their children. We also drop households with missing information (e.g. wage, time use, expenditure) and the outliers by trimming the households in the 1st and 99th percentiles of male and female wage distribution. Further, we also include information on singles to allow for the possibility that a married individual may consider a single of opposite gender as a potential partner. This selection procedure results in a sample of 4502 individuals consisting of 1460 couples, 616 single males and 966 single females.

Child custody laws. Child custody laws in the US are determined at the state level and therefore, there is quite a variation to be exploited. An overview of different child custody arrangements in the US states is illustrated in Figure 1.¹⁷ In 37 states (colored blue and green), the child custody law is based on single person custody arrangement and 7 states (colored yellow) use the joint custody law.¹⁸ Recall that under single person custody, the non-custodian parent is asked to transfer

¹⁷ Data on custodial arrangements is taken from the '2014 Shared Parenting Report Card A New Look At Child Welfare A State-by-State Ranking' by National Parents Organization. Data on guidelines to determine the child support payments is taken directly from the state legal guidelines (see e.g. Colorado Statues Title 14: Domestic Matters), all rules are retrieved to their status in 2015.

¹⁸ We do not consider the other 7 states (in grey) with child custody that is a hybrid version of different rules.

a child support payment to the custodian parent. The minimum amount of child support payment is determined by the state legal authorities using either of following two rules.

1. Income shares rule (32 states in blue).

Child support depends on the (estimate of the) pre-divorce (current) children's consumption. According to this rule, given the necessary amount of children's consumption to be provided, non-custodial parent is supposed to provide the payment at least equal to his share of income in the total household earnings multiplied by the estimated children's consumption.

2. Percentage rule (5 states in green).

Non-custodial parent is supposed to pay a fixed proportion of his income. The proportion depends only on the number of children.¹⁹



Figure 1: States with default child custody laws

Children's Consumption. Apart from the expenditures on childcare, PSID does not report other expenditures that are specific to children. Ignoring such children specific consumption (like food, clothing, education, etc.) would lead to underestimation of both current children's consumption and the minimum child support payment in states which use income share rule. In order to

¹⁹The specific percentage used to compute the payment could be either flat or varying. In the states using flat rules, the percentage does not change whether or not the income of non-custodial changes. In states using varying rules, the percentage changes as the non-custodial parent's income changes. In our application, we will focus on states with percentage rule which uses flat percentage to determine the payments.

avoid any bias in the intrahousehold allocation due to mis-measured children's consumption, we instead impute children's consumption using the '2015 expenditure on children by families' report provided by the United States Department of Agriculture (USDA). The USDA provides an annual report for the estimates of the cost of raising children from birth through age 17. Our motivation for using the USDA report instead of any other estimation method is due to the fact that these reports are primarily provided to determine the child support payments. The state legal authorities consult this report to form a typical expenditure on children which serves as a guideline to compute the minimum child support payment (see Pirog et al. (1998) and Lino (2001)). An additional advantage of using this report is that it provides the breakdown of total expenditure on children coming from different consumption categories like food, transportation, housing, health care, education, child care etc. We will use these detailed expenditure subcategories to form an estimate of children's daily-routine consumption and children's big-decision consumption which are needed for the joint custody model. Further details on these estimates are provided in Appendix B.

Set up. The setup of our empirical analysis is a standard labor supply setting. Table 1 gives the summary statistics of the sample of couples under consideration. Wages are hourly wage rates in dollars. Leisure is the average hours spent on leisure per week (computed as 112 - hours worked on labor market) and consumption is the dollars per week spent on household aggregate Hicksian consumption. Leisure is considered as a pure private assignable consumption. We assume that 50% of the Hicksian aggregate consumption is consumed privately by the household members. Recall that we do not observe who consumes what inside the household. Thus, the private Hicksian consumption forms the non-assignable component of household's total private consumption. The remaining 50% of the Hicksian aggregate is consumed publicly.

Finally, we discuss about the nature of children's consumption. In CDDV, children's consumption is assumed to be a public good of similar nature as the Hicksian aggregate public good. That is, in CDDV setting, utility from children's consumption can be easily internalized by a new partner upon remarriage. As discussed earlier, in case of no legislation, single person custody and joint custody model, the nature of children's consumption is assumed to be dependent on the matching. For all observed matchings, where both spouses are the biological parents of the children, we assume that children's consumption is a public good. However, this is not the case for potential matchings. Thus, upon remarriage, children's consumption is not a public consumption for new partners but it remain a public consumption between the ex-partners.

In order to evaluate individual's outside options, we need prices and income for all counterfactual situations (potential matches). We assume that the labor market productivity is independent of individual's marital status. Hence, price of leisure (in all matches) is individual's wage rate. Price of Hicksian good is normalized to one. The full income in a potential match is determined as the sum of potential labor income (which is 112 times individual wage rate) and non-labor income of the individuals. In the data, we do not observe individual non-labor income. In what follows, we will treat individual non-labor income as an unknown variable such that the sum of individual

	mean	std. dev.	\min	max
wage male	25.93	16.72	2.18	103.45
wage female	21.81	12.82	2.65	76.92
leisure male	67.45	10.67	22.00	102.00
leisure female	73.44	9.95	12.00	102.00
age male	39.75	11.75	19.00	84.00
age female	38.16	11.62	19.00	79.00
consumption	767.53	428.95	5.58	2840.00
children's consumption	290.01	290.75	0.00	2003.78
dummy for married	0.83	0.37	0.00	1.00
number of children	1.18	1.22	0.00	7.00

member's non-labor income add to the total non-labor income of the household.²⁰

Table 1: Summary statistics for couples

Finally, we define individual's consideration set which specifies the set of potential partners that the individual would consider as outside options. We construct the consideration sets based on age of individuals and their state of residence. There is ample empirical evidence in the literature that couples match assortatively based on age. Thus, it is unrealistic to assume that individuals of all age are on the same marriage market. We assume that a male (female) individual's marriage market contains all single and married females (males) who are at most 11 (7) years older and 7 (11) years younger. These age restrictions correspond to the 2.5th and 97.5th percentile of the distribution of age difference of the matched couples in the sample. Further, we restrict the potential partners to be from the same state to allow for the possibility of geographically restricted marriage markets. Table 2 gives summary statistics for the size of marriage markets. On average, males have about 43 potential partners and females have about 37 potential partners (married and single). We also see quite some heterogeneity in the size of individual's marriage market with the number of potential partners ranging from 0 to 143 for males and from 0 to 120 for females.²¹

Table 2: Summary statistics on the size of individual's marriage market

	males	females
mean	42.92	36.66
standard deviation	32.86	28.36
minimum	0	0
first quartile	19	15
median	35	31
third quartile	57	47
maximum	143	120

 $^{^{20}}$ Instead of fixing individual non-labor to be 50% of household non-labor income, we follow similar setting as in CDDV by restricting individual non-labor income to be between 40% to 60% of household non-labor income.

²¹ For individuals with zero potential partners, the identification is driven by their own individual rationality restriction and the no blocking pair restrictions for their current spouse. Note that the marriage markets we consider can be seen as inner bound approximations of the true marriage markets of the individuals. In principle, including additional potential partners will yield tighter bounds.

3.2 States with Single Person Sustody

First, we focus our analysis on a subsample of households residing in states with single person custody law (states with color blue and green in Figure 1). This subsample contains 1358 couples, out of which 1125 (82.84%) are from the states which use income share rule and 233 (17.16%) are from the states which use percentage rule. We begin by checking the rationalizability of observed matchings under the three models of stable matching (no legislation, single person custody and CDDV). Next, we identify and compare the share of female's private consumption (of Hicksian good) obtained from the models.

3.2.1 Rationalizability and Instability

As discussed in Section 2.5, we check the rationalizability of the observed data by computing the minimum divorce costs that are needed such that the current matchings can be rationalized. These divorce cost indices capture the minimum post-divorce income loss that can guarantee the rationalizability of the dataset. As discussed in Section 2.5, we interpret these as an economic measure of instability in observed marriages. Recall that a higher divorce costs implies a less stable matching. Table 3 summarizes the average divorce costs across family size, child support rules, and structural models. Columns 2-5 summarize the average divorce costs for the no legislation model. Columns 6-9 give the corresponding numbers for the single person custody model and columns 10-13 give the same for the CDDV model. The 'female' and 'male' columns correspond to the average divorce costs derived from the individual rationality conditions for married females and males correspondingly. The 'average' and 'maximum' columns (as discussed in Section 2.5) correspond to the average and maximum divorce costs derived from the no blocking pair conditions (over all potential matches) for all married couples. Finally, the first panel is for households from the states which use income shares rule and the second panel is for households from the states which use the percentage rule to compute minimum child support payments. Note that the difference between panels with income shares and percentage rules are only relevant for the single person custody case. This is because the no legislation and the CDDV models do not take into account the child custody law.

	no legislation				single person custody				CDDV			
	individual rationality no blocking pair		cking pair	individual rationality no		no blo	no blocking pair		individual rationality		no blocking pair	
	female	male	average	maximum	female	male	average	maximum	female	male	average	maximum
				income shares								
no child	0.04	0.36	0.75	5.99	0.04	0.36	0.77	6.03	0.04	0.36	1.04	7.13
one child	0.09	0.20	0.21	3.17	0.15	0.20	0.36	4.54	0.09	0.20	0.95	8.36
two children	0.03	0.35	0.21	2.17	0.07	0.35	0.33	3.87	0.03	0.35	1.06	8.72
>2 children	0.00	0.28	0.19	2.21	0.07	0.28	0.29	4.08	0.00	0.28	1.15	10.00
total	0.04	0.31	0.43	3.94	0.07	0.31	0.51	4.92	0.04	0.31	1.04	8.18
						percenta	age rule					
no child	0.00	0.36	0.73	6.00	0.00	0.36	0.75	6.03	0.00	0.36	1.05	7.23
one child	0.05	0.41	0.26	3.22	0.07	0.41	0.40	4.53	0.05	0.41	1.13	9.47
two children	0.00	0.51	0.23	2.28	0.01	0.51	0.34	3.80	0.00	0.51	1.16	10.82
>2 children	0.00	0.09	0.07	1.73	0.00	0.09	0.13	3.04	0.00	0.09	1.08	12.90
total	0.01	0.38	0.40	3.80	0.02	0.38	0.48	4.68	0.01	0.38	1.10	9.48

Table 3: Average divorce costs (in %) across family size, child support rules, and structural models

Table 3 shows the following results. First, taking into account the differential nature of children's consumption and custody laws suggest that couples with children are more stable as compared to couples without children (see 'maximum' divorce cost) in the no legislation and single person custody models. An opposite effect is suggested by the CDDV model which does not account for these factors. Second, the divorce costs corresponding to the no blocking pair restrictions ('average' and 'maximum') suggest that the CDDV model is more restrictive than the no legislation and the single person custody model. Third, all three models show that females need lower divorce costs than males to rationalize their decision of being married.

Table 3 summarizes the average divorce costs for different groups of couples (based on number of children). However, there is also quite some heterogeneity in the revealed divorce costs among couples within each group. In order to account for this variation in divorce costs when understanding the effect of children in marital instability, we follow the same exercise as in CDDV, by relating the revealed divorce costs with observable household characteristics. Table 4 shows the effect of children on the measure of instability obtained from the three models of stable matching. We perform two regressions for each model. In the first one, we use 'average' indices and in the second, we use 'maximum' indices as the dependent variable.

	no legislation		single pers	on custody	CDDV	
	average	maximum	average	maximum	average	maximum
number of children	-0.149***	-1.280***	-0.126***	-0.746***	0.0363**	0.774^{***}
	(0.0103)	(0.0594)	(0.0112)	(0.0577)	(0.0171)	(0.0702)
Observations	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$

Table 4: Economic measure of instability and number of children

Notes: OLS estimates. Brackets contain robust standard errors. *** indicates p < 0.01 and ** indicates p < 0.05. Each cell reports a coefficient from a separate regression. All regressions include fixed effects for region, marital status and child support rule. Regressions also control for average wage, absolute wage difference, average age, absolute age difference, marriage market size of male, marriage market size of female and log income.

Not surprisingly, we see different effects of number of children on marital instability across the three models.²² The no legislation and single-person custody models reveal a negative effect of the number of children on the marital instability whereas a positive effect applies to the estimates from the CDDV model. This result can be interpreted in the following way in the context of stable marriage models. In the no legislation and single person custody model, children's consumption is public within the current marriage, but it generates no utility for the potential partner. Thus, the higher is the budget share of household consumption on children's consumption, the higher is the economies of scale within current matchings and the lower is the economies of scale within potential matchings. This makes individuals with children less attractive to the outside options. Whereas, in

 $^{^{22}}$ We find several other relations between the instability measure and observed household characteristics which are common in the three models. These findings are similar to those shown in CDDV. In order to focus our discussion on the effect of taking into account children's consumption and custody law, we only show the estimates of number of children in the main text. The full set of estimates are reported in Appendix C.

the CDDV model, children's consumption can be easily shared with any potential partners just like other public goods like housing or transportation. Hence, higher children's consumption implies higher economies of scale, which makes individuals with children more attractive in the marriage market resulting in a positive dependence between marital instability and number of children.

3.2.2 Identification of Private Consumption

Using the divorce costs summarized above, we can construct new datasets such the they are rationalizable by the three models of stable matching. This allows us to use the stable matching restrictions to set identify the resource allocation in the household. Here, we focus on the identification of female's share of private Hicksian good consumption.²³ Table 5 presents the average lower and upper bound on the female's share of private consumption of Hicksian good by the number of children.²⁴ The column 'difference' is the average difference between the upper and the lower bounds. This reflects the informativeness of the bounds. Columns 2-4 give the result from the no legislation model, 5-7 give the result from the single person custody model and last three columns 8-10 give the corresponding result from the CDDV model. Panel 1 contains the results for the states with income shares rule and panel 2 contains that for the states with percentage rule.

	no legislation			sing	le person	custody	CDDV		
	lower	upper	difference	lower	upper	difference	lower	upper	difference
				inc	ome sha	res rule			
no child	0.25	0.47	0.22	0.25	0.47	0.22	0.23	0.43	0.21
one child	0.30	0.55	0.25	0.40	0.63	0.24	0.23	0.44	0.21
two children	0.27	0.55	0.28	0.38	0.62	0.24	0.23	0.40	0.17
> 2 children	0.31	0.65	0.34	0.50	0.71	0.20	0.22	0.38	0.16
total	0.27	0.53	0.26	0.35	0.57	0.22	0.23	0.42	0.19
				р	ercentag	e rule			
no child	0.25	0.46	0.21	0.25	0.46	0.21	0.21	0.42	0.21
one child	0.36	0.57	0.21	0.45	0.66	0.21	0.26	0.47	0.21
two children	0.34	0.63	0.29	0.45	0.68	0.23	0.21	0.46	0.25
> 2 children	0.43	0.75	0.32	0.56	0.79	0.24	0.32	0.47	0.16
total	0.32	0.57	0.25	0.39	0.61	0.22	0.23	0.45	0.22

Table 5: Bounds on female's share of private consumption

Let us first discuss the identifying power of the three models. First, all three models have significant identifying power. On average, the CDDV model gives the most precise bounds. Further, the bounds from the CDDV model are generally tighter for couples with children. The next best model, in terms of identifying power, is the single person custody model. The no legislation model

 $^{^{23}}$ Alternatively, we could have focused on identifying the "conditional" sharing rule which is defined as individual's share of total private consumption (i.e. leisure + private Hicksian consumption) conditional on the level of public consumption. Given the assumption of assignability of leisure, by knowing the bounds on share of private consumption in Hicksian good, we can easily construct the bounds on female's conditional sharing rule. Using the conditional sharing rule to compare the performance of the three models give similar qualitative conclusion (see Appendix D).

 $^{^{24}}$ Appendix C shows the box plots of these bounds by number of children in the household. These figures give a visualization of the empirical distribution of the bounds and confirm the findings reported in the main text.

gives the weakest identification. In contrast to the CDDV model, we see that the bounds from both the no legislation and the single person custody model are wider for couples with children.

Although there are differences in identifying power of the models, we can clearly see the effect of taking into account the differential nature of children's consumption and the child custody law on females share of private Hicksian consumption. First, for couples without children, the bounds from all three models are similar to each other. This is intuitive as the key difference between the three models is due to presence of children, so there is no direct effect on childless couples. Next, while female's share of private consumption is increasing with the number of children according to the no legislation and single person custody model, this effect is not observed in the CDDV model. Comparing the bounds obtained from the no legislation model with those from the CDDV model shows the effect of taking into account the differential nature of children's consumption. In particular, both lower and upper bounds from the no legislation model are higher than the bounds from the CDDV model. Further comparison of the no legislation and single person custody bounds highlights the effects of taking account of the child custody law. In particular, inclusion of child support transfers further increases the females' shares of private consumption for households with children. For instance, under the no legislation model, on average, a mother of two children residing in a state with income share rule may get between 27% - 55% of Hicksian private consumption. While under the single person custody model, this range is significantly shifted higher (38% - 62%).

	no legislation		single pers	on custody	CDDV		
	lower	upper	lower	upper	lower	upper	
number of children	0.0162^{**}	0.0545^{***}	0.0717^{***}	0.0711^{***}	0.00256	-0.00977	
	(0.00664)	(0.00700)	(0.00712)	(0.00674)	(0.00607)	(0.00673)	
Observations	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$	$1,\!358$	

Table 6: Female's share of private consumption of Hicksian good and household characteristics

Notes: OLS estimates. Brackets contain robust standard errors. *** indicates p < 0.01 and ** indicates p < 0.05. Each cell reports a coefficient from a separate regression. All regressions include fixed effects for region, states, marital status and child support rule. Regressions also control for log wage ratio, age difference, marriage market size of male, marriage market size of female and log income.

Given that there is quite some heterogeneity in the female's share of private consumption across households, we conduct regression analysis to investigate the connection between the resource allocation and number of children while accounting for the observed households characteristics. Table 6 report the estimates of effect of number of children on female's share of private Hicksian consumption.²⁵ The 'lower' ('upper') columns corresponds to the estimates obtained when we use lower (upper) bound as the dependent variable. Number of children affect female's shares differently for different models. A positive effect of number of children applies to the female's share from the no legislation and the single person custody model, whereas no such effect is present

 $^{^{25}}$ Similar to the findings for measure of marital instability in Table 4, we find several observable characteristics revealing similar effects from all three models. These findings, reported in Appendix C, are in line with those reported in CDDV.

for the estimates from the CDDV model. Further, quantitatively, the effect of children is much stronger in the single person custody model as compared to the no legislation model. Taking into account the differential nature of children's consumption and the custody law implies that each additional child leads to a 7% increase in female's share of private consumption. Intuitively, this is because, under single person custody model, females bargaining power is increased due to the increase in their post-divorce income from the child support payments. These finding confirm the results from Table 5, that it is important to take into account the differential nature of children's consumption and related custody laws when considering an individuals outside option, in order to avoid unbiased estimates.

3.3 States with joint custody

In this section, we focus on a subsample of 102 couples residing in the states with joint custody law (states colored in yellow in Figure 1). Recall that for the joint custody model, we need to separate children's consumption into daily routine type consumption (k) and big-decision type consumption (K).²⁶ The USDA provides detailed information of children's consumption for different budgetary components (housing, food, transportation, clothing, childcare and education, health care and miscellaneous costs). We use these estimates for food, transport, clothing and miscellaneous to form children's daily routine consumption and the rest (housing, health care, childcare and education) to form children's big decision consumption.

Similar to the previous section, we start from the analysis of rationalizability of the observed data by the no legislation, joint custody and CDDV model. Next, we use the computed divorce costs to form rationalizable data sets which can be used to set identify female's share of private Hicksian good consumption. The result using this data shows that the joint custody model has much weaker identifying power as compared to the CDDV model. This is to say that the bounds on female's share of private consumption obtained through the joint custody model is much wider than those obtained through the CDDV model.

Table 7: Average divorce costs (in %)

	no legislation			joint custody				CDDV				
	individua	al rationality	no blo	cking pair	individual rationality no blocking pai		cking pair	individual rationality		no blocking pair		
	female	male	average	maximum	female	male	average	maximum	female	male	average	maximum
no child	0.16	0.14	0.36	2.89	0.16	0.14	0.36	2.90	0.16	0.14	0.60	5.08
one child	0.00	0.74	0.43	2.83	0.00	0.74	0.52	3.78	0.00	0.74	1.27	8.33
two children	0.00	0.01	0.03	0.90	0.00	0.01	0.06	1.58	0.00	0.01	0.90	7.25
>2 children	0.00	0.52	0.08	0.75	0.00	0.52	0.13	1.35	0.00	0.52	1.25	8.25
total	0.07	0.24	0.25	2.10	0.07	0.24	0.28	2.51	0.07	0.24	0.86	6.54

Table 7 presents the mean 'average' and 'maximum' divorce costs corresponding to the no blocking pair restrictions (see section 2.5) that are needed to rationalize the data. Comparison of the 'maximum' divorce costs from different models suggests that CDDV model is more restrictive than both the no legislation and the joint custody model. Even with this difference, these instability

 $^{^{26}}$ See, for instance, the Clarification provided by the American Bar Association that summarizes and explains the applicability of the joint custody law in the US.

measures suggest the same conclusion as that obtained in the single person custody case. According to the no legislation and the joint custody model, couples with children are more stable than couples without children. An opposite effect holds for CDDV model.

	no legislation				joint cus	tody	CDDV		
	lower	upper	difference	lower	upper	difference	lower	upper	difference
no child	0.32	0.61	0.29	0.32	0.61	0.29	0.27	0.55	0.28
one child	0.18	0.40	0.22	0.10	0.49	0.39	0.18	0.40	0.22
two children	0.15	0.69	0.54	0.09	0.72	0.64	0.23	0.49	0.26
> 2 children	0.18	0.88	0.70	0.06	0.88	0.82	0.33	0.63	0.30
total	0.24	0.63	0.39	0.19	0.65	0.46	0.25	0.52	0.27

Table 8: Bounds on female's share of private consumption in states with joint custody

Table 8 presents the bounds on female's share of private Hicksian good consumption. Similar to single person custody case, the bounds for no legislation and joint custody models are wider than bounds for CDDV model. Informativeness of both models (no legislation and joint custody) is decreasing in the increase in number (expenditure) of children. However, while the bounds from single person custody model were more informative than those from the no legislation model, the bounds from the joint custody model are less informative then those from the no legislation model. Given the weakness in identification, we cannot make any inference about potential misidentification in intrahousehold allocations.

4 Concluding Remarks

We develop revealed preference conditions to analyze the implications of stable marriage markets on household consumption decisions while accounting for the differential nature of children's consumption and child custody law. In particular, we present the testable revealed preference restrictions under the exhaustive list of three cases of child custody (no legislation, single person custody and joint custody). These conditions allow for the identification of intrahousehold allocation. Unless children's consumption and custody law are incorporated structurally, the stable matching model (in theory) would misidentify the intrahousehold sharing.

To illustrate the performance of these conditions, we provide an empirical application to a survey of US households. This application confirms the theoretical concerns about the impact of ignoring children's consumption and custody laws while analyzing household consumption decisions via the stable marriage market restrictions. In particular, the results show that accounting these factors support previous empirical findings that couples with children are more stable as compared to childless couples. Whereas, ignoring these factors would suggest otherwise that couples with children are less stable. Further, comparing the intrahousehold allocation identified with and without these factors shows that number of children does not impact the resource allocation when these factors are not taken into account, while incorporating them (no legislation and single person custody) reveal the underlying positive correlation. In general, the post-divorce children's consumption may be dependent on the beliefs an individual has about the potential match of his/her current partner. In our conditions, we only employ restrictions that are not dependent on these beliefs. This allows us to make a minimal amount of assumptions about the belief structure of the individual. Extending our methodology to structurally incorporate these beliefs (individual has about the remarriage of his/her current partner) may provide additional identifying power. Given the weakness in identification from the joint custody model, enriching the analysis (and therefore, improving the identification) by introducing beliefs in the stability restrictions may be a promising avenue for future work.

Further, in this paper, we study the effect of the differential nature of children's consumption and custody laws on intrahousehold allocation. We did not aim at the identification of children's consumption following the divorce of their parents. In order to study the impact of divorce on the welfare of children, it is necessary to get an estimate of post-divorce children's consumption which is not possible in a cross-sectional setting. However, extending the framework to a panel setting would allow us to additionally exploit the testable implications of collective rationality. In such a setting, it would be possible to identify the (after divorce) children's consumption. This can also help in studying the impact of different child custody arrangements on the welfare of children. In particular, it can contribute to the ongoing debate on whether or not joint custody is a custody arrangement made in the best interest of children especially as compared to single person custody. We see this as another direction for fruitful future research.

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A Proofs

A.1 Proof of Proposition 1

Before we proceed with the proof, let us introduce some additional notation. Recall that, upon divorce, m and $\sigma(m)$ make independent contribution to children's consumption (voluntary contribution public good game). Let c^m and $c^{\sigma(m)}$ be the contributions that m and $\sigma(m)$ make to the children's consumption upon divorce. Given the public nature of children's consumption, we have $c^m + c^{\sigma(m)} = C^m = C^{\sigma(m)}$.

Individual Rationality. Conditions for m and w are symmetric in this case. Therefore, we present the derivations only for m. The conditions for w can be obtained in a similar way. The optimization problem for m when he is single looks as follows:

$$\begin{cases} u^m(q^m, Q, c^m + c^{\sigma(m)}_{m', \sigma(m)}) \to \max_{q^m, Q, c^m} \\ p_{m, \emptyset} q^m + P_{m, \emptyset} Q + c^m = y_{m, \emptyset} \end{cases}$$

for some $m'(\neq m) \in M \cup \{\emptyset\}$. Here m' is m's belief about potential rematching of $\sigma(m)$. The consumption decisions of both ex-partners $(m, \sigma(m))$ corresponds to the Nash equilibrium solution for the provision of children's consumption. In this case, m would maximize his utility given that $\sigma(m)$ is contributing $c_{m',\sigma(m)}^{\sigma(m)}$ to children's consumption. The first order conditions for his optimization problem gives,

$$\nabla_{q^m} u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}) = \lambda_{m,\emptyset} p_{m,\emptyset}$$
$$\nabla_Q u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}) = \lambda_{m,\emptyset} P_{m,\emptyset}$$
$$\nabla_C u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}) = \lambda_{m,\emptyset}$$

Let $(q_{m,\emptyset}, Q_{m,\emptyset}, c_{m,\emptyset}^m)$ be the solution to above maximization problem. Concavity of utility functions gives,

$$u^{m}(q_{m,\sigma(m)}^{m}, Q_{m,\sigma(m)}, C_{m,\sigma(m)}) - u^{m}(q_{m,\emptyset}, Q_{m,\emptyset}, C_{m,\emptyset})$$

$$\leq \lambda_{m,\emptyset} p_{m,\emptyset}(q_{m,\sigma(m)}^{m} - q_{m,\emptyset}) + \lambda_{m,\emptyset} P_{m,\emptyset}(Q_{m,\sigma(m)} - Q_{m,\emptyset}) + \lambda_{m,\emptyset}(C_{m,\sigma(m)} - C_{m,\emptyset})$$

Individual rationality for m implies,

$$\begin{split} 0 &\leq u^{m}(q_{m,\sigma(m)}^{m},Q_{m,\sigma(m)},C_{m,\sigma(m)}) - u^{m}(q_{m,\emptyset},Q_{m,\emptyset},C_{m,\emptyset}) \\ 0 &\leq \lambda_{m,\emptyset}p_{m,\emptyset}(q_{m,\sigma(m)}^{m} - q_{m,\emptyset}) + \lambda_{m,\emptyset}P_{m,\emptyset}(Q_{m,\sigma(m)} - Q_{m,\emptyset}) + \lambda_{m,\emptyset}(C_{m,\sigma(m)} - C_{m,\emptyset}) \\ 0 &\leq p_{m,\emptyset}(q_{m,\sigma(m)}^{m} - q_{m,\emptyset}) + P_{m,\emptyset}(Q_{m,\sigma(m)} - Q_{m,\emptyset}) + (C_{m,\sigma(m)} - C_{m,\emptyset}) \\ p_{m,\emptyset}q_{m,\emptyset} + P_{m,\emptyset}Q_{m,\emptyset} + C_{m,\emptyset} &\leq p_{m,\emptyset}q_{m,\sigma(m)}^{m} + P_{m,\emptyset}Q_{m,\sigma(m)} + C_{m,\sigma(m)} \\ p_{m,\emptyset}q_{m,\emptyset} + P_{m,\emptyset}Q_{m,\emptyset} + c_{m,\emptyset}^{m} + c_{m',\sigma(m)}^{\sigma(m)} &\leq p_{m,\emptyset}q_{m,\sigma(m)}^{m} + P_{m,\emptyset}Q_{m,\sigma(m)} + C_{m,\sigma(m)} \\ \end{split}$$

Budget constraint implies $p_{m,\emptyset}q_{m,\emptyset}^m + P_{m,\emptyset}Q_{m,\emptyset} + c_{m,\emptyset}^m = y_{m,\emptyset}$ and moreover, recall that $c_{m',\sigma(m)}^{\sigma(m)} > 0$. Further simplification gives the individual rationality condition shown in Proposition 1.

$$y_{m,\emptyset} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + C_{m,\sigma(m)}$$

No Blocking Pairs. We assume that every potential match (m, w) would make Pareto efficient decisions within themselves. The optimization problem for (m, w) can be written as follows:

$$\begin{cases} u^{m}(q^{m}, Q, C^{m}) + \mu u^{w}(q^{w}, Q, C^{w}) \to \max_{q^{m}, q^{w}, Q, c^{m}, c^{w}} \\ p_{m,w}(q^{m} + q^{w}) + P_{m,w}Q + c^{m} + c^{w} \le y_{m,w} \end{cases}$$

for some $\mu > 0$. Here, $C^m = c^m + c_{m',\sigma(m)}^{\sigma(m)}$ and $C^w = c^w + c_{\sigma(w),w'}^{\sigma(w)}$ for some $m'(\neq m) \in M$ and $w'(\neq w) \in W$. The first order conditions gives,

$$\begin{aligned} \nabla_{q^m} u^m(q^m_{m,w}, Q_{m,w}, C^m_{m,w}) &= \lambda_{m,w} p_{m,w} \\ \nabla_{q^w} \mu u^w(q^w_{m,w}, Q_{m,w}, C^w_{m,w}) &= \lambda_{m,w} p_{m,w} \\ \nabla_Q u^m(q^m_{m,w}, Q_{m,w}, C^m_{m,w}) &+ \mu \nabla_Q u^w(q^w_{m,w}, Q_{m,w}, C^w_{m,w}) &= \lambda_{m,w} \\ \nabla_{C^m} u^m(q^m_{m,w}, Q_{m,w}, C^m_{m,w}) &= \lambda_{m,w} \\ \nabla_{C^w} \mu u^w(q^w_{m,w}, Q_{m,w}, C^w_{m,w}) &= \lambda_{m,w} \end{aligned}$$

Let $P_{m,w}^w = \frac{\mu \nabla u^w(q_{m,w}^w, Q_{m,w}, C_{m,w}^w)}{\lambda_{m,w}}$ and $P_{m,w}^m = P_{m,w} - P_{m,w}^w$. Concavity of u^m and u^w implies,

$$u^{m}(q_{m,\sigma(m)}^{m}, Q_{m,\sigma(m)}, C_{m,\sigma(m)}) - u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m,w}^{m})$$

$$\leq \lambda_{m,w}(p_{m,w}(q_{m,\sigma(m)}^{m} - q_{m,w}^{m}) + P_{m,w}^{m}(Q_{m,\sigma(m)} - Q_{m,w}) + (C_{m,\sigma(m)} - C_{m,w}^{m}))$$

$$u^{w}(q_{\sigma(w),w}^{w}, Q_{\sigma(w),w}, C_{\sigma(w),w}) - u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w})$$

$$\leq \lambda_{m,w}(p_{m,w}(q_{\sigma(w),w}^{w} - q_{m,w}^{w}) + P_{m,w}^{w}(Q_{\sigma(w),w} - Q_{m,w}) + (C_{\sigma(w),w} - C_{m,w}^{w}))$$

No blocking pairs conditions require,

$$u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m,w}^{m}) \leq u^{m}(q_{m,\sigma(m)}^{m}, Q_{m,\sigma(m)}, C_{m,\sigma(m)}^{m})$$
$$u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) \leq u^{w}(q_{\sigma(w),w}^{w}, Q_{\sigma(w),w}, C_{\sigma(w),w}^{w})$$

Thus,

$$0 \le p_{m,w}(q_{m,\sigma(m)}^m - q_{m,w}^m) + P_{m,w}^m(Q_{m,\sigma(m)} - Q_{m,w}) + (C_{m,\sigma(m)} - C_{m,w}^m)$$

$$0 \le p_{m,w}(q_{\sigma(w),w}^w - q_{m,w}^w) + P_{m,w}^w(Q_{\sigma(w),w} - Q_{m,w}) + (C_{\sigma(w),w} - C_{m,w}^w)$$

$$p_{m,w}q_{m,w}^m + P_{m,w}^mQ_{m,w} + C_{m,w}^m \le p_{m,w}q_{m,\sigma(m)}^m + P_{m,w}^mQ_{m,\sigma(m)} + C_{m,\sigma(m)}$$

$$p_{m,w}q_{m,w}^w + P_{m,w}^wQ_{m,w} + C_{m,w}^w \le p_{m,w}q_{\sigma(w),w}^w + P_{m,w}^wQ_{\sigma(w),w} + C_{\sigma(w),w}$$

Adding last two inequalities and using the adding up constraints, $P_{m,w}^m + P_{m,w}^w = P_{m,w}$, $C_{m,w}^m = c^m + c_{m',\sigma(m)}^{\sigma(m)}$ and $C_{m,w}^w = c^w + c_{\sigma(w),w'}^{\sigma(w)}$ gives the no blocking pairs restriction shown in Proposition

 $y_{m,w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^w Q_{\sigma(w),m} + C_{m,\sigma(m)} + C_{\sigma(w),w}$

A.2 Proof of Proposition 2

Let $\tau_{m,w}^{m',\sigma(m)}$ be the child support amount m transfers to $\sigma(m)$ if he forms a couple with w and expects his ex-partner $\sigma(m)$ to form a couple with m'. By the assumption of perfect enforcement of custody law we have $\tau_{m,w}^{m',\sigma(m)} \geq T_{m,\sigma(m)}$, where $T_{m,\sigma(m)}$ is the minimum transfers a non-custodian parent is obliged to pay to a custodian parent. Denote by $D_{m',\sigma(m)}(\tau)$ the demand for children's consumption $\sigma(m)$ exhibits if she forms a couple with m' $(m' \neq m)$ and receives a transfer τ from m. Let $D_{m',\sigma(m)}^{-1}(C)$ be the inverse of this demand function. Moreover, we assume $D_{m',\sigma(m)}$ to be smooth for all $(m',\sigma(m))$, hence the inverse is smooth as well. Furthermore, we assume that $D_{m',\sigma(m)}$ is a concave function and $D'_{m',\sigma(m)}(\tau) \geq 1$. This assumption essentially means that $\sigma(m)$ is compliant. That is, she does not use the money designated for a children's consumption for her personal consumption. This can be guaranteed, for instance, by the presence of alternate possibility of m to buy the children consumption at the market price of one.

Individual Rationality for $m \in M$. m gets utility from the consumption of private and public goods and children's consumption (which is produced through cash transfers to his ex-partner $\sigma(m)$). Although, children's consumption cannot be brought directly from the market, it can be controlled by m through a transfer to $\sigma(m)$. We assume that the consumption decisions, upon divorce, can be represented as the subgame-perfect equilibrium of a two-stage game. In the first stage, m optimizes his utility function conditional on the best-response taken by $\sigma(m)$ for every cash transfer. In the second stage, $\sigma(m)$ optimizes her utility function given the transfer by m. The optimization problem of m, when single, looks like:

$$u^{m}(q^{m}, Q, C^{m}) \to \max_{q^{m}, Q, \tau}$$
$$p_{m, \emptyset}q^{m} + P_{m, \emptyset}Q + \tau \leq y_{m, \emptyset}$$
$$C^{m} = D_{m', \sigma(m)}(\tau)$$
$$\tau \geq T_{m, \sigma(m)}$$

Let $(q_{m,\emptyset}^m, Q_{m,\emptyset}, \tau_{m,\emptyset}^{m',\sigma(m)})$ be the solution to this problem. First order conditions give,

$$\nabla_{q^m} u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}^m) = \lambda_{m,\emptyset}^1 p_{m,\emptyset}$$
$$\nabla_Q u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}^m) = \lambda_{m,\emptyset}^1 P_{m,\emptyset}$$
$$\nabla_C u^m (q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}^m) = \frac{(\lambda_{m,\emptyset}^1 - \lambda_{m,\emptyset}^2)}{D'_{m',\sigma(m)}(\tau_{m,\emptyset}^{m',\sigma(m)})}$$

where $\lambda_{m,\emptyset}^1 \geq 0$ and $\lambda_{m,\emptyset}^2 \geq 0$ are Lagrange multipliers for the budget constraint and the minimum transfer constraint. Let $\varphi_{m,\emptyset}^{m',\sigma(m)} = \frac{1}{D'_{m',\sigma(m)}(\tau_{m,\emptyset}^{m',\sigma(m)})}$. Since, $1 \leq D'_{m',\sigma(m)}$, we have, $0 < \varphi_{m,\emptyset}^{m',\sigma(m)} \leq 1$. We further assume that the minimum transfer constraint $\tau \geq T_{m,\sigma(m)}$ is not binding $(\lambda_{m,\emptyset}^2 = 0)$.²⁷ Concavity of the utility function implies,

$$u^{m}(q_{m,\sigma(m)}^{m},Q_{m,\sigma(m)},C_{m,\sigma(m)}) - u^{m}(q_{m,\emptyset},Q_{m,\emptyset},C_{m,\emptyset})$$

$$\leq \lambda_{m,\emptyset}^{1}p_{m,\emptyset}(q_{m,\sigma(m)}^{m} - q_{m,\emptyset}) + \lambda_{m,\emptyset}^{1}P_{m,\emptyset}(Q_{m,\sigma(m)} - Q_{m,\emptyset}) + \lambda_{m,\emptyset}^{1}\varphi_{m,\emptyset}^{m',\sigma(m)}(C_{m,\sigma(m)} - C_{m,\emptyset})$$

By individual rationality of m,

$$\begin{split} 0 &\leq u^{m}(q_{m,\sigma(m)}^{m},Q_{m,\sigma(m)},C_{m,\sigma(m)}) - u^{m}(q_{m,\emptyset},Q_{m,\emptyset},C_{m,\emptyset}) \\ 0 &\leq p_{m,\emptyset}(q_{m,\sigma(m)}^{m} - q_{m,\emptyset}) + P_{m,\emptyset}(Q_{m,\sigma(m)} - Q_{m,\emptyset}) + \varphi_{m,\emptyset}^{m',\sigma(m)}(C_{m,\sigma(m)} - C_{m,\emptyset}) \\ p_{m,\emptyset}q_{m,\emptyset} + P_{m,\emptyset}Q_{m,\emptyset} &\leq p_{m,\emptyset}q_{m,\sigma(m)}^{m} + P_{m,\emptyset}Q_{m,\sigma(m)} + \varphi_{m,\emptyset}^{m',\sigma(m)}(C_{m,\sigma(m)} - C_{m,\emptyset}) \\ y_{m,\emptyset} - \tau_{m,\emptyset}^{m',\sigma(m)} &\leq p_{m,\emptyset}q_{m,\sigma(m)}^{m} + P_{m,\emptyset}Q_{m,\sigma(m)} + \varphi_{m,\emptyset}^{m',\sigma(m)}(C_{m,\sigma(m)}^{m} - C_{m,\emptyset}) \end{split}$$

In order to simply the last term on the right hand of the inequality, first recall that $0 < \varphi_{m,\emptyset}^{m',\sigma(m)} \leq 1$. Thus, we can write $\varphi_{m,\emptyset}^{m',\sigma(m)} C_{m,\sigma(m)}^m \leq C_{m,\sigma(m)}^m$. Next,

$$\tau = D_{m',\sigma(m)}^{-1}(C) = D_{m',\sigma(m)}^{-1}(0) + \int_0^\tau \frac{1}{D'_{m',\sigma(m)}(x)} dx \le \frac{C}{D'_{m',\sigma(m)}(\tau)}$$

Here, we make a technical assumption that $D^{-1}(0) = 0$. The latter inequality is implied by concavity of $D_{m,w}$ (that is, $D'_{m,w}(\tau) \leq D'_{m,w}(\tau')$ for every $\tau' < \tau$.) This gives,

$$\begin{aligned} \tau_{m,\emptyset}^{m',\sigma(m)} &= D_{m',\sigma(m)}^{-1}(C_{m,\emptyset}^m) \leq \frac{C_{m,\emptyset}^m}{D'_{m',\sigma(m)}(\tau_{m,\emptyset}^{m',\sigma(m)})} \\ \tau_{m,\emptyset}^{m',\sigma(m)} &\leq \varphi_{m,\emptyset}^{m',\sigma(m)}C_{m,\emptyset}^m \end{aligned}$$

Substituting these inequalities and further simplification gives the individual rationality condition shown in Proposition 2.

$$y_{m,\emptyset} - \tau_{m,\emptyset}^{m',\sigma(m)} \le p_{m,\emptyset}q_{m,\sigma(m)}^m + P_{m,\emptyset}Q_{m,\sigma(m)} + C_{m,\sigma(m)}^m - \tau_{m,\emptyset}^{m',\sigma(m)}$$
$$y_{m,\emptyset} \le p_{m,\emptyset}q_{m,\sigma(m)}^m + P_{m,\emptyset}Q_{m,\sigma(m)} + C_{m,\sigma(m)}^m$$

Individual Rationality for $w \in W$. Recall that the consumption decisions of $w \in W$ are made in the second stage where she receives the unconditional income transfer $(\tau_{\sigma(w),w'}^{\emptyset,w})$ from $\sigma(w)$. The

 $^{^{27}}$ Later, we will present the conditions without this assumption. We show that with binding minimum transfers contraint, the resulting conditions are weaker in nature. Thus, using those conditions for identification would gives less informative bounds.

optimization problem of w, when single, looks as follows:

$$\begin{cases} u^w(q^w, Q, C) \to \max_{q^w, Q, C} \\ p_{\emptyset, w} q^w + P_{\emptyset, w} Q + C \le y_{\emptyset, w} + \tau^{\emptyset, w}_{\sigma(w), w'} \end{cases}$$

Let $(q_{\emptyset,w}^w, Q_{\emptyset,w}, C_{\emptyset,w}^w)$ be the solution to the above problem. First order conditions imply,

$$\nabla_{q^w} u^w(q^w_{\emptyset,w}, Q_{\emptyset,w}, C^w_{\emptyset,w}) = \lambda_{\emptyset,w} p_{\emptyset,w}$$
$$\nabla_Q u^w(q^w_{\emptyset,w}, Q_{\emptyset,w}, C^w_{\emptyset,w}) = \lambda_{\emptyset,w} P_{m,\emptyset}$$
$$\nabla_C u^w(q^w_{\emptyset,w}, Q_{\emptyset,w}, C^w_{\emptyset,w}) = \lambda_{\emptyset,w}$$

Using concavity of u^w and individual rationality condition for w (in a similar way as used before), we get the following inequality:

$$y_{\emptyset,w} + \tau^{\emptyset,w}_{\sigma(w),w'} \le p_{\emptyset,w}q^w_{\sigma(w),w} + P_{\emptyset,w}Q_{\sigma(w),w} + C^w_{\sigma(w),w}$$

Finally, using the assumption of perfect enforcement of law $(\tau_{\sigma(w),w'}^{\emptyset,w} \ge T_{\sigma(w),w})$, we get the individual rationality conditions for w as shown in Proposition 2.²⁸

$$y_{\emptyset,w} + T_{\sigma(w),w} \le p_{\emptyset,w} q_{\sigma(w),w}^w + P_{\emptyset,w} Q_{\sigma(w),w} + C_{\sigma(w),w}^w$$

No Blocking Pairs. The optimization problem of a potential match (m, w) looks as follows:

$$u^{m}(q^{m}, Q, C^{m}) + \mu u^{w}(q^{w}, Q, C^{w}) \rightarrow \max_{q^{m}, q^{w}, Q, \tau, C^{w}}$$

$$p_{m,w}(q^{m} + q^{w}) + P_{m,w}Q + \tau + C^{w} \leq y_{m,w} + \tau^{m,w}_{\sigma(w),w'}$$

$$C^{m} = D_{m',\sigma(m)}(\tau)$$

$$\tau \geq T_{m,\sigma(m)}$$

²⁸ Note that we can also use $\tau_{\sigma(w),w'}^{\emptyset,w}$ as unknown transfers (with minimum binding constraint) and still get linear conditions. However, this would make no difference from computational point of view, because the algorithms to solve the linear inequalities would equalize $\tau_{\sigma(w),w'}^{\emptyset,w}$ to $T_{\sigma(w),w}$ in order to maximize the stability. Moreover, the unknown transfer $\tau_{\sigma(w),w'}^{\emptyset,w}$ depends on the belief about the potential match (w') of $\sigma(w)$. This requires us to make significant additional assumptions in order to apply conditions without adding anything to the empirical content.

Here m' is the belief of m about potential rematch of his ex-partner $\sigma(m)$. Similarly, w' indicates the potential rematch of w's ex-partner $\sigma(w)$. The first order condition would look as follows.

$$\begin{aligned} \nabla_{q^{m}} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m,w}^{m}) &= \lambda_{m,w}^{1} p_{m,w}; \\ \nabla_{q^{w}} \mu u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) &= \lambda_{m,w}^{1} p_{m,w}; \\ \nabla_{Q} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m,w}^{m}) &+ \mu \nabla_{Q} u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) &= \lambda_{m,w}^{1} P_{m,w}; \\ \nabla_{C^{m}} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m,w}^{m}) &= \frac{\lambda_{m,w}^{1} - \lambda_{m,w}^{2}}{D'_{m',w}(\tau)} \\ \nabla_{C^{w}} \mu u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) &= \lambda_{m,w}^{1}; \end{aligned}$$

Denote $\varphi_{m,w}^{m',\sigma(m)} = \frac{1}{D'_{m',\sigma(m)}(\tau_{m,w}^{m',\sigma(m)})}$. Since $D'_{m',\sigma(m)}(\tau_{m,w}^{m',\sigma(m)}) \ge 1$, we have $0 < \varphi_{m,w}^{m',\sigma(m)} \le 1$. Let $P_{m,w}^w = \frac{\mu \nabla u^w(q_{m,w}^w, Q_{m,w}, C_{m,w}^w)}{\lambda_{m,w}}$ and $P_{m,w}^m = P_{m,w} - P_{m,w}^w$. Finally, since the minimum transfers constraint is assumed to be non-binding, we have $\lambda_{m,w}^2 = 0$. Using concavity of utility functions and no blocking pair restrictions, we have the following inequality,

$$y_{m,w} - \tau_{m,w}^{m',\sigma(m)} + \tau_{\sigma(w),w'}^{m,w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^m Q_{\sigma(w),w} + C_{\sigma(w),w}^w + \varphi_{m,w}^{m',\sigma(m)}(C_{m,\sigma(m)}^m - C_{m,w}^m)$$

Using similar arguments as in individual rationality conditions for m, we have

$$\varphi_{m,w}^{m',\sigma(m)}C_{m,w}^{m} \ge \tau_{m,w}^{m',\sigma(m)}$$
$$\varphi_{m,w}^{m',\sigma(m)}C_{m,\sigma(m)}^{m} \le C_{m,\sigma(m)}^{m}$$
$$\tau_{\sigma(w),w'}^{m,w} \ge T_{\sigma(w),w}$$

Using these inequalities, we get the no blocking pairs condition shown in Proposition 2.

$$y_{m,w} + T_{\sigma(w),w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^m Q_{\sigma(w),w} + C_{\sigma(w),w}^w + C_{m,\sigma(m)}^m$$

Binding Minimal Transfers. Now, we present the conditions when we assume that the minimum transfer constraints are binding. In this case $\lambda_{m,w}^2 > 0$ and $\tau_{m,w}^{m',\sigma(m)} = T_{m,\sigma(m)}$. Concavity of u^m and individual rationality of m implies,

$$y_{m,\emptyset} - T_{m,\sigma(m)} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + \frac{1 - \frac{\lambda_{m,\emptyset}^2}{\lambda_{m,\emptyset}^1}}{D_{m',\sigma(m)}(\tau_{m',\sigma(m)}^{m,\emptyset}))} \Big[C_{m,\sigma(m)}^m - C_{m,\emptyset} \Big]$$

Note that, $\nabla_{C^m} u^m(q_{m,\emptyset}^m, Q_{m,\emptyset}, C_{m,\emptyset}^m)) \leq \frac{\lambda_{m,w}^1 - \lambda_{m,w}^2}{D'_{m',w}(\tau))}$. Since u^m is strictly increasing in every component and $D'_{m',w}(\tau) \geq 1$, it must be the case that $\lambda_{m,w}^2 \leq \lambda_{m,w}^1$. Hence, $0 \leq \frac{1 - \frac{\lambda_{m,w}^2}{\lambda_{m,w}^1}}{D_{m',w}(\tau)} \leq 1$.

Simplifying above inequality gives,

$$y_{m,\emptyset} - T_{m,\sigma(m)} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + C_{m,\sigma(m)}^m$$

Individual rationality condition for $w \in W$ is the same as for the case of non-binding constraints on minimum transfers. Using the same arguments given above, we can derive the no blocking pairs conditions in case minimum transfers are binding.

$$y_{m,w} - T_{m,\sigma(m)} + T_{\sigma(w),w} \le p_{m,w}(q_{m,\sigma(m)}^m + q_{\sigma(w),w}^w) + P_{m,w}^m Q_{m,\sigma(m)} + P_{m,w}^w Q_{\sigma(w),w} + C_{m,\sigma(m)}^m + C_{\sigma(w),w}^w$$

A.3 Proof of Proposition 3

Individual Rationality. Under joint custody, individual rationality conditions for $m \in M$ and $w \in W$ are symmetric. Here we only show the derivation for $m \in M$, corresponding condition for w can be obtained in a similar way. The optimization problem of m, when single, looks as follows:

$$\begin{cases} u^m(q^m, Q, C^m) \to \max_{q^m, Q, k^m, K^m} \\ p_{m, \emptyset} q^m + P_{m, \emptyset} Q + k^m + \alpha_m K^m \le y_{m, \emptyset} \\ C^m = k^m + k^{\sigma(m)} + K^m + K^{\sigma(m)} \end{cases}$$

Recall that children's daily routine consumption (k) are brought non-cooperatively and children's big-decision consumption (K) are brought cooperatively by the ex-partners $(m, \sigma(m))$. In this setting, the prices that m faces for children's consumption is 1 for k and α_m for K. First order conditions give,

$$\begin{split} \nabla_{q^m} u^m(q^m_{m,\emptyset},Q_{m,\emptyset},k^m_{m,\emptyset}+k^{\sigma(m)}+K^m_{m,\emptyset}+K^{\sigma(m)}) &= \lambda_{m,\emptyset}p_{m,\emptyset} \\ \nabla_Q u^m(q^m_{m,\emptyset},Q_{m,\emptyset},k^m_{m,\emptyset}+k^{\sigma(m)}+K^m_{m,\emptyset}+K^{\sigma(m)}) &= \lambda_{m,\emptyset}P_{m,\emptyset} \\ \nabla_{k^m} u^m(q^m_{m,\emptyset},Q_{m,\emptyset},k^m_{m,\emptyset}+k^{\sigma(m)}+K^m_{m,\emptyset}+K^{\sigma(m)}) &= \lambda_{m,\emptyset} \\ \nabla_{K^m} u^m(q^m_{m,\emptyset},Q_{m,\emptyset},k^m_{m,\emptyset}+k^{\sigma(m)}+K^m_{m,\emptyset}+K^{\sigma(m)}) &= \lambda_{m,\emptyset}\alpha_m \end{split}$$

As before, using concavity of u^m and individual rationality condition for m, we get,

$$y_{m,\emptyset} \le p_{m,\emptyset} q_{m,\sigma(m)}^m + P_{m,\emptyset} Q_{m,\sigma(m)} + k_{m,\sigma(m)} + \alpha_m K_{m,\sigma(m)}$$

No Blocking Pairs. The optimization problem for a potential match $(m, w) \in M \times W$ looks as follows:

$$u^{m}(q^{m}, Q, C^{m}) + \mu u^{w}(q^{w}, Q, C^{w}) \rightarrow \max_{q^{m}, q^{w}, Q, k^{m}, k^{w}, K^{m}, K^{w}}$$

$$p_{\emptyset, w}q^{w} + P_{\emptyset, w}Q + k^{m} + k^{w} + \alpha_{m}K^{m} + (1 - \alpha_{\sigma(w)})K^{w} \leq y_{m, w}$$

$$C^{m} = k^{m} + k^{\sigma(m)} + K^{m} + K^{\sigma(m)}$$

$$C^{w} = k^{w} + k^{\sigma(w)} + K^{w} + K^{\sigma(w)}$$

Let $(q_{m,w}^m, q_{m,w}^w, Q_{m,w}, C_{m,w}^m, C_{m,w}^w)$ be the outcome of this optimization problem. First order conditions for this optimization problem gives,

$$\nabla_{q^{m}} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m',w}^{m}) = \lambda_{m,w} p_{m,w};$$

$$\nabla_{q^{w}} \mu u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) = \lambda_{m,w} p_{m,w};$$

$$\nabla_{Q} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m',w}^{m})) + \mu \nabla_{Q} u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m,w}^{w}) = \lambda_{m,w} P_{m,w};$$

$$\nabla_{k^{m}} u^{m}(q_{m,w}^{m}, Q_{m,w}, C_{m',w}^{m})) = \lambda_{m,w};$$

$$\nabla_{K^{w}} \mu u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m',w}^{m})) = \lambda_{m,w} \alpha_{m};$$

$$\nabla_{K^{w}} \mu u^{w}(q_{m,w}^{w}, Q_{m,w}, C_{m',w}^{w})) = \lambda_{m,w} (1 - \alpha_{\sigma(w)});$$

Let $P_{m,w}^w = \frac{\mu \nabla u^w(q_{m,w}^w, Q_{m,w}, C_{m,w}^w)}{\lambda_{m,w}}$ and $P_{m,w}^m = P_{m,w} - P_{m,w}^w$. As before, using concavity of u^m and u^w and the no blocking pair restriction we get the condition shown in Propsoition 3.

$$y_{m,w} \le p_{m,w}(q_{m,\sigma(m)}^{m} + q_{\sigma(w),w}^{w}) + P_{m,w}^{m}Q_{m,\sigma(m)} + P_{m,w}^{w}Q_{\sigma(w),m} + k_{m,\sigma(m)} + \alpha_{m}K_{m,\sigma(m)} + k_{\sigma(w),w} + (1 - \alpha_{\sigma(w)})K_{\sigma(w),w}$$

B Estimating children's consumption from the USDA report

The USDA report for cost of raising children is based on the 2011 - 2015 Consumer Expenditure Survey data. This report provides average expenditures on a child from birth till age 17 for both a married-couple and a single-parent families with two children. These estimates are provided at the regional level for all married couples living in an urban area and at national level for all single-parent households and all married-couple households living in a rural area. For each group of households, the cost of children estimates are further separately estimated based on the income level and age group of children. There are three categories based on before-tax income level (less than \$59,200, \$59,200 - \$107,400 and more than \$107,400) and six categories based on age group of children (0 -2, 3 - 5, 6 - 8, 9 - 11, 12 - 14 and 15 - 17). For each category of households, the USDA provides the total annual expenditure in dollars on children. Further details are provided on the split-up of this cost based on different budgetary components like housing, food, transportation, clothing, health care, child care and education and miscellaneous.

In order to use these estimates for our dataset, we use the following procedure. First, for each category of households (based on marital status, region of residence, income category and age of

children), we use the given USDA dollar estimates to compute the average fraction of household income that goes to children's consumption. Further, using the detailed split up of different budgetary components, we compute the fraction of these costs that goes to the expenditures of type 'daily routine' and the rest that goes to the expenditures of type 'big decisions' (we need this split up only for the joint custody model). Next, using the before-tax income of the households reported in the PSID data and (based on same categorization), we impute children's consumption for our data, by multiplying the family income with fractions computed in the previous step. The estimates given in the USDA report are for families with two children. We use the adjustment factors provided in the report in order to compute the expense for families with less than or more than two children. For a one child family, we multiple the above expenditures by 1.27. To estimate the expenses for a family with three or more children, we multiply the above fractions by 0.76. Total children's consumption in a family is then given by the sum of each child expense. Lastly, we drop households for which total estimated children's consumption is greater than the reported household expenditure in the PSID.

C Additional Empirical Results

C.1 Economic instability and household characteristics

Table 9 gives the full sets of estimates for the regressions of our measures of economic instability ('average' and 'maximum' divorce costs) on observed household characteristics (corresponding to Tables 3 and 4). Other than the different effect of number of children (discussed in the main text), most of the other effects are qualitatively similar to the ones reported in CDDV.

C.2 Box plots of the bounds

Figures 2 and 3 give a visual summary of the distribution of lower and upper bounds on female's share of private consumption for states with income share and percentage rule respectively. The distribution of female's share clearly increases with number of children when we use the single person custody as the identifying model. This effect is smaller for the no legislation model and absent for the CDDV model. The figures reveal that the findings are true not just for average (as shown in Table 5) but for the majority of the sample.

	no legi	islation	single pers	on custody	CD	DV
	average	maximum	average	maximum	average	maximum
average wage	0.00383	-0.0375**	0.00295	-0.0321^{**}	0.00843	-0.0444^{**}
	(0.00375)	(0.0165)	(0.00411)	(0.0158)	(0.00555)	(0.0202)
absolute wage difference	0.0363^{***}	0.106^{***}	0.0407^{***}	0.0890^{***}	0.0551^{***}	0.0843^{***}
	(0.00257)	(0.00691)	(0.00268)	(0.00646)	(0.00294)	(0.00766)
average age	0.0142^{***}	0.0554^{***}	0.0130^{***}	0.0555^{***}	0.00444^{*}	0.0255^{***}
	(0.00195)	(0.00810)	(0.00198)	(0.00791)	(0.00247)	(0.00880)
absolute age difference	-0.0107^{**}	-0.0300	-0.0104^{**}	-0.0330	-0.00647	-0.0314
	(0.00463)	(0.0238)	(0.00480)	(0.0227)	(0.00625)	(0.0272)
marriage market size male	-0.000559	0.00708	-0.000586	0.0117^{*}	0.00359^{*}	0.0193^{**}
	(0.00127)	(0.00649)	(0.00135)	(0.00612)	(0.00185)	(0.00801)
marriage market size female	0.00107	0.0219^{***}	0.00116	0.0237^{***}	-0.00298	0.0379^{***}
	(0.00149)	(0.00785)	(0.00158)	(0.00737)	(0.00223)	(0.00989)
number of children	-0.149***	-1.280^{***}	-0.126***	-0.746***	0.0363^{**}	0.774^{***}
	(0.0103)	(0.0594)	(0.0112)	(0.0577)	(0.0171)	(0.0702)
married	-0.00609	0.116	-0.00626	0.324	0.00611	0.0672
	(0.0335)	(0.208)	(0.0351)	(0.201)	(0.0561)	(0.243)
log income	-0.682***	-1.888***	-0.702***	-1.941^{***}	-1.302^{***}	-2.727***
	(0.103)	(0.485)	(0.109)	(0.458)	(0.153)	(0.580)
percentage rule	-0.0343	-0.369*	-0.0517	-0.573***	0.00287	0.409^{*}
	(0.0384)	(0.190)	(0.0404)	(0.177)	(0.0609)	(0.247)
North Central	-0.0652	-0.379	-0.0644	-0.533**	-0.0625	-0.455^{*}
	(0.0532)	(0.247)	(0.0558)	(0.244)	(0.0722)	(0.263)
South	-0.0470	-0.503**	-0.0339	-0.539**	-0.0152	0.0141
	(0.0548)	(0.236)	(0.0576)	(0.233)	(0.0722)	(0.251)
West	-0.145***	-1.082***	-0.160***	-1.358^{***}	-0.237***	-1.680***
	(0.0557)	(0.258)	(0.0582)	(0.254)	(0.0748)	(0.285)
Constant	5.286***	18.04^{***}	5.524***	18.57^{***}	10.83***	27.27^{***}
	(0.759)	(3.623)	(0.800)	(3.423)	(1.145)	(4.344)
Observations	1,358	1,358	1,358	1,358	$1,\!358$	1,358
R-squared	0.457	0.419	0.456	0.329	0.416	0.428

Table 9: Instability and household characteristics

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Figure 2: Box plots of lower (left) and upper (right) bounds of female's share in states with income shares rule



Figure 3: Box plots of lower (left) and upper (right) bounds of female's share in states with percentage rule



C.3 Female's share of private consumption and household characteristics

Table 10 gives the full sets of estimates for the regressions of lower and upper bounds on female's share of private consumption on observed household characteristics (corresponding to Tables 5 and 6). Other than the different effect of number of children (discussed in the main text), most of the other effects are qualitatively similar to the ones reported in CDDV.

D Robustness check using conditional sharing rule

In this section, we use the conditional sharing rule as a basis for the comparative analysis. Conditional sharing rule is defined as an individual's share of total private consumption in the household conditional on the level of public consumption. In our context, for individual i, it is defined as $\frac{w_i l_i + q^i}{w_i l_i + w_j l_j + q}$ where $i \neq j$, w_i is wage rate, l_i is leisure and q^i is the private Hicksian good consumption of i. Since leisure is an assignable good, it is evident that the bounds on conditional sharing rule

	no legis	slation	single pers	on custody	CD	DV
	lower	upper	lower	upper	lower	upper
$\log(w_f/w_m)$	0.369^{***}	0.444***	0.388***	0.433***	0.342***	0.443***
	(0.0123)	(0.0125)	(0.0125)	(0.0127)	(0.0126)	(0.0124)
$age_m - age_f$	0.00483^{**}	0.000129	0.00198	-0.000322	0.00400^{**}	0.00431^{**}
	(0.00194)	(0.00190)	(0.00202)	(0.00190)	(0.00178)	(0.00194)
married	-0.0235	-0.0427^{*}	-0.0251	-0.0313	-0.0107	0.00381
	(0.0214)	(0.0219)	(0.0220)	(0.0212)	(0.0194)	(0.0217)
log(income)	-0.00524	0.0627^{***}	-0.0414**	0.0615^{***}	0.0363^{**}	0.151^{***}
	(0.0182)	(0.0192)	(0.0189)	(0.0189)	(0.0163)	(0.0190)
percentage rule	0.0340	-0.303***	0.171^{***}	-0.272^{***}	-0.119	-0.429^{***}
	(0.0503)	(0.102)	(0.0539)	(0.105)	(0.144)	(0.120)
number of children	0.0162^{**}	0.0545^{***}	0.0717^{***}	0.0711^{***}	0.00256	-0.00977
	(0.00664)	(0.00700)	(0.00712)	(0.00674)	(0.00607)	(0.00673)
North Central	-0.0534	0.248	-0.339*	0.191	0.0640	0.530^{***}
	(0.128)	(0.174)	(0.176)	(0.179)	(0.202)	(0.177)
South	-0.148	-0.126	-0.327*	-0.0497	-0.228	-0.0114
	(0.140)	(0.174)	(0.186)	(0.163)	(0.161)	(0.163)
West	-0.291^{**}	0.276^{*}	-0.494***	0.263^{*}	-0.214	0.451^{***}
	(0.116)	(0.150)	(0.165)	(0.156)	(0.195)	(0.152)
marriage market size male	-0.00206***	-0.00204**	-0.00217^{***}	-0.00229^{***}	-0.00279^{***}	-0.00393***
	(0.000787)	(0.000811)	(0.000779)	(0.000754)	(0.000752)	(0.000826)
marriage market size female	0.00245^{***}	0.00230^{**}	0.00260^{***}	0.00322^{***}	0.00249^{***}	0.00400***
	(0.000936)	(0.000960)	(0.000923)	(0.000897)	(0.000888)	(0.000963)
Constant	0.440^{**}	0.130	0.870^{***}	0.114	0.175	-0.769^{***}
	(0.185)	(0.207)	(0.226)	(0.208)	(0.189)	(0.199)
Observations	$1,\!358$	$1,\!358$	1,358	$1,\!358$	1,358	1,358
R-squared	0.485	0.548	0.519	0.565	0.477	0.545

Table 10: Female's share of private consumption of Hicksian good and household characteristics

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

can be calculated directly from the bounds on q^i (we used the bounds on q^i as our key parameter in the main text).

Table 11 shows the average lower and upper bounds on female's conditional sharing rule across different household size for a sample of households from the states with single person custody law. Panel 1 is for households residing in states with income share rule and panel 2 is for those with percentage rule. We get similar results as those shown in the main text. For couples without children, the bounds on conditional sharing rule are the same from all three models. For couples with children, we get different bounds from all three models. In fact, in many instances, the bounds obtained from the three models are non overlapping. For instance, for a mother with more than two children in an income share rule states, the average conditional sharing rule from the CDDV model is between 47% - 49%, whereas, it is between 50% - 51% from the single person custody model. These bounds again point towards the possible misidentification if we ignore child custody laws or differential nature of children's consumption. However, also note that the extent of misidentification looks smaller in this case. That is, the bounds are clearly shifted but on absolute level it looks as if the shift is only marginal. This is because the non-assignable Hicksian private consumption makes only a small fraction of the total private consumption in the sample (on average about 11%). In general, the effect of misidentification will increase with the increase in share of non-assignable private consumption.

	no legi	islation	single p	erson custody	CD	DV
	lower	upper	lower	upper	lower	upper
			incom	e shares rule		
no child	0.44	0.47	0.44	0.47	0.44	0.46
one child	0.49	0.52	0.50	0.53	0.49	0.51
two children	0.46	0.48	0.47	0.49	0.46	0.47
> 2 children	0.48	0.51	0.50	0.51	0.47	0.49
total	0.46	0.49	0.47	0.49	0.46	0.48
			perc	entage rule		
no child	0.44	0.46	0.44	0.46	0.43	0.46
one child	0.50	0.53	0.52	0.54	0.50	0.52
two children	0.46	0.48	0.47	0.49	0.45	0.47
> 2 children	0.54	0.56	0.55	0.57	0.53	0.54
total	0.47	0.50	0.48	0.50	0.46	0.48

Table 11: Bounds on conditional sharing rule for a sample from the states with single person custody law

Next, we relate the bounds on conditional sharing rule with observed household characteristics. For this we conduct interval regressions where we explicitly use the lower and upper bounds as dependent variables. Table 12 shows the result of three interval regressions conducted using the three sets of bounds obtained from the no custody, single person custody and CDDV model. These results support the conclusions we make from the regression analysis shown in the main text (Table 6). In particular, there are many similar effects from the three models (e.g. relation between conditional sharing rule and wage ratio, region and size of marriage markets). Although, the effect of number of children is not as dramatic as the one we find through the regressions shown in the main text, quantitatively the effect of children is more strong in the single person custody and the No legislation model as compared to the CDDV model.

	no legislation	single person custody	CDDV
$\log(w_f/w_m)$	0.233***	0.233^{***}	0.231^{***}
	(0.00304)	(0.00306)	(0.00302)
age_m - age_f	-1.24e-05	-0.000182	2.81e-05
-	(0.000277)	(0.000263)	(0.000281)
married	-0.00494	-0.00405	-0.00398
	(0.00320)	(0.00311)	(0.00334)
$\log(\text{income})$	0.00249	4.63e-05	0.0113^{***}
	(0.00278)	(0.00280)	(0.00292)
percentage rule	0.0153	0.0173	0.0135
	(0.0103)	(0.0107)	(0.00987)
number of children	0.00865^{***}	0.0114***	0.00646^{***}
	(0.00106)	(0.00105)	(0.00109)
North Central	-0.0164	-0.0317**	-0.00789
	(0.0124)	(0.0160)	(0.0119)
South	-0.0107	-0.0226	-0.00560
	(0.00928)	(0.0138)	(0.00831)
West	-0.0247***	-0.0349***	-0.0250***
	(0.00681)	(0.0131)	(0.00766)
marriage market size male	-0.000319***	-0.000353***	-0.000430***
	(0.000119)	(0.000108)	(0.000117)
marriage market size female	0.000339**	0.000427^{***}	0.000380***
	(0.000140)	(0.000128)	(0.000138)
Constant	0.493***	0.523***	0.414***
	(0.0236)	(0.0256)	(0.0245)
Observations	1,358	1,358	1,358
Robus	t standard error	s in parentheses	-

Table 12: Interval regression of conditional sharing rule on household characteristics

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1