# Does Income Assistance Increase Disposable Income?\*

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### Abstract

This paper theoretically analyzes how income assistance/welfare benefits affect the aggregate disposable income of the benefit recipients when the government cannot observe the recipients' earning capability. If the recipients' earning capabilities are uniformly distributed, the model shows that the means-tested linear income assistance benefits do not increase the aggregate disposable income of the benefit recipients at all, regardless of the benefit size. Under a more realistic distribution of earning capabilities, the aggregate disposable income can decrease. The expenditure minimizing benefit reduction rate is strictly between zero and one.

Keywords: Income Assistance; Disposable Income; Moral Hazard; Adverse Selection.

JEL Codes: H24, D82, H20

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# 1 Introduction

The main goal of the income assistance/welfare  $\text{programs}^1$  is to increase the disposable income of the recipients. Surprisingly, however, few studies have theoretically analyzed the effects of income assistance benefits on the *aggregate* disposable income of the benefit recipients.

It is well known that if the government cannot observe workers' earning capabilities, income assistance (or welfare) benefits can induce the benefit recipients to work/earn less than their capability, called the moral hazard problem. Also, even the capable workers can reduce their work hours/wages to become eligible for the (means-tested) benefits, called the adverse selection problem. Then, the income assistance benefits can increase the disposable income (i.e. the sum of wage income and welfare benefits) for some benefit recipients, but decrease it for the others.

This paper provides a theoretical model to show that if the (means-tested) income assistance benefit is linear to the wage income of the recipient and if the earning capabilities are uniformly distributed, the increase in the disposable income for some benefit recipients is exactly cancelled out by the decrease in the disposable income for the others. That is, the income assistance benefits do not increase the *aggregate* disposable income at all, regardless of the benefit size.

Intuitively, since the earning capabilities are not observable to the government, workers can adjust their wage income to choose whether to receive the benefit or not. Then, the benefit recipient with the highest earning capability would be the one who is just indifferent between receiving the benefit and not receiving it.

Now note that individual disposable income is determined by the *marginal* utility of earning wage income. Since the aggregate disposable income is the integral of individual disposable incomes, the aggregate disposable income of the benefit recipients is determined by the *level* of utility by the benefit recipient with the highest earning capability. Because the level of utility by that recipient is the same whether or not he or she receives the benefit, it must mean that the aggregate disposable income of the benefit recipients is the same whether or not he or she receives the benefit, it must mean that the aggregate disposable income of the benefit recipients is the same as the aggregate disposable income without the benefits.

Moreover, because income assistance benefits reduce wage income mostly

<sup>&</sup>lt;sup>1</sup>In this paper, I do not distniguish among welfare benefits, income assistance, income transfer, or tax reduction, and use them interchangeably.

for the recipients with relatively higher capability, if there exist relatively more higher ability workers among the benefit recipients, the aggregate disposable income can decrease. For example, if the earning capability distribution is single-peaked (e.g. log normal distribution) and if the benefit recipients are distributed on the left (or lower) side of the distribution, income assistance benefits can reduce the aggregate disposable income of the benefit recipients.

It is worth emphasizing that these results do not imply that the income assistance programs are not effective, because the income assistance programs can raise the disposable income of those with relatively lower earning capabilities among the benefit recipients. Therefore, even when the aggregate disposable income remains the same or even decreases, if the society cares enough about the disposable income of the least capable, the income assistance benefits can still be justified.

This paper does not focus on deriving the "optimal" income assistance programs because the objectives of an income assistance program can be so diverse. Given that income assistance benefits do not increase the aggregate disposable income, however, one might be interested in minimizing the required budgets/expenditure. If the earning capability is observable, the budget minimizing benefit reduction rate with respect to earning capability is one. That is, the benefit must decrease as much as the capability increases. However, if the earning capability is not observable, I show that the budget minimizing benefit reduction rate with respect to wage income is less than one.

There exist long debates on the effects of welfare programs on poverty both politically and academically. The critics of the welfare programs argue that the welfare programs poster dependencies on the benefits and reduce work incentives, thereby trap the welfare recipients in poverty.<sup>2</sup> The empirical evidence is also mixed. Kenworthy (1999), Schoeni and Blank (2000), and Bavier (2002), for example, show that welfare programs have significant effects on reducing poverty. However, Weber et al. (2004), Gundersen and Ziliak (2004), and Borjas (2016) find no significant effects. <sup>3</sup>

The theoretical literature provides many textbook examples where welfare benefits can either increase or decrease *individual* labor supply and (dispos-

<sup>&</sup>lt;sup>2</sup>Murray (1984) argues "We tried to provide more for the poor and produce more poor instead." (p.9)

<sup>&</sup>lt;sup>3</sup>There are also many empirical studies on the effects of welfare programs on labor supply, but not on the disposable income. See Moffitt (2002) for a survey.

able) income (e.g. Moffitt 2002). However, as far as I know, no previous theoretical studies have examined the effects of income assistance programs on the *aggregate* disposable income of the benefit recipients.<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 provides a benchmark case where earning capabilities are observable. In section 4, I analyze how workers choose the benefits and their wage income when the earning capabilities are not observable. Then, section 5 analyzes the effects of income assistance benefits on the individual and the aggregate disposable incomes. In section 6, I analyze the required welfare expenditure and the budget-minimizing benefit reduction rates. Section 7 concludes with the summary and the discussion on the directions for future studies.

# 2 Basic Model

Suppose that workers are risk-neutral and that their utility function is given as follows:

$$U(w) = w + B - \frac{1}{k}c(w) \tag{1}$$

where w is the wage income (or labor supply); B is the income assistance benefit; and  $\frac{1}{k}c(w)$  is the cost of earning wage w where c(0) = 0, c' > 0, c'' > 0, c'(0) = 0, and  $c'(\infty) = \infty$ . The cost function captures both the cost of efforts to earn wage income and the opportunity cost for leisure.

Note that the (marginal) cost of earning wage w decreases in k. Therefore, I interpret k as earning capability. As shown below, without the income assistance benefits, workers will earn wage income equal to their earning capability k. For now, I assume that k is uniformly distributed over an interval [0, 1]. Later, I will consider other distributions. Also, assuming that wage income is the only source of income, w + B can be defined as the disposable income. As shown below, both the wage and the benefit are the functions of k. Thus, the aggregate disposable income can be defined as follows:

$$D = \int_0^1 (w(k) + B(k))dk$$

 $<sup>^{4}</sup>$ In other words, the previous literature has not studied the distribution of individual types which can induce the welfare benefits to increase or decrease the aggregate disposable income.

If there exist no welfare benefits (i.e. B(k) = 0), then, from utility maximization, it is straightforward to show that the optimal wage is:

$$w^* = c'^{-1}(k) \equiv g(k),$$
 (2)

where  $g \equiv c'^{-1}$ . That is, without welfare benefits, a worker with earning capability k will work enough to earn wage equal to g(k) where g(0) = 0 and g' > 0. Then, without the welfare benefits, the aggregate disposable income is

$$D^N = \int_0^1 g(k)dk \tag{3}$$

For simplicity, I assume that the main goal of the income assistance is to guarantee the minimum income level (denoted by  $\underline{w}$ ) for everyone. I also assume that the government provides the income assistance to those below the minimum income level only, that is, the benefits are means-tested.<sup>5</sup> Let us define  $\underline{k}$  such that

$$\underline{w} = g(\underline{k}). \tag{4}$$

Then, from (2), individuals with earning capability less than  $\underline{k}$  must be supported by an income assistance program.<sup>6</sup>

As discussed in the beginning, I do not focus on deriving the "optimal" income assistance program because the objectives of the income assistance can be so diverse. For example, while the economists tend to focus on the *utility* of a benefit recipient, the policy makers tend to care more about the disposable *income* or labor supply of a benefit recipient. Also, some may care relatively more about the least capable recipients, while others may put equal weights on all benefit recipients. Therefore, I will focus on the effects of income assistance on the individual and the aggregate (disposable) income of the benefit recipients and the required welfare expenditure, without going into the discussion on the social welfare function.

## **3** Symmetric Information

For a benchmark, consider the symmetric information case where the government can also observe each worker's earning capability k. From (2), workers

<sup>&</sup>lt;sup>5</sup>The main results of this paper do not change even if the eligibility standard income is different from the minimum guaranteed disposable income  $\underline{w}$ .

<sup>&</sup>lt;sup>6</sup>In reality, many income assistance programs are based on the household income. For simplicity, however, I do not distinguish between an individual and a household.

with earning capability k can earn the wage  $w^* = g(k)$  by themselves. Then, one can consider the following income assistance program:

$$B^*(k) = \begin{cases} \frac{w}{w} - g(k) & \text{if } k \le \underline{k} \\ 0 & \text{if } k > \underline{k} \end{cases}$$
(5)

It is straightforward to show that  $B^*(k)$  minimizes the welfare expenditure/budget while guaranteeing everyone with the minimum disposable income  $\underline{w}$ . Note that because the benefit depends on the earning capability konly,  $B^*(k)$  does not change workers' incentives to earn wages. Thus, the aggregate disposable income under  $B^*(k)$  is

$$D^* = \int_0^1 w^*(k) + B^*(k)dk = \int_0^{\underline{k}} (g(k) + \underline{w} - g(k))dk + \int_{\underline{k}}^1 g(k)dk.$$
(6)

Then, by the definition of  $\underline{k}$ ,  $D^* - D^N = \int_0^{\underline{k}} (\underline{w} - g(k)) dk = c(\underline{w}) > 0.^7$  That is, the income assistance program  $B^*(k)$  increases the aggregate disposable income of the benefit recipients. Also, the required welfare expenditure is  $\int_0^1 B^*(k) dk = \int_0^{\underline{k}} (\underline{w} - g(k)) dk = D^* - D^N$  as well. To summarize,

**Proposition 1** When workers earning capabilities (k) is observable,  $B^*(k)$  increases the aggregate disposable income of the benefit recipients by  $c(\underline{w})$ . Also, the required aggregate welfare expenditure is  $c(\underline{w})$ .

**Proof.** From the discussion above.

Note that if the government raises the minimum income level  $\underline{w}$ , it raises the benefit level  $B^*(k)$  for a given k, and also, from (4), increases  $\underline{k}$  so that more workers can receive the benefits. Intuitively, with  $B^*(k)$ , since the workers' incentives to earn wage income are not affected by the income assistance benefits, the more the government spend on the benefits by raising  $\underline{w}$ , the more the aggregate disposable income of the benefit recipients increase. While this result may seem obvious, as I will show below, it no longer holds when k is not observable.

<sup>&</sup>lt;sup>7</sup>The last equality is from the formular  $\int f^{-1}(y)dy = yf^{-1}(y) - F \circ f^{-1}(y) + C$  and  $g = c'^{-1}$ .

**Example 1** (Symmetric Information) Suppose that  $c(w) = \frac{1}{2}w^2$ . Then,  $w^* = k$  and  $\underline{k} = \underline{w}$ . From Figure 1, both the increase in the aggregate disposable income and the aggregate benefit expenditure can be represented by the area  $E = \frac{1}{2}\underline{w}^2$ .

[Figure 1 here]

## 4 Asymmetric Information and Wage Income

Now suppose that the government cannot observe each worker's earning capability k, but that workers themselves know their earning capability. Thus, there exists an information asymmetry problem. Note that the income assistance program  $B^*(k)$  in (5) is no longer feasible because the government cannot observe k any more.

### 4.1 Linear Benefits

Even though the government cannot observe earning capability k, it can still observe a worker's wage income w. Thus, I consider the linear income assistance benefit,  $B^{L}(w)$ , as follows:

$$B^{L}(w) = \begin{cases} \frac{w}{0} - bw & \text{if } w \leq \underline{w} \\ 0 & \text{if } w > \underline{w} \end{cases}$$
(7)

where  $0 \le b \le 1.^8$ 

Note that as wage income increases, the benefit decreases by b. Thus, b represents the benefit reduction rate or phase-out rate. If b = 0, the benefit is fixed and does not decrease in wage income, as in the basic pension for the elderly in Korea. If b = 1, the benefit decreases as much as the wage income increases, as in the national basic livelihood security payment in Korea.

With the linear benefits, *if* a worker is eligible for the benefits (i.e.  $w \leq \underline{w}$ ), the optimal wage would be determined by the following first order condition:

$$U'(w) = (1-b) - \frac{1}{k}c'(w) = 0$$

<sup>&</sup>lt;sup>8</sup>More general benefit functions are not easily tractible, and remain as the topics for future research.

or

$$w_1^L(k) = g((1-b)k).$$
(8)

Note that compared with the no-benefit (or symmetric information) case in (2), the wage income is smaller as long as b > 0. That is, if income assistance benefits strictly decrease in wage income, the benefit recipients work/earn less, known as the moral hazard problem.

### 4.2 Benefit Choice

To analyze the wage income and the benefit size, however, one must check whether the wage income  $w_1^L(k)$  in (8) is eligible for the benefit (i.e.  $g((1-b)k) \leq \underline{w})$ , and whether the worker will choose the benefits (i.e. choose to earn wages less than or equal to  $\underline{w}$ ) in the first place.

Let us define  $k_1$  such that

$$g((1-b)k_1) = \underline{w}.$$
(9)

Since g' > 0, workers with  $k \leq k_1$  can earn the wage  $w_1^L(k)$  and still qualified for the benefits.

Let us define the level of utility when a worker with  $k \ (\leq k_1)$  receives the benefit as

$$U_1(k) = \underline{w} + (1-b)g((1-b)k) - \frac{1}{k}c(g((1-b)k))$$
(10)

Instead, if a worker earns a wage greater than  $\underline{w}$  and does not receive the benefits, he would earn  $w^* = g(k)$  which must be greater than  $\underline{w}$ . Let us define the level of utility when a worker does not receive the benefits as

$$U^{*}(k) = g(k) - \frac{1}{k}c(g(k)).$$
(11)

**Lemma 1** There exists  $k_0 \in [\underline{k}, 1)$  such that  $U^*(k) \geq U_1(k)$  if  $k \geq k_0$ .

#### **Proof.** See appendix.

That is, even with the information asymmetry problem, workers with high enough earning capability would not receive the income assistance benefits, because they have to reduce their wage income too much to be eligible for the benefits. Throughout the paper, I assume that  $\underline{w}$  is low enough that some high capability workers will not choose the benefits, that is  $k_0 < 1$ .

If  $k_0 \leq k_1$ , then workers with  $k \leq k_0$  would prefer receiving the benefits, and make wage income  $w_1^L(k)$  which is small enough to be eligible for the benefits.

In the following lemma, I derive the condition for  $k_0 \leq k_1$ .

**Lemma 2** There exist  $b_0 \in (0,1)$  such that  $k_0 \leq k_1$  if  $b \geq b_0$ .

**Proof.** See appendix.

Intuitively, if b is large enough  $(b > b_0)$ , the size of the benefit becomes too small for higher earning capable workers. Thus, only those workers with very low earning capability  $(k < k_0)$  will choose to receive the benefits. Also, because the benefit decreases rapidly as wage increases, workers have less incentive to earn wage income. Thus, the optimal wages of the benefit recipients  $(w_1^L(k))$  are low enough to be eligible for the benefits  $(k_0 < k_1)$ .

### 4.3 Wage Income

From lemma 1 and 2, there are two cases to consider. First, suppose that  $b \ge b_0$  (i.e.  $k_0 \le k_1$ ). Then, as discussed above, if  $k \le k_0$ ,  $U_1 \ge U^*$  and  $w_1^L(k) = g((1-b)k) \le \underline{w}$ . That is, workers with  $k \le k_0$  prefer receiving the benefits and earn  $w_1^L(k)$ . Also,  $w_1^L(k)$  is feasible as it is low enough to be eligible for the benefit.

If  $k > k_0$ , a worker would prefer not receiving the benefits and earn  $w^*(k)$  only. Also, since  $k_0 \ge \underline{k}$  from lemma 1,  $w^*(k) > \underline{w}$ . That is,  $w^*(k)$  is not eligible for the benefits if  $k > k_0$ .

Second, now suppose that  $b < b_0$  (i.e.  $k_0 > k_1$ ). If  $k \le k_1$ , from lemma 1, the optimal wage of the benefit recipients  $w_1^L(k)$  qualifies for the benefits and the worker prefers receiving the benefits.

If  $k > k_1$ , however, a worker may prefer receiving the benefits, but the optimal wage of the benefit recipients  $w_1^L(k)$  is greater than  $\underline{w}$  and does not qualify for the benefits. Therefore, in order to qualify for the benefits, the worker would have to reduce the wage to  $\underline{w}$ . Let us define the level of utility in this case as follows:

$$U_2 = \underline{w} + (1-b)\underline{w} - \frac{1}{k}c(\underline{w}).$$

A worker would reduce his wage to  $\underline{w}$  in order to receive the benefits if it is still better than not receiving the benefits, that is  $U_2 \ge U^*$ .

**Lemma 3** If  $b < b_0$  and  $k > k_1$ , there exist  $k_2 \in (k_1, 1)$  such that  $U^*(k) \geq U_2(k)$  if  $k \geq k_2$ .

### **Proof.** See appendix.

That is, if  $k_1 < k \leq k_2$ , then  $U^*(k) \leq U_2(k)$  and the worker will earn just  $\underline{w}$  in order to be eligible for the benefit.

Finally, if  $k > k_2$ , the worker would not choose the benefits and earn  $w^* = g(k)$ . Again, I assume that  $\underline{w}$  is low enough that  $k_2 < 1$ . That is, some high capability workers will not choose the benefits.

To summarize, I can fully characterize the workers' wage income when the means-tested linear benefit  $B^{L}(w)$  is available as follows:

**Proposition 2** With linear benefits  $B^L(w)$  in (7), there exist  $b_0 \in (0, 1)$  such that the wage income by a worker with earning capability k is determined as follows:

(i) If  $b_0 \leq b \leq 1$ , then there exist  $\underline{k} \leq k_0 < 1$  such that

$$w^{L}(k) = \begin{cases} g((1-b)k) & \text{if } k \le k_{0} \\ g(k) & \text{if } k > k_{0} \end{cases}$$
(12)

(ii) If  $0 \le b < b_0$ , then there exist  $\underline{k} \le k_1 < k_2 < 1$  such that

$$w^{L}(k) = \begin{cases} g((1-b)k) & \text{if } k \le k_{1} \\ \underline{w} & \text{if } k_{1} < k \le k_{2} \\ g(k) & \text{if } k > k_{2} \end{cases}$$
(13)

where  $\underline{k}$  and  $k_1$  are defined by (4) and (9), respectively. Also,  $k_0$  and  $k_2$  are defined by lemma 1 and 3, respectively.

**Proof.** From the discussion above.

Note that as long as b > 0, the income assistance benefits strictly decrease in wage income. Therefore, those who receive the benefits earn less wages than the symmetric information case, known as the moral hazard problem. Moreover, both  $k_0$  and  $k_1$  in proposition 2 are larger than  $\underline{k}$ . Therefore, those workers who can earn wages greater than  $\underline{w}$  are receiving the benefits and earn wages less than  $\underline{w}$ , known as the adverse selection problem. Thus, both the moral hazard and the adverse selection problems reduce wage income of the benefit recipients. Then, it is a *priori* ambiguous whether the disposable income (i.e. the sum of wage income and the income assistance benefits) would increase for a benefit recipients.

## 5 Income Assistance and Disposable Income

From proposition 2, the disposable income of a benefit recipient is  $w^L(k) + B^L(w^L(k)) = \underline{w} + (1-b)w^L(k)$ . Since the wage income without the benefits is  $w^* = g(k)$ , the change in the *individual* disposable income due to the income assistance benefit can be defined as

$$\Delta(k) = (\underline{w} + (1 - b)w^{L}(k)) - g(k).$$
(14)

Then, I can characterize the change in individual disposable income as in the following proposition.

**Proposition 3**  $\Delta(0) > 0$ ,  $\Delta'(k) < 0$ ,  $\Delta(k_0) < 0$ , and  $\Delta(k_2) < 0$ .

**Proof.** See appendix.

From continuity, among the benefit recipients, the disposable income of those workers with relatively lower earning capability increases. However, the disposable income of those with relatively higher earning capabilities decreases.

Intuitively, workers with relatively lower earning capability earn lower wage income without the benefits. Thus, the moral hazard problem due to the income assistance benefit (i.e. decrease in the wage income) is smaller in absolute magnitude. Also, the size of the benefit is larger for lower wage incomes. Therefore, the disposable income increases for workers with relatively lower earning capability. By the same intuition, the disposable income decreases for workers with relatively higher earning capability.

The following theorem shows that the increase in the disposable income among the relatively lower capability benefit recipients is exactly cancelled out by the decrease in the disposable income among the relatively higher capability benefit recipients.

**Theorem 1** If k is uniformly distributed, the aggregate disposable income of the benefit recipients do not increase for all  $\underline{w}$  and  $b \in [0, 1]$ .

#### **Proof.** See appendix.

Note that the theorem holds for all  $\underline{w}$  and  $b \in [0, 1]$ .<sup>9</sup> If the government relaxes the eligibility condition for the income assistance benefits by raising  $\underline{w}$ , more workers will receive the benefits. Thus, one might think that the aggregate disposable income of all the benefit recipients would increase. Surprisingly, however, theorem 1 states that regardless of the level of the benefit  $(\underline{w})$  or the slope of the benefit function (b), the income assistance benefits do not increase the aggregate disposable income at all, as if one is filling a bottomless pit.

This result is in contrast with the symmetric information outcome in proposition 1. When the workers' earning capabilities are observable, greater income assistance benefits can further increase the aggregate disposable income. However, when the workers' earning capabilities are not observable, the aggregate disposable income does not increase regardless of the size of the benefit.

For intuition, one needs to make two observations. First, from (2) or (9), individual wage income is determined by the (inverse) marginal utility (or cost) of earning wage income. Since the size of the welfare benefit is determined by the individual wage income as in (7), one can observe that *individual* disposable income is determined by the *marginal* utility of earning wage income. Then, the *aggregate* disposable income, which is the integral of individual disposable income, must be determined by the *level* of utility by the highest capability benefit recipient ( $k = k_0$  or  $k_2$  in proposition 2).

Second, because the earning capability is not observable, workers can choose whether or not to receive the benefits. Thus, the highest capability benefit recipient is the one who is just indifferent between benefits and no benefits (lemma 1 and 3). That is, the level of utility by the highest capability benefit recipient is the same between receiving the benefits and not receiving

<sup>&</sup>lt;sup>9</sup>Recall, however, that throughout the paper, I assume  $\underline{w}$  is low enough that some workers do not choose the benefits, that is,  $k_0 < 1$  and  $k_2 < 1$ .

the benefits. Then, by the first observation above, the aggregate disposable income with income assistance benefits must be the same as the aggregate disposable income without them.

**Example 2** (Decreasing Benefits) Suppose that  $c(w) = \frac{1}{2}w^2$  and b = 1. That is, the benefits decrease as much as the wage income increase. If a worker receives the benefit, his utility function is  $U(w) = w + \underline{w} - w - \frac{1}{2k}w^2 = \underline{w} - \frac{1}{2k}w^2$ . Therefore, the optimal wage for the benefit recipient is  $w_1^L = 0$  (which is eligible for the benefit) and the utility level is  $U_1 = \underline{w}$ . If the worker does not receive the benefits, from example 1, the optimal wage is  $w^* = k$  and the utility level is  $U^* = \frac{1}{2}k$ . Therefore, a worker would receive the benefit if  $U_1 \ge U^*$  or  $k \le 2\underline{w}$ .

#### [Figure 2 here]

In figure 2, the thick solid line represents the disposable income (= wage + benefit), and the thin solid line represent the wage income only. The dashed line represents the wage income when there exists no income assistance program. Note that for workers with  $k \leq \underline{w}$ , the disposable income increases by area E. However, for those workers with  $\underline{w} < k \leq 2\underline{w}$ , the disposable income decreases by area D. Since  $E = D = \frac{1}{2}\underline{w}^2$ , the aggregate disposable income does not change regardless of  $\underline{w}$ .

**Example 3** (Fixed Benefits) Suppose that  $c(w) = \frac{1}{2}w^2$  and b = 0. That is, the benefits do not decrease in wage income as long as the wage is less than  $\underline{w}$ . If a worker receives the benefit, his utility function is  $U(w) = w + \underline{w} - \frac{1}{2k}w^2$ . Therefore, the optimal wage for the benefit recipient is  $w_1^L = k$  And if  $k \leq \underline{w}$ ,  $w_1^L$  qualifies for the benefits. If  $k > \underline{w}$ , to receive the benefits, workers would earn just  $\underline{w}$ , and their utility level is  $U_2 = \underline{w} + \underline{w} - \frac{1}{2k}\underline{w}^2$ . From example 2, without the benefit, the utility level is  $U^* = \frac{1}{2}k$ . Therefore, a worker would reduce their wage to  $\underline{w}$  in order to receive the benefit if  $U_2 \geq U^*$  or  $k \leq (2 + \sqrt{3})\underline{w} \approx 3.73\underline{w}$ .

[Figure 3 here]

In figure 3, for workers with  $k \leq 2\underline{w}$ , the disposable income increases by area E' + E. However, for those workers with  $2\underline{w} < k \leq 3.73\underline{w}$ , the disposable income decreases by area D'. Since E + E' = D', the aggregate disposable income does not change regardless of  $\underline{w}$ .

In example 2 (b = 1), the benefits decreases as much as wage income. Thus, worker who receive the benefits would make zero wages. That is, with larger b, relatively more severe moral hazard problems arise. In example 3 (b = 0), the benefits do not decrease in wages. Thus, the benefit recipients would like to earn up to their earning capability. That is, there is less moral hazard problem. However, since the benefits do not decrease in income, those who have relatively higher ability would earn wage income just enough to qualify for the benefit at lower marginal cost and receive the benefits. That is, with smaller b, there exist relatively less severe moral hazard problems, but more severe adverse selection problems. Therefore, the aggregate disposable income does not increase regardless of b.

Theorem 1 depends on a key assumption that k is uniformly distributed in the economy. More realistically, suppose that the distribution of k, denoted by f(k), is single-peaked (e.g. normal or log-normal distribution). Also, the benefit recipients are distributed on the left (or lower) side of the distribution.

Then, among the benefit recipients, there will be relatively more high ability workers, that is, f'(k) > 0. From proposition 3, since disposable income decreases for relatively high ability workers, if there exists relatively more high ability workers, the aggregate disposable income must decrease.

**Theorem 2** If the distribution of k is single-peaked, and the benefit recipients are distributed on the left side of the distribution, the aggregate disposable income of the benefit recipients decreases for all  $\underline{w}$  and  $b \in [0, 1]$ .

**Proof.** From the discussion above.

Even though one of the main goals of an income assistance program is to increase the disposable income of the benefit recipients, theorem 1 and 2 show that the aggregate disposable income of all the benefit recipients does not increase, and may well decrease. If one cares about the aggregate disposable income only, this is a very surprising and pessimistic result. On the other hand, the disposable income of the least capable workers (the original benefit target group in the symmetric information case) does increase due to the income assistance program (e.g.  $k < \underline{w}$  in figures 2 or 3). Thus, if the society cares enough about the disposable income of the least capable, the income assistance programs may still be justified.

# 6 Welfare Expenditure

So far I have not considered the "optimality" of an income assistance program because the objectives of an income assistance program can be so diverse. Given that income assistance programs do not increase the aggregate disposable income, however, this section considers the objective of "minimizing the welfare expenditure/budget while guaranteeing everyone with the minimum income level  $\underline{w}$ ".

That is, since the welfare expenditure must be financed by taxes which generate deadweight loss, if the income assistance programs do not increase the aggregate disposable income, one could consider the program with the minimum expenditure as optimal. Even if one does not agree with this objective, it would also be instructive to understand how the budgetary requirement changes with the benefit reduction rates b, for example.

Recall that in the symmetric information case, the income assistance program  $B^*(k)$  minimizes the welfare expenditure while guaranteeing everyone with the minimum income level  $\underline{w}$ . That is,  $B^*(k)$  is the first-best. Since  $B^*(k)$  depends on earning capability k, not on wage income w, the first-best benefit reduction rate with respect to wage income is zero, but the benefit reduction rate with respect to earning capability (k) is one. The following proposition shows that when the earning capability is not observable, the second-best (i.e. expenditure minimizing) benefit reduction rate with respect to wage income is strictly between zero and one.

**Proposition 4** Within the class of the linear income assistance programs  $B^{L}(w)$ , the optimal benefit reduction rate is strictly between zero and one.

#### **Proof.** See appendix.

Note that the linear income assistance programs  $B^L(w)$  guarantees everyone with the minimum income level  $\underline{w}$ . Therefore, I can focus on minimizing the required welfare expenditure/budget. Intuitively, if the benefit reduction rate b decreases a little bit, "all the workers" who receive the benefits will have stronger incentives to earn wage income, which reduces the welfare expenditure. However, since the size of the benefit will become larger, "some additional workers" with higher ability will reduce their wage income in order to be eligible for the benefits, which increases the welfare expenditure. Let us call the first effect as the intramarginal effect, and the second as the marginal effect.

When b = 1, it is optimal for the workers to earn zero wage income. Thus, if *b* decreases slightly, the increase in the size of the benefit is almost zero, or the second-order effect. Since the marginal effect is almost zero, there is just the negative (first-order) intramarginal effect. That is, if b = 1, a slight reduction of *b* will decrease the welfare expenditure.

When b = 0, if b increases slightly, workers will reduce their wage income, but the resulting increase in the welfare expenditure should be very small exactly because b is still very close to zero. Since the intramarginal effect is very small, or the second-order effect, there will be just the negative (firstorder) marginal effect. That is, if b = 0, a slight increase in b will reduce the welfare expenditure. Therefore, the optimal (expenditure minimizing) benefit reduction rate should be strictly between zero and one.

**Example 4** Suppose that  $c(w) = \frac{1}{2}w^2$  and  $\underline{w} = 0.1$ . From lemma 2,  $b_0 = 2 - \sqrt{2}$ . If  $b \ge b_0$ , from proposition 2(i), the welfare expenditure is  $E_1 = \int_0^{k_0} (0.1 - b(1-b)k)dk$  since g(k) = k. If  $b < b_0$ , the welfare expenditure is  $\int_0^{k_1} (0.1 - b(1-b)k)dk + \int_{k_1}^{k_2} (0.1 - b(0.1))dk$ . Then, figure 4 shows the relationship between the welfare expenditure and benefit reduction rate b, where the welfare expenditure is minimized at  $b = \frac{2}{3}$ .

[Figure 4 here]

## 7 Conclusion

It is well-known that income assistance or welfare benefits can decrease labor supply and wage income because of the moral hazard and the adverse selection problems. Therefore, income assistance benefits can increase the disposable income for some, but decrease it for others. However, it has been unclear whether the aggregate disposable income of all the benefit recipients will increase or decrease. This paper shows that the extent of the moral hazard and the adverse selection problems are far more severe than one might have expected. Thus, if the earning capabilities are uniformly distributed, the *aggregate* disposable income of the benefit recipients does not increase at all regardless of the size of the benefits. Moreover, if the distribution of earning capabilities is single-peaked, the aggregate disposable income can even decrease.

As emphasized in the beginning, these results do not necessarily imply that income assistance programs are ineffective. From proposition 3 (and examples 2 and 3), for those benefit recipients with relatively lower capabilities, the income assistance benefits do increase their disposable income. However, this paper shows that a policy maker cannot simply assume that the welfare benefits will increase the aggregate (or average) disposable income of the benefit recipients.

I should note that these results are based on the linear benefit function and the risk-neutral workers. It would be interesting for future research how these results can extend to more general benefit/utility functions. In particular, a typical earned income tax credit (EITC) has an increasing, fixed, and then decreasing benefit structure. Also, a worker may be able to select one benefit among different benefit functions. It would be also interesting to analyze the dynamic effects such as human capital accumulation from working, which is an important rationale for the EITC.

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#### Appendix

**Proof of Lemma 1** Define F(k) such that

$$F(k) \equiv U^*(k) - U_1(k).$$
 (A.1)

Then, from the envelope theorem,  $\frac{\partial F}{\partial k} = \frac{1}{k^2}(c(g(k)) - c(g((1-b)k)) > 0.$ If  $\underline{w}$  is small enough, F(1) > 0 since  $U^*$  is the maximum of  $w - \frac{1}{k}c(w)$ . Also,  $\lim_{k\to 0} F(k) = -\underline{w} < 0$  since g(0) = 0 and  $\lim_{k\to 0} \frac{1}{k}c(g(k)) = \lim_{k\to 0} \frac{c'g'}{1} = \lim_{k\to 0} \frac{kg'}{1} = 0$  from L'Hospital's rule. Therefore, there exist a unique  $k_0 \in (0, 1)$  such that  $F(k_0) \ge 0$  iff  $k \ge k_0$ .

Also,  $k_0 \ge \underline{k}$  since  $F(\underline{k}) = -(1-b)g((1-b)\underline{k}) - \frac{1}{k}(c(g(\underline{k})) - c(g((1-b)\underline{k})) < 0.$ 

**Proof of Lemma 2** Since  $\frac{\partial F}{\partial k} > 0$  and  $F(k_0) = 0$ ,  $k_0 \le k_1$  iff  $F(k_1) \ge 0$ . Since  $k_1$  is a function of b from (9), define

$$F_b(b) = F(k_1(b)) = g(k_1(b)) - \frac{1}{k_1}c(g(k_1(b))) - \underline{w} - (1-b)\underline{w} + \frac{1}{k_1(b)}c(\underline{w}).$$

If b = 0, then  $g(k_1) = \underline{w}$ . Thus,  $F_b(0) = -\underline{w} < 0$ . Also, if  $b \to 1$ , then  $k_1 \to \infty$ . Thus,  $\lim_{b\to 1} F_b(b) > 0$  if  $\underline{w}$  is small enough, since  $g(k) - \frac{1}{k}c(g(k)) > 0$  and increasing in k for all k > 0. Since  $\frac{\partial F_b}{\partial b} = \frac{1}{k^2}(c(g(k_1)) - c(\underline{w})) + \underline{w} > 0$ , there exist  $b_0 \in (0, 1)$  such that  $F(k_1) \ge 0$  iff  $b \ge b_0$ .

**Proof of Lemma 3** Define H(k) such that

$$H(k) = U^* - U_2 = g(k) - \frac{1}{k}c(g(k)) - \left(\underline{w} + (1-b)\underline{w} - \frac{1}{k}c(\underline{w})\right).$$
 (A.2)

Note that  $\frac{\partial H}{\partial k} = \frac{1}{k^2}(c(g(k)) - c(\underline{w})) > 0$  if  $k > k_1$ . Also from lemma 2,  $H(k_1) = F(k_1) < 0$  if  $b < b_0$ . And H(1) > 0 if  $\underline{w}$  is small enough. Therefore, if  $b < b_0$ , there exist  $k_2 \in (0, 1)$  such that  $H(k) \ge 0$  iff  $k \ge k_2$  and  $H(k_2) = 0$ . Also note that  $k_2 > k_1$  iff  $H(k_1) = F(k_1) < 0$ . Thus, from lemma 2,  $k_2 > k_1$  iff  $b < b_0$ .

**Proof of Theorem 1** First, suppose that  $b \ge b_0$ . Note that

$$\int_{k_0}^{1} g(k)dk = G(1) - k_0 g(k_0) + c(g(k_0))$$

where  $G() = \int g(k) dk$ . Likewise,

$$\int_{0}^{k_{0}} [\underline{w} + (1-b)g((1-b)k)]dk = \underline{w}k_{0} + (1-b)\int_{0}^{k_{0}} g((1-b)k)dk$$
$$= \underline{w}k_{0} + \int_{0}^{(1-b)k_{0}} g(z)dz$$
$$= \underline{w}k_{0} + (1-b)k_{0}g((1-b)k_{0}) - c(g((1-b)k_{0}))dk$$

Therefore, from the definition  $k_0$ , the aggregate disposable income is

$$D_1 \equiv \int_0^{k_0} [\underline{w} + (1-b)g((1-b)k)]dk + \int_{k_0}^1 g(k)dk = G(1) = D^N,$$

where  $D^N$  is the disposable income when there exists no benefits, as defined in (3).

Second, now suppose that  $b < b_0$ . Note that

$$\begin{aligned} \int_{k_2}^1 g(k)dk &= G(1) - k_2 g(k_2) + c(g(k_2)) \\ \int_{k_1}^{k_2} [\underline{w} + (1-b)\underline{w}]dk &= [\underline{w} + (1-b)\underline{w}](k_2 - k_1) \end{aligned}$$
$$\begin{aligned} \int_0^{k_1} [\underline{w} + (1-b)g((1-b)k)]dk &= \underline{w}k_1 + (1-b)\int_0^{k_1} g((1-b)k)dk \\ &= \underline{w}k_1 + \int_0^{(1-b)k_1} g(z)dz \\ &= wk_1 + (1-b)k_1g((1-b)k_1) - c(g((1-b)k_1)) \end{aligned}$$

From the definitions of  $k_1$  and  $k_2$ , the aggregate disposable income is

$$D_2 \equiv \int_0^{k_1} [\underline{w} + (1-b)g((1-b)k)]dk + \int_{k_1}^{k_2} [\underline{w} + (1-b)\underline{w}]dk + \int_{k_2}^1 g(k)dk = G(1) = D^N.$$

Therefore, for all b and  $\underline{w}$ , the linear income assistance benefits  $B^{L}(w)$  does not increase the aggregate disposable income

**Proof of Proposition 3** Suppose that  $b \ge b_0$ . For the benefit recipients (i.e.  $k \le k_0$ ), from proposition 2(i),  $\Delta(k) = (\underline{w} + (1-b)g((1-b)k) - g(k))$ . Note

that  $\Delta'(k) = (1-b)^2 g'(k) - g'(k) < 0$  since  $b \ge b_0 > 0$ . Also  $\Delta(0) = \underline{w} > 0$ and  $\Delta(k_0) = -\frac{1}{k} (c(g(k)) - c(g((1-b)k)) < 0$ . Suppose that  $b < b_0$ . For  $k \le k_1$ ,  $\Delta(k) = (\underline{w} + (1-b)g((1-b)k) - g(k))$ .

Suppose that  $b < b_0$ . For  $k \le k_1$ ,  $\Delta(k) = (\underline{w} + (1-b)g((1-b)k) - g(k))$ . From above,  $\Delta(0) > 0$  and  $\Delta'(k) < 0$ . For  $k_1 < k \le k_2$ ,  $\Delta(k) = (\underline{w} + (1-b)\underline{w}) - g(k)$ . Thus,  $\Delta'(k) = -g'(k) < 0$  and, from definition of  $k_2$ ,  $\Delta(k_2) = -\frac{1}{k} (c(g(k_2)) - c(g(\underline{k}))) < 0$  since  $k_2 > \underline{k}$ .

**Proof of Proposition 4** Suppose  $b > b_0$ . Then, from (7) and proposition 2, the welfare budget/expenditure is

$$E_1 = \int_0^{k_0} [\underline{w} - bg((1-b)k)]dk.$$

Then, from the Leibniz's rule,

$$\frac{\partial E_1}{\partial b} = (\underline{w} - bg((1-b)k_0))\frac{\partial k_0}{\partial b} - \int_0^{k_0} [g((1-b)k) - bkg'((1-b)k)]dk$$

Also, from (A.1),

$$\frac{\partial k_0}{\partial b} = -\frac{g((1-b)k_0)}{\frac{1}{k^2}(c(g(k_0)) - c(g((1-b)k_0)))}.$$

Therefore,

$$\left. \frac{\partial E_1}{\partial b} \right|_{b=1} = \int_0^{k_0} kg'(0)dk > 0.$$

That is, when b = 1, reducing b slightly will decrease the welfare expenditure E.

Now suppose that  $b < b_0$ . Then, from (7) and proposition 2, the welfare budget/expenditure is

$$E_2 = \int_0^{k_1} [\underline{w} - bg((1-b)k)]dk + \int_{k_1}^{k_2} [\underline{w} - b\underline{w}]dk.$$

Then, from the Leibniz's rule,

$$\begin{aligned} \frac{\partial E_2}{\partial b} &= \left(\underline{w} - bg((1-b)k_1)\right) \frac{\partial k_1}{\partial b} - \int_0^{k_1} \left[g((1-b)k) - bkg'((1-b)k)\right] dk \\ &+ \left[\underline{w} - b\underline{w}\right] \frac{\partial k_2}{\partial b} - \left[\underline{w} - b\underline{w}\right] \frac{\partial k_1}{\partial b} - \int_{k_1}^{k_2} \underline{w} dk \\ &= \left(\underline{w} - b\underline{w}\right) \frac{\partial k_1}{\partial b} - \int_0^{k_1} \left[g((1-b)k) - bkg'((1-b)k)\right] dk + \left[\underline{w} - b\underline{w}\right] \frac{\partial k_2}{\partial b} \\ &- \left[\underline{w} - b\underline{w}\right] \frac{\partial k_1}{\partial b} - \underline{w}(k_2 - k_1) \\ &= -\int_0^{k_1} \left[g((1-b)k) - bkg'((1-b)k)\right] dk + \left[\underline{w} - b\underline{w}\right] \frac{\partial k_2}{\partial b} - \underline{w}(k_2 - k_1) \end{aligned}$$

Note that from (A.2),  $\frac{\partial H}{\partial b} = \underline{w} > 0$ . Therefore,  $\frac{\partial k_2}{\partial b} < 0$ . Then,

$$\left. \frac{\partial E_2}{\partial b} \right|_{b=0} = -\int_0^{k_1} g(k)dk + \underline{w}\frac{\partial k_2}{\partial b} - \underline{w}(k_2 - k_1) < 0.$$

That is, when b = 0, increasing b slightly will decrease the welfare expenditure.

Therefore, the expenditure minimizing benefit reduction rate must be between zero and one.















Figure 4 Welfare Expenditure and b ( $c(w) = \frac{1}{2}w^2$ )